

# Workshop: Quantitative Symplectic Geometry: May 8-12, 2017

Events for:  
Monday, May 8th - Friday, May 12th

## Monday, May 8th

9:30am **Lisa Traynor - SCGP 102**

**Title:** The Length and Width of Lagrangian Cobordisms

**Abstract:** Lagrangian cobordisms between Legendrian submanifolds arise in Relative Symplectic Field Theory. In recent years, there has been much progress on answering qualitative questions such as: For a fixed pair of Legendrians, does there exist a Lagrangian cobordism between them? One can also ask quantitative questions: What is the "length" or "width" of a Lagrangian cobordism? I will give examples of pairs of Legendrians where Lagrangian cobordisms are flexible in that the non-cylindrical region can be arbitrarily short; I will also give examples of other pairs of Legendrians where Lagrangian cobordisms are rigid in that there is a positive lower bound to their length. In addition, I will give some calculations of the relative Gromov width of particular Lagrangian cobordisms. This is joint work with Joshua M. Sabloff.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Egor Shelukhin - SCGP 102**

**Title:** Persistence modules with operators

**Abstract:** Intersecting with classes in the ambient homology gives a natural structure on the persistence modules of Morse and Floer theory. We discuss this structure and present new applications to Hofer's geometry and the  $C^0$ -geometry of Morse functions. This is a joint work with Leonid Polterovich and Vukasin Stojisavljevic.

12:00pm **Lunch - SCGP Cafe**

2:15pm **Julian Chaidez - SCGP 102**

**Title:** Computing EHZ Capacities Of 4d Polytopes

**Abstract:** The Ekeland-Hofer-Zehnder of a convex domain with smooth boundary in  $\mathbb{R}^{2n}$  is the minimal symplectic action of a Reeb orbit on the boundary. This capacity extends uniquely to  $C^0$  convex domains such as polytopes. It was proven by Artstein-Avidan and Ostrover that this extended capacity can also be computed in terms of so-called "generalized Reeb orbits" that minimize action.

3:15pm **Tea - SCGP Lobby**

4:00pm **Ana Rita Pires - SCGP 102**

**Title:** Infinite Staircases in Symplectic Embeddings

**Abstract:** McDuff and Schlenk studied an embedding capacity function, which describes when a 4-dimensional ellipsoid can symplectically embed into a 4-ball. The graph of this function includes an infinite staircase related to the odd index Fibonacci numbers. Infinite staircases have been shown to exist also in the graphs of the embedding capacity functions when the target manifold is a polydisk or the ellipsoid  $E(2,3)$ . This talk describes joint work with Cristofaro-Gardiner, Holm, and Mandini, in which we use ECH capacities and Ehrhart theory to show that infinite staircases exist for these and a few other target manifolds. I will also explain why we conjecture that these are the only such target manifolds.

**Tuesday, May 9th**

9:00am **Michael Entov - SCGP 102**

**Title:** Unobstructed symplectic packing of tori by ellipsoids

**Abstract:** I will discuss why any finite collection of disjoint (not necessarily equal) ellipsoids admits a symplectic embedding to an even-dimensional torus equipped with a Kahler form as long as the total symplectic volume of the ellipsoids is less than the volume of the torus.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Olguta Buse - SCGP 102**

**Title:** Symplectic packing stability beyond four dimensions

**Abstract:** A classical question in symplectic geometry is to decide if a symplectic manifold can be symplectically fully filled by any large enough number of balls. A first answer was provided by P. Biran in the case of 4-dimensional manifolds with cohomologically rational symplectic forms. In collaboration with R. Hind we showed that this is also the case for all manifolds with rational cohomology class, for all compact 4-manifolds, and for several other symplectic domains (the latter two cases are based on joint work with R. Hind and E. Opshtein). I will explain how this phenomenon arises as an application of ECH, allowing one to show that rescalings of all sufficiently elongated ellipsoids (of "thin shape") can fully fill symplectic manifolds with rational cohomology class. Time permitting I will discuss how to further employ properties of ECH to probe the question of whether such behavior can be extended to other situations.

11:30am **Lunch - SCGP Cafe**

1:00pm **SCGP Weekly Talk: Michael Hutchings - SCGP 102**

**Title:** Title: Measuring space and time in symplectic geometry

**Abstract:** A basic question in symplectic geometry is to determine when one symplectic manifold with boundary (such as a domain in  $\mathbb{R}^{2n}$ ) can be embedded into another, preserving the symplectic structure. Another basic question is to understand periodic orbits of Hamiltonian vector fields. It turns out that these two questions are closely related: periodic orbits of Hamiltonian vector fields (recast as Reeb vector fields) on the boundaries of symplectic manifolds give rise to symplectic embedding obstructions. We will explain how this relation works, and discuss some recent results and conjectures about symplectic embeddings and periodic orbits of Reeb vector fields.

2:30pm **Yaron Ostrover - SCGP 102**

**Title:** The symplectic size of a random convex body

**Abstract:** We will discuss symplectic measurements of convex domains in the classical phase space. We will be interested in particular in the expected value of certain symplectic capacities of random convex bodies, and the computational complexity of estimating symplectic capacities.

