Strategy to compute soft terms:

(i) For your favorite string compactification, derive the 4D effective action of light fields at the Kaluza-Klein scale M_{KK}. (cf: Jan's lecture)

(ii) Stabilize the moduli at a vacuum with nearly vanishing C.C (cf: Shamit's lecture) and compute SUSY-breaking auxiliary components of moduli and other matter and gauge fields to identify the dominant source of SUSY breaking.

(iii) Compute the effective interactions between SUSY-breaking fields and the visible gauge and matter fields, which are generated at the messenger scale $M_{\rm mess}$.
(For high messenger scale $M_{\rm mess} \sim M_{KK}$ or $M_{\rm string}$, this can be done in the step (i).)

(iv) Compute the soft masses of visible sector fields at $M_{\rm mess}$.

(v) Take into account the RG evolution of soft masses from $M_{\rm mess}$ down to the TeV scale.

(vi) Apply your results for TeV scale phenomenology.

Don't forget that you can start from any step with suitable assumptions, and still do an interesting physics.

Generic features of SUSY breaking and its mediation (Step (ii) and (iii))

In (locally) supersymmetric 4D effective field theory, there can be three type of SUSY-breaking order parameters for spontaneous breakdown of $N = 1$ SUSY.

- Auxiliary F-component of chiral scalar superfield:

$$X^i = X_0^i + \sqrt{2}\theta\tilde{X}^i + \theta\theta \mathbf{F}^i$$

- Auxiliary D-component of real vector superfield:

$$V_A = -\theta\sigma^\mu\bar{\theta}A_\mu + i\left(\theta\theta\bar{\theta}\bar{\lambda}_A - \bar{\theta}\bar{\theta}\theta\lambda_A\right) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\mathbf{D_A}$$

- Auxiliary components of supergravity multiplet:

For later convenience, we use the chiral compensator formulation of 4D SUGRA, in which there can be two SUSY-breaking (but Poincare-invariant) auxiliary components in SUGRA multiplets, i.e. F-components of the chiral density superfield $\mathcal{E}$ and the compensator superfield C:

$$\begin{aligned} 2\mathcal{E} &= \sqrt{-g}\left[\,1 + i\theta\sigma^\mu\bar{\psi}_\mu - \theta\theta\left(\mathbf{M}^* + \bar{\psi}_\mu\sigma^{\mu\nu}\bar{\psi}_\nu\right)\,\right] \\ C &= C_0 + \sqrt{2}\theta\tilde{C} + \theta\theta\mathbf{F}^C \end{aligned}$$

In compensator formulation of 4D SUGRA, we have the super-Weyl invariance parametrized by a chiral superfield τ:

$$\mathcal{E} \rightarrow e^{6\tau}\mathcal{E}, \quad C \rightarrow e^{-2\tau}C,$$

which assures that a liner combination of $\ln C$ and $\ln\mathcal{E}$ is a gauge degree of freedom.

One can then consider a super-Weyl gauge transformation:

$$\mathcal{E} \rightarrow e^{6(\sqrt{2}\theta\tilde{\tau}+\theta\theta F^{\tau})}\mathcal{E}, \quad C \rightarrow e^{-2(\sqrt{2}\theta\tilde{\tau}+\theta\theta F^{\tau})}C,$$

to arrive at

$$M^* = 0, \quad \tilde{C} = 0.$$

In this gauge choice, SUSY breaking by full supergravity multiplets can be described entirely by

$$C = C_0 + \theta\theta F^C$$

within an effective global SUSY formulation in which the SUGRA multiplet describing curved superspace are all replaced by their vacuum values:

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \psi_\mu = H_\mu = 0, ...$$

Then, SUSY breaking in generic 4D SUGRA can be studied with the following form of effective action for SUSY breaking hidden sector defined on rigid $N=1$ superspace:

$$\begin{aligned}\mathcal{L}_{\rm H} &= \int d^2\theta d^2\bar{\theta}\, CC^* \left[-3\exp\left(-\frac{1}{3}K_0(X^*, X, V_A)\right)\right] \\ &\quad + \left(\int d^2\theta\, \frac{1}{4} f_A(X, C)\mathcal{W}^{A\alpha}\mathcal{W}^A_\alpha + C^3 W_0(X) + {\rm c.c.}\right)\end{aligned}$$

(Note a change of notation $K \to -3e^{-K/3}$. In this new notation, K can be identified as the Kähler potential of matter fields in the Einstein frame.)

