

# G2 manifolds Workshop Talk Schedule

Events for:  
Tuesday, September 2nd - Friday, September 5th

## Tuesday, September 2nd

8:30am **Registration/breakfast - SCGP Lobby/Cafe**

9:30am **Diarmuid Crowley - SCGP 102**

**Title:** New invariants in  $G_2$  topology

**Abstract:** The  $\nu$ -invariant is a  $\mathbb{Z}/48$ -valued invariant of  $G_2$ -structures on 7-manifolds  $M$  up to homotopies and diffeomorphisms. In this talk I will report on joint work with Johannes Nordström about computations of the  $\nu$ -invariant for twisted connected sum  $G_2$ -manifolds. I will also report on work of Johannes Nordström and Sebastian Goette computing the  $\nu$ -invariant for other  $G_2$ -manifolds: this will be presented in detail in the following talk by Johannes Nordström. When  $M$  is 2-connected, the  $\nu$ -invariant leads to a complete classification of  $G_2$ -structures on  $M$  up to homotopies and diffeomorphisms. For many examples of interest the  $\nu$ -invariant is a complete invariant.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Johannes Nordstrom - SCGP 102**

**Title:** Disconnecting the  $G_2$  moduli space

**Abstract:** I will describe how the homotopy invariant of  $G_2$ -structures on closed 7-manifolds introduced in Diarmuid Crowley's talk can be defined analytically. This intrinsic definition makes it possible to compute the invariant for a class of closed  $G_2$ -manifolds generalising the well-known twisted connected sums, leading to examples of closed 7-manifolds where one can use the homotopy theory of  $G_2$ -structures to distinguish between connected components of the moduli space of holonomy  $G_2$  metrics. Moreover, the analytic definition leads to a more refined invariant that can in some cases even distinguish between  $G_2$ -metrics whose associated  $G_2$ -structures are homotopic. This talk is based on joint work with Diarmuid Crowley and Sebastian Goette.

11:30am **AST-105 - SCGP 103**

12:00pm **Lunch - SCGP Cafe**

1:15pm **Ian Hambleton - SCGP 102**

**Title:** Smooth group actions on 4-manifolds and Yang-Mills gauge theory

**Abstract:** An equivariant version of the Yang-Mills moduli spaces can provide information and "hidden" constraints for smooth actions of finite groups on 4-manifolds. I will discuss this setting and present some sample results illustrating the difference between smooth and topological group actions.

2:15pm **Short Break**

2:30pm **Marisa Fernandez - SCGP 102**

**Title:** Formality in cosymplectic and Sasakian geometries

**Abstract:** In this talk, we give conditions under which a mapping torus, not necessarily symplectic, has a non-zero Massey product. We apply this to prove that there are non-formal compact cosymplectic manifolds of dimension  $m$  ( $=2n+1$ ) and with first Betti number  $b$  if and only if  $m=3$  and  $b \geq 2$ , or  $m \geq 5$  and  $b \geq 1$ . On the other hand, we prove that all higher Massey products on any simply connected Sasakian manifold vanish. Nevertheless, for every  $n \geq 3$ , we exhibit the first examples of simply connected compact Sasakian manifolds of dimension  $2n+1$  which are non-formal because they have a non-zero triple Massey product. (Joint work with G. Bazzoni, I. Biswas, V. Munoz and A. Tralle)

3:30pm **Tea Time - SCGP Lobby/Patio**

4:00pm **Matthias Kreck: G2 manifolds talk & Math's Geometry/Topology seminar - SCGP 103**

**Title:** From Kaluza-Klein manifolds to Calabi-Yau manifolds

**Abstract:** TBA

**Wednesday, September 3rd**

8:30am **Breakfast - SCGP Cafe**

9:30am **Anna Fino - SCGP 102**

**Title:**  $G_2$  structures and Ricci solitons

**Abstract:** In this talk we present some general results about  $G_2$ -structures whose underlying Riemannian metric is Einstein, as well as recent results on the existence of left invariant closed  $G_2$  forms determining a Ricci soliton metric on nilpotent Lie groups. For each one of these structures, we prove a long time existence and uniqueness of solution for the Laplacian flow and we show that the solution converges to a flat  $G_2$ -structure. This talk is based on joint work with Marisa Fernandez and Victor Manero.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Jason Lotay - SCGP 102**