3:30pm **Tea - SCGP Lobby**

4:00pm **Discussion Session - SCGP 102**

**Title:** Discussion Session

## Wednesday, May 10th

9:30am **Dusa McDuff - SCGP 102**

**Title:** The stabilized symplectic embedding problem

**Abstract:** I will discuss some recent work (mostly joint with Dan Cristofaro-Gardiner and Richard Hind) on the stabilized symplectic embedding problem for ellipsoids into balls. The main tools come from embedded contact homology.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Felix Schlenk - SCGP 102**

**Title:** PDEs and symplectic embedding obstructions

**Abstract:** For some PDEs (such as certain periodic Schrödinger equations and the KdV equation) the evolution can be described as a symplectic flow on an infinite dimensional symplectic vector space. And for some of these PDEs, this flow essentially leaves invariant finite dimensional symplectic subspaces. Every symplectic rigidity theorem in  $\mathbb{R}^{2n}$  that holds for all  $n$  then makes a statement on the evolution of this PDE. This is well known for the non-squeezing theorem (implying the absence of asymptotically stable equilibria), but any other symplectic embedding obstruction holding in all dimensions (such as Gromov's 2-ball theorem or the recent results on rigidity under stabilization) has another application to these PDEs. Afternoon free. Evening banquet.

12:00pm **Lunch - SCGP Cafe**

3:15pm **Tea - SCGP Lobby**

## Thursday, May 11th

9:30am **Michael Usher - SCGP 102**

**Title:** Knotted symplectic embeddings between domains in  $\mathbb{R}^4$

**Abstract:** I will discuss a proof that many toric domains  $X$  in  $\mathbb{R}^4$  admit symplectic embeddings  $f$  into dilates of themselves which are knotted in the strong sense that there is no symplectomorphism of the target that takes  $f(X)$  to  $X$ . For instance  $X$  can be taken equal to a polydisk  $P(1,1)$ , or to any convex toric domain that both is contained in  $P(1,1)$  and properly contains a ball  $B^4(1)$ ; by contrast a result of McDuff shows that  $B^4(1)$  (or indeed any four-dimensional ellipsoid) cannot have this property. The embeddings are constructed based on recent advances on symplectic embeddings of ellipsoids, though in some cases a more elementary construction is possible. The fact that the embeddings are knotted is proven using filtered  $S^1$ -equivariant symplectic homology. This is joint work with Jean Gutt.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Georgios Dimitroglou Rizell - SCGP 102**

**Title:** A quantitative perspective on the classification of Lagrangian tori

**Abstract:** We present classification results for Lagrangian tori while taking quantitative considerations into account. In this manner we obtain a characterisation of product tori inside the unit ball up to Hamiltonian isotopy. In particular, we show that an extremal Lagrangian torus inside the unit four-ball is entirely contained in the boundary, and that it is Hamiltonian isotopic to the monotone product torus contained inside the same. This builds upon joint work with E. Goodman and A. Ivrii.

12:00pm **Lunch - SCGP Cafe**

2:15pm **Vinicius Gripp Ramos - SCGP 102**

**Title:** Symplectic embeddings of Lagrangian products

**Abstract:** Lagrangian products form a class of symplectic manifolds whose geometry is related to the dynamics of a billiard table determined by the factors. In this talk, I will explain this relation and how ECH capacities can be used to find sharp obstructions to a large class of symplectic embedding problems of lagrangian products. This is joint work with Daniele Sepe.

3:15pm **Tea - SCGP Lobby**

4:00pm **Discussion Session - SCGP 102**

**Title:** Discussion Session

**Friday, May 12th**

9:30am **Viktor Ginzburg - SCGP 102**

**Title:** Lusternik-Schnirelmann Theory, the Shift Operator and Closed Reeb Orbits

**Abstract:** In this talk we focus on the role of Lusternik-Schnirelmann theory in multiplicity results for closed Reeb orbits. We develop a variant of this theory for the shift operator in the equivariant Floer and symplectic homology and prove that spectral invariants are strictly decreasing under the action of the shift operator when periodic orbits are isolated. We then show how this fact is used in the proofs of multiplicity results for simple closed Reeb orbits without non-degeneracy. The talk is based on a joint work with Basak Gurel and Iâ€™ll try to give at least some technical details of the constructions and proofs.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Kai Cieliebak - SCGP 102**

**Title:** Poincare duality for free loop spaces

**Abstract:** The Chas-Sullivan loop product on the homology of a free loop space and the Goresky-Hingston product on its cohomology fit together to a product on a larger space. This space satisfies a kind of Poincare duality and thus explains various dualities between loop space homology and cohomology observed over the past years.

12:00pm **Lunch - SCGP Cafe**

2:15pm **Mark Mclean - SCGP 102**

**Title:** The size of a neighborhood of a Lagrangian in  $C^n$ .

**Abstract:** I will talk about a quantitative result of mine with Strom Borman from 2013. Every Lagrangian inside a symplectic manifold has a small neighborhood symplectomorphic to an open subset of its cotangent bundle. One can ask, how big can this neighborhood be? One way of measuring this is finding the largest symplectically embedded ball so that the Lagrangian restricted to the ball is linear through the origin. For any Lagrangian inside Euclidean space satisfying a particular topological condition we can bound the size of this ball in terms of its diameter. In fact we have a bound in terms of its displacement energy. We use a tool called wrapped Floer cohomology.

3:15pm **Tea - SCGP Lobby**

4:00pm **Jean Gutt - SCGP 102**

**Title:** Equivariant symplectic capacities

**Abstract:** We study obstructions to symplectically embedding a cube (a polydisk with all factors equal) into another symplectic manifold with boundary of the same dimension. We find sharp obstructions in many cases, including all "convex toric domains" and "concave toric domains" in  $C^n$ . The proof uses analogues of the Ekeland-Hofer capacities, which are conjecturally equal to them, but which are defined using  $S^1$ -equivariant symplectic homology. This is joint work with Michael Hutchings.