There is still a residual Weyl-invariance which often plays a useful role:

$$\eta_{\mu\nu} \to e^{2(\tau_0+\tau_0^*)}\eta_{\mu\nu}, \quad C \to e^{-2\tau_0}C, \quad \theta^\alpha \to e^{-\tau_0+2\tau_0^*}\theta^\alpha$$
$$(\tau_0 = \text{complex constant})$$

From this superspace lagrangian, one can derive the equations of motion for the SUSY-breaking auxiliary components, which can be easily solved to yield the following on-shell expressions of auxiliary components:

$$F^i = K^{i\bar{j}} F_{\bar{j}} = -e^{K/2} K^{i\bar{j}} \left(\frac{\partial W}{\partial X^i} + \frac{\partial K}{\partial X^i} \right)^* = -e^{K/2} K^{i\bar{j}} \left(D_j W \right)^*$$

$$\left(K_{i\bar{j}} = \partial_{X^i} \partial_{X^{j*}} K, \;\; K^{i\bar{j}} K_{k\bar{j}} = \delta^i_j \right)$$

$$\frac{F^C}{C_0} = m^*_{3/2} + \frac{1}{3} F^i \partial_{X^i} K \qquad \left(m_{3/2} = e^{K/2} W \right)$$

$$D_A = -\frac{1}{\mathrm{Re} f_A} \left(\frac{\partial K}{\partial V_A} \right)_{V_A = 0}$$

and also the resulting scalar potential

$$V_{\text{potential}} = e^K \left[K^{i\bar{j}} D_i W \left(D_j W \right)^* - 3|W|^2 \right] + \frac{1}{2\mathrm{Re}\left(f_A \right)} \left(\frac{\partial K}{\partial V_A} \right)^2_{V_A = 0}$$

in the Einstein metric frame with $C_0 = e^{K/6}$, in which there is no kinetic mixing between X^i and $g_{\mu\nu}$.

One of the key steps to compute soft masses is to compute 1st the VEVs of the F and D components by minimizing this scalar potential, and identify what are the dominant sources of soft masses.

In most cases, internal gauge symmetries in SUGRA model can be realized in a way that $CC^*e^{-K/3}$, C^3W and $f_A\mathcal{W}^{A\alpha}\mathcal{W}^A_\alpha$ are separately invariant under the following form of infinitesimal superspace gauge transformation:

$$\begin{aligned}\delta e^V &= -i\Lambda^\dagger e^V + ie^V\Lambda \quad \left(V = V_A T^A,\ \Lambda = \Lambda_A T^A\right)\\ \delta Z^m &= \Lambda^A \eta_A^m(Z) \quad \left(Z^m = (C, X^i)\right)\end{aligned}$$

Here $\eta_A = \eta_A^m \frac{\partial}{\partial Z^m}$ are the holomorphic Killing vector fields for gauge transformation, obeying

$$[\eta_A, \eta_B] = -f_{ABC}\eta_C, \quad [\eta_A, \bar{\eta}_B] = 0.$$

Some simple examples:

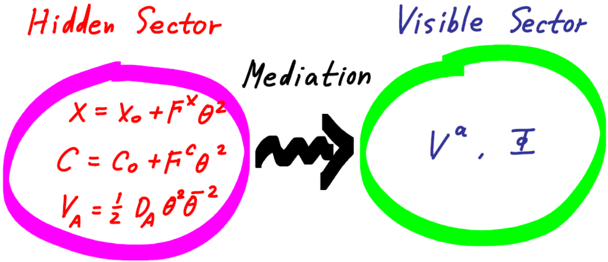
* Ordinary linear gauge transformation: $\eta_A^i = i(T_A)^i_j X^j$
* Shift $U(1)$ gauge symmetry, e.g. the transformation of some moduli under anomalous $U(1)$ gauge symmetry: η_A^i = constants
* $U(1)_R$ gauge symmetry: $\eta_A^C = -iq_C C$ $\left(3q_C = \text{constant FI term}\right)$

Then, the D-term is given by

$$\begin{aligned} D_A &= -\frac{1}{\mathrm{Re} f_A}\left(\frac{\partial K}{\partial V_A}\right) = -\frac{i\eta_A^i\left(\partial_i \ln W + \partial_i K\right)}{\mathrm{Re} f_A} \\ &= -\frac{i}{\mathrm{Re} f_A}\frac{\eta_A^i D_i W}{W} = \frac{i g_A^2 \eta_A^i F_i}{m_{3/2}} \end{aligned}$$

Note that this correlation between D and F terms can not be read off in the global SUSY limit.