**Title:** Coupled flows and calibrated geometry

**Abstract:** A key proposal for solving the difficult problem of finding calibrated special Lagrangian representatives in homology classes in Calabi-Yau manifolds is to use mean curvature flow. This programme rests on the crucial and surprising fact that here mean curvature flow preserves the Lagrangian condition. I will discuss generalisations of this phenomenon to the symplectic and  $G_2$  settings, where the submanifold flow preserves a distinguished class of submanifolds only once it is coupled to a deformation of the ambient structure, thus revealing a natural flow for the symplectic or  $G_2$  structure as well as for the submanifolds.

12:00pm **Lunch - SCGP Cafe**

1:15pm **Ronan Conlon - SCGP 102**

**Title:** An affine Calabi-Yau manifold with irregular tangent cone at infinity

**Abstract:** An asymptotically conical (AC) Calabi-Yau manifold is a non-compact Ricci-flat Kahler manifold modelled on a Ricci-flat Kahler cone at infinity. I will present a new example of an AC Calabi-Yau manifold with asymptotic model an irregular Ricci-flat Kahler cone. This example in particular provides the first example of an affine Ricci-flat Kahler manifold of Euclidean volume growth with irregular tangent cone at infinity. This is joint work with Hans-Joachim Hein (UMD).

2:15pm **Short Break**

2:30pm **Yohsuke Imagi - SCGP 102**

**Title:** Singularities of Special Lagrangian Submanifolds

**Abstract:** Special Lagrangian submanifolds are calibrated submanifolds of Calabi-Yau manifolds (not of  $G_2$ -manifolds). They are area-minimizing with respect to the Calabi-Yau metric and Lagrangian with respect to its Kahler form. The moduli space of compact special Lagrangian submanifolds (of a fixed Calabi-Yau manifold) is smooth by a theorem of Mclean, but it need not be compact because some special Lagrangian submanifolds tend to singular special Lagrangians, which are currents or varifolds rather than submanifolds. The space of all compactly-supported special Lagrangian (integral) currents (without boundary and with a fixed homology class in a fixed Calabi-Yau manifold) is compact by a theorem of Federer and Fleming, but it seems very difficult to find a "nice" structure on the space of special Lagrangian currents; by nice I mean something like manifolds-with-corner which enables one to define counting invariants of special Lagrangians, ideally. To do so we have to develop a deep theory on singularities of special Lagrangians, which is interesting itself and will be also important in other problems (including the SYZ conjecture for instance). I've studied two kinds of isolated singularities of special Lagrangians: one is modelled on Clifford  $T^2$ -cones and the other is modelled on the union of transversely-intersecting two planes. Let  $X$  be a compact special Lagrangian 3-fold with Clifford  $T^2$ -cone singularities. I've determined a neighbourhood of  $X$  in the space of special Lagrangian currents; I'll give a sketch of the proof in the talk. I want to do something similar for the singularities modelled on the union of two planes, but it's more difficult and seems to require something new. Joyce, Oliveira dos Santos and I have proved a uniqueness theorem for "exact" special Lagrangians in  $C^m$  asymptotic at infinity to the union of two planes. Exactness is a sufficient condition for Lagrangian Floer cohomology to be well-defined, which is essential to our proof.

3:30pm **Tea Time - SCGP Lobby/Patio**

4:00pm **Jake Solomon - SCGP 102**

**Title:** Geometry of the space of positive Lagrangians

**Abstract:** A Lagrangian submanifold of a Calabi-Yau manifold is called positive if the real part of the holomorphic volume form restricted to it is positive. A Hamiltonian isotopy class of positive Lagrangian submanifolds admits a Riemannian metric with non-positive curvature. Its universal cover admits a functional, with critical points special Lagrangians, that is strictly convex with respect to the metric. Solutions of the geodesic equation, both smooth (with A. Yuval) and viscosity (with Y. Rubinstein), will be discussed. Mirror symmetry relates these phenomena to analogous phenomena for the space of Hermitian metrics on a holomorphic vector bundle and the space of Kahler metrics.