Once some fields $\{X^i, C, V_A\}$ develop SUSY breaking vacuum values, the MSSM soft terms are determined by **a mediation mechanism** generating local effective interactions between $\{X^i, C, V_A\}$ and the MSSM superfields $\{V_a, \Phi^I\}$ at the messenger scale $M_{\rm mess}$:



All low energy consequences of the mediation mechanism can be described by an (Wilsonian) effective lagrangian at $M_{\rm mess}$, which includes the local interactions between MSSM and SUSY-breaking fields.

Wilsonian effective lagrangian at $M_{\rm mess}$:

$$\mathcal{L}(M_{\rm mess}) = \mathcal{L}_{\rm H} + \mathcal{L}_{\rm V}$$

$$\mathcal{L}_{\rm H} = \int d^2\theta d^2\bar{\theta}\, CC^* \left[-3\exp\left(-\frac{1}{3}K_0(X^*, X, V)\right)\right]$$
$$+ \left(\int d^2\theta\, \frac{1}{4}\tilde{f}_A(Z)\mathcal{W}^{A\alpha}\mathcal{W}^A_\alpha + C^3 W_0(X) + \text{c.c.}\right)$$

$$\mathcal{L}_{\rm V} = \int d^2\theta d^2\bar{\theta}\, CC^* \left[Y_I(Z, Z^*, V_A)\Phi_I^*\Phi_I + \left(X_H(Z, Z^*, V_A)H_u H_d + \text{c.c}\right)\right]$$
$$+ \left(\int d^2\theta\, \frac{1}{4}\tilde{f}_a(Z)\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + C^3\left(\tilde{\mu}(\Phi)H_u H_d + \frac{\lambda_{IJK}}{6}\Phi_I\Phi_J\Phi_K\right) + \text{c.c}\right)$$

$Z = \{X, C\}, V_A$: SUSY-breaking hidden sector superfields

Φ_I, V_a : Visible sector gauge and matter superfields

$$-3e^{-K/3} = -3e^{-K_0/3} + Y_I\Phi_I^*\Phi_I + X_H H_u H_d + \ldots$$
$$\Rightarrow \quad K = K_0 + e^{K_0/3}Y_I\Phi_I^*\Phi_I + e^{K_0/3}X_H H_u H_d + \ldots$$

In many cases, these local interactions are generated at $M_{\rm string}$ or M_{KK}, and then $M_{\rm mess}$ can be taken as a high value close to $M_{\rm string}$ or M_{KK}.

In some cases, soft terms are generated through radiative corrections, e.g. anomaly mediation and gauge mediation, and then the renormalization scheme for this Wilsonian action should be specified for unambiguous calculation of the physical (1PI) soft masses.

For instance, here we take a C-independent regularization scheme, for which

Kaplunovsky,Louis

$$\tilde{f}_a(Z) = f_a(X) - \frac{3}{8\pi^2}\left(\mathrm{tr}(T_a^2(\mathrm{adj})) - \sum_I \mathrm{tr}(T_a^2(\Phi_I))\right)\ln C$$

in order for the super-Weyl invariance is a gauge symmetry of the regulated quantum amplitudes.

We can also choose a renormalization convention for which

$$\text{Wilsonian } \; Y_I \; \text{ at } \; M_{\rm mess} \; = \; \text{1PI } \; Y_I \; \text{ at } \; p = M_{\rm mess}.$$

- 1PI gauge coupling superfield at the external momentum $p < \Lambda_{\rm mess}$:

Novikov,Shifman,Vainshtein,Zakharov; Kaplunovsky,Louis; KC,Nilles

$$
\begin{aligned}
\mathcal{F}_a(p^2) &= \mathrm{Re}(\tilde{f}_a(Z)) + \frac{b_a}{16\pi^2}\ln\left(\frac{M_{\rm mess}^2}{p^2}\right) \\
&- \frac{1}{8\pi^2}\sum_I \mathrm{tr}(T_a^2(\Phi_I))\ln(CC^*Y_I) + \frac{1}{8\pi^2}\mathrm{tr}(T_a^2(\mathrm{adj}))\ln\mathcal{F}_a \\
&= \mathrm{Re}(f_a(X)) + \frac{b_a}{16\pi^2}\ln\left(\frac{CC^*M_{\rm mess}^2}{p^2}\right) \\
&-\frac{1}{8\pi^2}\sum_I \mathrm{tr}(T_a^2(\Phi_I))\ln\left(Y_I(p^2/CC^*)\right) + \frac{1}{8\pi^2}\mathrm{tr}(T_a^2(\mathrm{adj}))\ln\mathcal{F}_a(p^2/CC^*)
\end{aligned}
$$

Here Y_I is the 1PI matter wave function factor, and $p^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$ corresponds to the rigid SUSY limit of the curved superspace D'Alambertian operator which involves the auxiliary component M.

In our particular gauge choice $M = 0$, $p^2 = \eta^{\mu\nu}\partial_\mu\partial_\nu$ does not contribute to SUSY breaking, so can be regarded as a constant external momentum. In other gauge choice with $M \neq 0$, the full superspace expression of p^2 should be used to correctly incorporate the SUSY breaking by SUGRA multiplet. Of course, all physical results should be independent of the gauge choice.

$$
\begin{aligned}
\mathcal{F}_a(p^2) &= \mathrm{Re}(f_a(X)) + \frac{b_a}{16\pi^2}\ln\left(\frac{CC^* M_{\mathrm{mess}}^2}{p^2}\right) \\
&\quad -\frac{1}{8\pi^2}\sum_I \mathrm{tr}(T_a^2(\Phi_I))\ln\left(Y_I(p^2/CC^*)\right) + \frac{1}{8\pi^2}\mathrm{tr}(T_a^2(\mathrm{adj}))\ln\mathcal{F}_a(p^2/CC^*)
\end{aligned}
$$

Note that the 1PI gauge coupling superfield is invariant under

$$
\eta_{\mu\nu} \to e^{2(\tau_0+\tau_0^*)}\eta_{\mu\nu}, \quad C \to e^{-2\tau_0}C, \quad \theta^\alpha \to e^{-\tau_0+2\tau_0^*}\theta^\alpha,
$$

and it implies that any p^2-dependence comes through the invariant combination

$$
\ln(CC^*/p^2) = \ln C + \ln C^* - \ln p^2,
$$

and therefore

$$
\frac{\partial}{\partial \ln C} = -\frac{d}{d\ln p^2}
$$

* Lowest component of $\mathcal{F}_a|_{C_0=e^{K_0/6}} = 1/g_a^2(p) =$ 1PI gauge coupling constant at the external gauge boson momentum p:

$$\begin{aligned}\frac{1}{g_a^2(p)} &= \mathrm{Re}(f_a(X)) + \frac{b_a}{16\pi^2}\ln\left(\frac{e^{K_0/3}M_{\rm mess}^2}{p^2}\right) \\ &- \frac{1}{8\pi^2}\sum_I \mathrm{tr}(T_a^2(\Phi_I))\ln(e^{-K_0/3}Z_I) - \frac{1}{8\pi^2}\mathrm{tr}(T_a^2(\mathrm{adj}))\ln g_a^2(p)\end{aligned}$$

* F-component of $\mathcal{F}_a$ for $C_0 = e^{K_0/6} =$ 1PI gaugino mass at the external gaugino momentum p:

$$\frac{M_a(p)}{g_a^2(p)} = F^m\partial_m\mathcal{F}_a|_{C=C^*=e^{K_0/6}} \quad \left(\partial_m = \frac{\partial}{\partial Z^m}\ \text{ for }\ Z^m = (C, X^i)\right)$$

$$= \frac{F^i\partial_i\big(\mathrm{Re}f_a(X)\big) - \frac{1}{8\pi^2}\sum_I \mathrm{tr}(T_a^2(\Phi_I))F^i\partial_i\ln(Y_I) + \frac{1}{16\pi^2}\big(b_a + \sum_I \mathrm{tr}(T_a^2(\Phi_I)\gamma_I\big)\frac{F^C}{C_0}}{1 - \frac{1}{8\pi^2}\mathrm{tr}(T_a^2(\mathrm{adj}))g_a^2}$$

$$\big(\gamma_I = d\ln Y_I/d\ln p\big)$$

The last piece is the gaugino mass that arises from the auxiliary component of the SUGRA multiplet, i.e. F^C, which can not be captured in tree level approximation, and is called anomaly mediation.

- 1PI soft masses at $p = M_{\rm mess}$:

$$* \; M_a(M_{\rm mess}) = F^m \partial_m \ln \mathcal{F}_a \quad \left(\partial_m = \frac{\partial}{\partial Z^m} \;\; \text{for} \;\; Z^m = (C, X^i)\right)$$

$$* \; m_I^2(M_{\rm mess}) = -F^m F^{\bar{m}} \partial_m \partial_{\bar{m}} \ln Y_I - \frac{1}{2} D_A \frac{\partial}{\partial V_A} \ln Y_I$$

$$* \; A_{IJK}(M_{\rm mess}) = -F^m \partial_m \ln \left(\frac{\lambda_{IJK}}{Y_I Y_J Y_K} \right)$$

$$* \; \mu(M_{\rm mess}) = \frac{1}{CC^* \sqrt{Y_{H_u} Y_{H_d}}} \left(C^3 \tilde{\mu} + F^{\bar{m}} \partial_{\bar{m}} (CC^* X_H) \right)_{C=C^*=e^{K_0/6}}$$