**Thursday, September 4th**

8:30am **Breakfast - SCGP Cafe**

9:30am **Sergei Gukov - SCGP 102**

**Title:** Singularities of G2 manifolds: geometry and physics

**Abstract:** TBA

10:30am **Coffee Break - SCGP Cafe**

11:00am **Dave Morrison - SCGP 102**

**Title:** Singular limits of G2 metrics and non-abelian gauge symmetry in M-theory

**Abstract:** TBA

11:30am **AST-105 - SCGP 103**

12:00pm **Lunch - SCGP Cafe**

1:15pm **Gordon Kane - SCGP 102**

**Title:** M-theory Compactified on a G2 Manifold â€œ Connections to our real 4D world

**Abstract:** TBA

2:15pm **Short Break**

2:30pm **Antonella Grassi - SCGP 102**

**Title:** Elliptic fibrations in F-theory via deformations

**Abstract:** TBA

3:30pm **Tea Time - SCGP Lobby/Patio**

4:00pm **Mirjam Cvetič - SCGP 102**

**Title:** Elliptic Fibrations with Higher Rank Mordell-Weil Groups: F- $\hat{A}$ -Theory Compactifications with Higher Rank Abelian Symmetries

**Abstract:** TBA

6:00pm **Workshop Banquet**

## Friday, September 5th

8:30am **Breakfast - SCGP Cafe**

9:30am **Thomas Walpuski - SCGP 102**

**Title:** G<sub>2</sub>-instantons and the Seiberg-Witten equation with multiple spinors

**Abstract:** TBA

10:30am **Coffee Break - SCGP Cafe**

11:00am **Yalong Cao - SCGP 102**

**Title:** Donaldson-Thomas theory for Calabi-Yau four-folds

**Abstract:** Let  $X$  be a compact complex Calabi-Yau four-fold. Under certain assumptions, we define Donaldson-Thomas type deformation invariants ( $DT_{\{4\}}$  invariants) by studying moduli spaces of solutions to the Donaldson-Thomas equations on  $X$ . We also study sheaves counting problem on local Calabi-Yau four-folds. We relate  $DT_{\{4\}}$  invariants of  $K_{\{Y\}}$  to the Donaldson-Thomas invariants of the associated Fano three-fold  $Y$ . In some special cases, we prove a  $DT_{\{4\}}/GW$  correspondence for  $X$ . When the Calabi-Yau four-fold is toric, we use the virtual localization formula to define the equivariant  $DT_{\{4\}}$  invariants. There is a related work by D.Borisov and D.Joyce. We will mention their work and compare it with ours. This is a joint work with Naichung Conan Leung.

12:00pm **Lunch - SCGP Cafe**

1:15pm **Goncalo Oliveira - SCGP 102**

**Title:** Monopoles on G<sub>2</sub> manifolds

**Abstract:** TBA

2:15pm **Short Break**

2:30pm **David Baraglia - SCGP 102**

**Title:** Associative and coassociative fibrations

**Abstract:** I will present some results and some open problems concerning calibrated fibrations of  $G_2$  and  $\text{Spin}(7)$ -manifolds whose fibers are associative, coassociative or Cayley. In the coassociative and Cayley cases such fibrations of compact manifolds must have singularities, but curiously this doesn't seem to be the case for associative fibrations. I will also discuss the case of semi-flat calibrated fibrations. These are associative, coassociative or Cayley fibrations whose fibers are flat tori.

3:30pm **Tea Time - SCGP Lobby/Patio**

4:00pm **Misha Verbitsky - SCGP 102**

**Title:** Kahler structure on the knot space of a  $G_2$ -manifold

**Abstract:** A knot space in a manifold  $M$  is a space of oriented immersions from a circle  $S^1$  to  $M$  up to  $\text{Diff}(S^1)$ . Brylinski has shown that a knot space of a Riemannian threefold is formally Kahler. An elementary construction allows one to construct a Hermitian almost complex structure on the space of knots inside a 7-manifold  $M$  if its structure group is reduced to  $G_2$ . I prove that this Hermitian structure is formally Kaehler if  $M$  has holonomy  $G_2$ , and the formal integrability is equivalent to the holonomy condition.