(2nd term = Giudice-Masiero)

$$\begin{aligned} * \; B\mu(M_{\rm mess}) = & -\frac{1}{CC^* \sqrt{Y_{H_u} Y_{H_d}}} \Big[F^m \left(\partial_m (C^3 \tilde{\mu}) - C^3 \tilde{\mu} \partial_m \ln (C^2 C^{*2} Y_{H_u} Y_{H_d}) \right) \\ & + \quad F^m F^{\bar{n}} \left(\partial_m \partial_{\bar{n}} (CC^* X_H) - \partial_{\bar{n}} (CC^* X_H) \partial_m \ln (C^2 C^{*2} Y_{H_u} Y_{H_d}) \right) \\ & + \quad \frac{1}{2} D_A \frac{\partial}{\partial V_A} (CC^* X_H) \Big]_{C=C^*=e^{K_0/6}} \end{aligned}$$

$$
\begin{aligned}
m_I^2(M_{\rm mess}) &= -F^m F^{\bar{m}} \partial_m \partial_{\bar{m}} \ln Y_I - \frac{1}{2} D_A \frac{\partial}{\partial V_A} \ln Y_I \\
&= -\left|\frac{F^C}{C_0}\right|^2 \frac{d^2 \ln Y_I}{d(\ln p^2)^2} - \left(\frac{F^C}{C_0} F^{\bar{i}} \partial_{\bar{i}} + \frac{F^{C*}}{C_0^*} F^i \partial_i\right) \frac{d \ln Y_I}{d \ln p^2} \\
&\quad -F^i F^{\bar{j}} \partial_i \partial_{\bar{j}} \ln Y_I - \frac{1}{2} D_A \frac{\partial}{\partial V_A} \ln Y_I \\
&= -\frac{1}{4}\left|\frac{F^C}{C_0}\right|^2 \frac{d\gamma_I}{d \ln p} - \frac{1}{2}\left(\frac{F^C}{C_0} F^{\bar{i}} \partial_{\bar{i}} + \frac{F^{C*}}{C_0^*} F^i \partial_i\right) \gamma_I \\
&\quad -F^i F^{\bar{j}} \partial_i \partial_{\bar{j}} \ln Y_I - \frac{1}{2} D_A \frac{\partial}{\partial V_A} \ln Y_I
\end{aligned}
$$

1st and 2nd terms represent the effects of F^C, which can not be captured in classical approximation.

The parts associated with F^C might be subleading in some cases, but still they are the model-independent parts.

So far, we have derived the on-shell expressions of the SUSY breaking auxiliary components starting from the superspace (off-shell) formulation, and also express the soft terms in terms of the superspace matter wavefunction coefficients $Y_I = e^{-K_0/3} Z_I$, rather than in terms of the matter Kähler metric Z_I:

$$\begin{aligned} -3e^{-K/3} &= -3e^{-K_0(Z,Z^*)/3} + Y_I(Z,Z^*)\Phi_I^*\Phi_I + \dots \quad \left(Y_I = e^{-K_0/3}Z_I\right) \\ K &= K_0(Z,Z^*) + Z_I(Z,Z^*)\Phi_I^*\Phi_I + \dots, \end{aligned}$$

Note that sequestering is transparently defined with Y_I:

$$\frac{\partial f_a}{\partial Z} = \frac{\partial Y_I}{\partial Z} = 0 \quad \Rightarrow \quad Z \;\; \text{is sequestered from the visible sector}$$

In some case, we need to couple the $N = 1$ SUGRA to a sector with less SUSY. One important examples is the low energy consequence of anti-brane which does not respect the $N = 1$ SUSY of the main sector. ($N = 1$ SUSY is non-linearly realized on anti-brane even at M_{string}.) In such case, there is no reason that the conventional on-shell expressions of auxiliary components are all valid, and one should rederive them starting from the appropriate off-shell (superspace) formulation.

Some popular mediation schemes

Currently there are four widely discussed mediation schemes, some of which naturally give flavor-conserving (and CP-conserving besides the Higgs B-parameter) soft masses:

- String dilaton and/or moduli mediation (Gravity mediation)
- Gauge mediation
- Anomaly mediation
- D-term mediation

Each of these mediation schemes gives distinctive superparticle spectrum different from each other.

- **Dilaton and/or moduli mediation**

Kaplunovsky,Louis; Brignole,Ibanez,Munoz

Dilaton or volume modulus mediation is a particular type of gravity mediation, preserving flavor and CP at least at the leading order in small coupling expansion.

Gravity mediation through "generic Planck scale suppressed interactions" does not give flavor and CP conserving soft terms:

$$\int d^2\theta d^2\bar{\theta}\left(c_{IJ}\frac{XX^*}{M_{Pl}^2}+d_{IJ}\frac{X}{M_{Pl}}\right)\Phi_I^*\Phi_J+\int d^2\theta\frac{X}{M_{Pl}}\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha$$

To examine the possibility of flavor and CP conserving gravity mediation, one needs a UV completion of the relevant Planck scale suppressed interactions.

String theory is the only known theory which might allow a systematic calculation of the Planck scale suppressed couplings between X and the MSSM fields.

Dilaton and/or volume moduli superfields $\{T\}$ in compactified string theory can provide a flavor and CP conserving gravity mediation.

Kaplunovsky,Louis; Brignole,Ibanez,Munoz

At leading order in the weak string coupling or large volume expansion, the couplings between T and the MSSM matter and gauge fields take the form:

$$\int d^4\theta CC^*(T+T^*)^{n_I}\Phi_I^*\Phi_I + \int d^2\theta \frac{1}{4} T W^{a\alpha} W^a_\alpha$$
$$\Longrightarrow \quad Y_I = (T+T^*)^{n_I}, \quad f_a = T$$

Typically the modular weights n_I are flavor-universal rational numbers, and the couplings of T and matter fields are constrained by the axionic shift symmetry: $T \to T + i\alpha$, which assures

$$\partial_T \lambda_{IJK} = 0$$

$\Longrightarrow$ Flavor and CP conserving soft masses at $M_{\rm mess} \sim M_{GUT}$:

$$\begin{aligned} M_a &= F^m \partial_m \ln \mathrm{Re}(f_a) = \frac{F^T}{T+T^*}, \\ m_I^2 &= -F^m F^{\bar{n}} \partial_m \partial_{\bar{n}} \ln Y_I = n_I \left| \frac{F^T}{T+T^*} \right|^2, \\ A_{IJK} &= -F^m \partial_m \ln \left(\frac{\lambda_{IJK}}{Y_I Y_J Y_K} \right) = -(n_I + n_J + n_K) \frac{F^T}{T+T^*} \end{aligned}$$

However, generically there are other moduli which have flavor non-universal couplings to the MSSM matter fields, for instance the complex structure moduli which determine the hierarchical structure of Yukawa couplings.

Flux compactification provides a natural set-up for such flavor non-universal moduli decoupled from SUSY breaking:

KC,Nilles,Falkowski,Olechowski; Conlon,Quevedo,Suruliz

Typically "flavon moduli" ($= U$) and "SUSY-breaking moduli" ($= T$) have different topological origins, e.g. 3-cycle and 4-cycle, so U can get a heavy mass from flux, while T is untouched by flux.

$\Longrightarrow$ Split moduli masses:

$$m_U \gg m_T \;\to\; F^U \sim \frac{m_T}{m_U} F^T \ll F^T$$

- **(Minimal) Gauge Mediation**

Dine,Fischler,Srednicki; Dimopoulos,Raby; Dine,Nelson,Shirman

Gauge-charged messenger $\Psi + \Psi^c$ with a Yukawa coupling to SUSY-breaking field X:

$$\int d^2\theta \, X\Psi\Psi^c \qquad \left(X = X_0 + \theta^2 F^X\right)$$

Effective action at $M_{\rm mess} = X_0$ after $\Psi + \Psi^c$ are integrated out:

$$\int d^4\theta \left(1 - \frac{2}{(16\pi^2)^2} N_\Psi \sum_a g_a^4(M_{\rm mess}) C_a(\Phi_I) \left(\ln \frac{X^* X}{M_{\rm mess}^2}\right)^2\right) \Phi_I^* \Phi_I$$

$$+ \int d^2\theta \frac{1}{4} \left(\frac{1}{g_a^2(M_{\rm mess})} - \frac{1}{8\pi^2} N_\Psi \ln \frac{X}{M_{\rm mess}}\right) \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha \qquad \left(C_a(\Phi)\mathbf{1} = \sum_a T_a^2(\Phi)\right)$$

$$\Rightarrow \quad Y_I(M_{\rm mess}) = 1 - \frac{2}{(16\pi^2)^2} N_\Psi \sum_a g_a^4(M_{\rm mess}) C_a(\Phi_I) \left(\ln \frac{X^* X}{M_{\rm mess}^2}\right)^2$$

$$f_a(M_{\rm mess}) = \frac{1}{g_a^2(M_{\rm mess})} - \frac{1}{8\pi^2} N_\Psi \ln \frac{X}{M_{\rm mess}}$$

$$\frac{\partial \lambda_{IJK}(M_{\rm mess})}{\partial \ln X} = 0$$

$$
\begin{aligned}
\Rightarrow \quad M_a(M_{\text{mess}}) &= F^m \partial_m \text{Re}(f_a) = -\frac{g_a^2(M_{\text{mess}})}{16\pi^2} N_\Psi \frac{F^X}{X_0} \\
m_I^2(M_{\text{mess}}) &= -F^m F^{\bar{n}} \partial_m \partial_{\bar{n}} \ln Y_I = \frac{2}{(16\pi^2)^2} N_\Psi \sum_a g_a^4(M_{\text{mess}}) C_a(\Phi_I) \left| \frac{F^X}{X_0} \right|^2 \\
A_{IJK}(\Phi_0) &= -F^m \partial_m \ln \left(\frac{\lambda_{IJK}}{Y_I Y_J Y_K} \right) = \mathcal{O}\left(\frac{g^4}{(16\pi^2)^2} \frac{F^X}{X_0} \right)
\end{aligned}
$$

Example:

$$\int d^4\theta CC^* \left(\Phi\Phi^* - \frac{(\Phi\Phi^*)^2}{4\Lambda_1^2}\right) + \int d^2\theta\, C^3 \left(\Lambda_2^2\Phi + (\lambda\Phi + M)\Psi\Psi^c\right)$$

$$\Longrightarrow \quad X = \lambda\Phi + M = X_0 + \theta^2 F^X$$

$$M_{\rm mess} \equiv X_0 = \lambda\Phi_0 + M = \lambda\frac{3\Lambda_1^2}{\Lambda_2^2} m_{3/2} + M,$$

$$F^X = \lambda F^\Phi = \lambda\Lambda_2^2$$

$$m_{\rm soft} \sim \frac{g^2}{16\pi^2}\frac{F^X}{X_0} \gg m_{3/2}$$

(Minimal) gauge mediation automatically gives flavor and CP conserving soft masses (except for B).

On the other hand, generically the scheme involves more mass scales other than M_{Pl} and the SUSY breaking scale, e.g. Λ_1 and M in this example, and a fully satisfactory model should explain the origin of those mass scales.

- **Anomaly Mediation** Randall,Sundrum; Giudice,Luty,Murayama,Rattazzi

C_0 in $C = C_0 + \theta^2 F^C$ is equivalent to the conformal factor of the spacetime metric, indicating that SUSY breaking by F^C appears through the breaking of scale invariance, including the breaking by scale anomaly. More specifically, the residual Weyl-invariance under

$$\eta_{\mu\nu} \to e^{2(\tau_0+\tau_0^*)}\eta_{\mu\nu}, \quad C \to e^{-2\tau_0}C, \quad \theta^\alpha \to e^{-\tau_0+2\tau_0^*}\theta^\alpha$$
$$(\tau_0 = \text{complex constant})$$

requires that any IPI RG running over $\ln p^2 = \ln(\eta^{\mu\nu}\partial_\mu\partial_\nu)$ should appears through the Weyl-invariant combination

$$\ln\left(\frac{p^2}{CC^*}\right),$$

leading to the relation

$$\frac{\partial}{\partial \ln C} = \equiv -\frac{d}{d\ln p^2}.$$

Roughly, anomaly mediation can be considered as gauge mediation with UV regulators play the role of the gauge-charged messenger.

$$
\begin{aligned}
\Rightarrow \quad M_a(M_{\rm mess}) &= F^C \partial_C \ln \mathcal{F}_a \;=\; \frac{F^C}{C_0}\frac{d\ln g_a^2(p)}{d\ln p^2}\bigg|_{p=M_{\rm mess}} \;=\; \frac{F^C}{C_0}\frac{\beta_a}{2g_a^2}\bigg|_{p=M_{\rm mess}} \\
A_{IJK}(\Lambda_{\rm mess}) &= -F^C\partial_C \ln\left(\frac{\lambda_{IJK}}{Y_I Y_J Y_K}\right) \;=\; -\frac{F^C}{C_0}\,\frac{d\ln(Y_I Y_J Y_K)}{d\ln p^2}\bigg|_{p=M_{\rm mess}} \\
&= -\frac{1}{2}\frac{F^C}{C_0}\left(\gamma_I + \gamma_J + \gamma_K\right)\Big|_{p=M_{\rm mess}} \qquad \left(\gamma_I \equiv \frac{d\ln Y_I}{d\ln p}\right) \\
m_I^2(\Lambda_{\rm mess}) &= -F^C F^{C*}\partial_C\partial_{C^*}\ln Y_I \;=\; -\left|\frac{F^C}{C_0}\right|^2 \frac{d^2\ln Y_I(p)}{d(\ln p^2)^2}\bigg|_{p=M_{\rm mess}} \\
&= -\left|\frac{F^C}{C_0}\right|^2 \frac{\dot{\gamma}_I}{4}\bigg|_{p=M_{\rm mess}}
\end{aligned}
$$

$M_{\rm mess}$ for anomaly mediation can be as high as $M_{\rm Planck}$.

For pure anomaly mediation, these relations are valid at any momentum scale.

However, pure anomaly mediation (within the MSSM) gives tachyonic slepton masses, which should be avoided by introducing other contribution to slepton masses.

Anomaly mediation also automatically gives flavor and CP conserving soft terms (except for B).

In order for the anomaly mediation to be a dominant source of soft terms, other mediations should be sequestered enough:

$$F^X \partial_X \ln Y_I,\ F^X \partial_X \ln f_a \ \lesssim \ \frac{1}{8\pi^2} \frac{F^C}{C_0} \ \sim \ \frac{m_{3/2}}{8\pi^2}$$

$$\left(\frac{F^C}{C_0} = m_{3/2}^* + \frac{1}{3} F^X \partial_X K \right)$$

- ***D*-term mediation:**

We already derived a relation between D and F in generic SUGRA: $D_A = ig_A^2 \eta_A^i F_i / m_{3/2}$.

For generic vacuum configuration satisfying the equations of motion

$$\frac{\partial V}{\partial X^i} = 0 \quad \left(V = e^K \left[K^{i\bar{j}} D_i W (D_j W)^* - 3|W|^2\right] + \frac{1}{2}\mathrm{Re}(f_A) D_A^2\right),$$

we can find another relation between the VEVs of D and F-terms:

$$\left\langle \left(K_{i\bar{j}} F^i F^{j*} - m_{3/2}^2 + M_A^2\right) \mathrm{Re}(f_A) D_A \right\rangle = \left\langle F^i F^{j*} \partial_i \partial_{\bar{j}} \left(\mathrm{Re}(f_A) D_A\right) + \frac{i\eta_A^i \partial_i f_A}{4} D_A^2 \right\rangle$$

$$M_A^2 = \eta_A^i \eta_A^{j*} K_{i\bar{j}} = U(1)_A \text{ gauge boson mass}$$

For $M_A^2 \gg m_{3/2}^2$, the above relation implies

$$D_A \sim \frac{q_i |F^i|^2}{M_A^2}$$

Note that for anomalous $U(1)$ symmetry, $\eta_A^i \partial f_A \neq 0$ and M_A^2 includes the Stuckelberg mass $\sim M_{\mathrm{Planck}}$:

$$f_A = T, \quad \eta_A^T = i\delta_{GS} \quad \Rightarrow \quad M_A^2 = \delta_{GS}^2 M_{\mathrm{Planck}}^2 \partial_T \partial_{\bar{T}} K + \ldots$$

Example: Models with anomalous $U(1)_A$

Binetruy,Dudas; Dvali,Pomarol; Arkani−Hamed,Dine,Martin

$$U(1)_A: \; V_A \to V_A - i(\Lambda - \Lambda^*), \;\; T \to T - i\delta_{GS}\Lambda, \;\; X \to e^{-i\Lambda}X, \;\; \Phi_I \to e^{-iq_I\Lambda}\Phi_I$$

$$\left(\delta_{GS} = \mathcal{O}(1/8\pi^2) \text{ for the Green-Schwarz anomaly cancelation}\right)$$

$$K = K_0(T + T^* - \delta_{GS}V_A) + X^* e^{-V_A} X + \Phi_I^* e^{-q_I V_A}\Phi_I,$$

$$\Longrightarrow \quad D_A \sim \frac{(\partial_T \partial_{T^*} K_0)^2}{\partial_T K_0(\partial_T K_0 - \delta_{GS}\partial_T\partial_{T^*}K_0)} \left|F^T\right|^2$$

$$m_I^2 = \frac{1}{2} q_I D_A$$

Depending upon the form of K_0 and also how to stabilize the D-flat direction $\left(\propto T - \delta_{GS} \ln X \right)$, D_A can be a dominant source of soft scalar mass.