Compactification, Model Building, and Fluxes

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Lecture 1. Model building in IIA: Intersecting brane worlds

We review the construction of chiral four-dimensional compactifications of type IIA string theory with intersecting D6-branes. Such models lead to four-dimensional theories with non-abelian gauge interactions and charged chiral fermions. We discuss the application of these techniques to building of models with spectrum as close as possible to the Standard Model, and review their main phenomenological properties.

1 Introduction

String theory has the remarkable property that it provides a description of gauge and gravitational interactions in a unified framework consistently at the quantum level. It is this general feature (beyond other beautiful properties of *particular* string models) that makes this theory interesting as a possible candidate to unify our description of the different particles and interactions in Nature.

Now if string theory is indeed realized in Nature, it should be able to lead not just to 'gauge interactions' in general, but rather to gauge sectors as rich and intricate as the gauge theory we know as the Standard Model of Particle Physics. In these lecture we describe compactifications of string theory where sets of D-branes lead to gauge sectors close to the Standard Model. We furthermore discuss the interplay of such D-brane systems with flux compactifications, recently introduced to address the issues of moduli stabilization and supersymmetry breaking.

Before starting, it is important to emphasize that there are other constructions in string theory which are candidates to reproduce the physics of the Standard Model at low energies, which do not involve D-branes. For instance, compactifications of heterotic string on Calabi-Yau threefolds, M-theory compactifications on G_2-holonomy spaces, etc. We emphasize D-brane models because of their simplicity, and also because they are often related to these other compactifications via string dualities. Hence, they provide a simple introduction from which the interested reader may jump onto the big picture.

This first lecture introduces D-branes and their properties, and deals with model building using intersecting D-branes. Useful reviews for this lecture are for example [1].

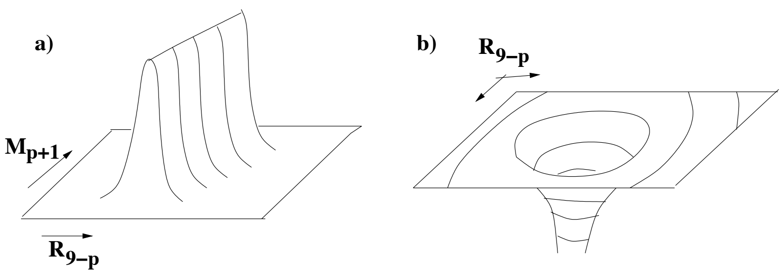

Figure 1: Two pictures of the p-brane as a lump of energy. The second picture shows only the transverse directions, where the p-brane looks like point-like.

These lectures are organized as follows. In section 2 we quickly review properties of D-branes and their world-volume dynamics. In section 3 we describe that configurations of intersecting D6-branes naturally lead to four-dimensional chiral fermions, and discuss their spectrum and supersymmetry. In section 4 we construct compactifications of type IIA string theory to four dimensions, including configurations of intersecting D6-branes. We provide explicit descriptions of toroidal compactifications of this kind, and generalizations to more general Calabi-Yau compactifications. In section 5 we introduce further ingredients to improve these models, namely orientifold 6-planes. We describe their properties, discuss configurations of D6-branes and O6-planes, and describe how to include them in compactifications in section 5.3. These techniques are exploited in section 6 to construct models whose chiral spectrum is that of the standard model, and in section 7 to describe supersymmetric chiral compactifications with intersecting branes. Appendix A provides some details on the computation of open string spectra for parallel and intersecting D-branes.

2 Overview of D-branes

2.1 Properties of D-branes

The study of string theory beyond perturbation theory has led to the introduction of new objects in string theory, D-branes. For a complementary description of D-branes and their properties see [2, 3].

Type II string theories contains certain 'soliton-like' states in their spectrum, with $p+1$ extended dimensions, the p-branes. They were originally found as solutions of the low-energy supergravity equations of motion. This is schematically shown in figure

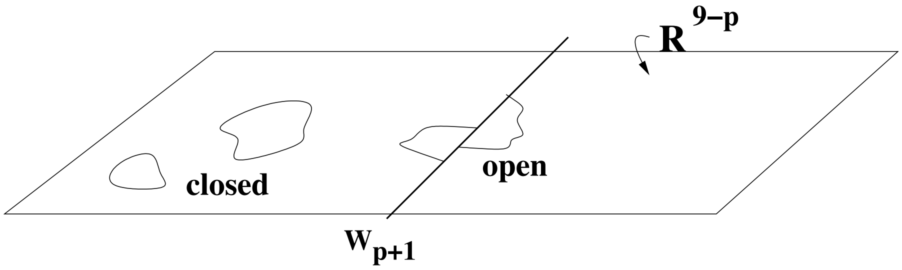

Figure 2: String theory in the presence of a Dp-brane. The closed string sector describes the fluctuations of the theory around the vacuum (gravitons, dilaton modes, etc), while the sector of open strings describes the spectrum of fluctuations of the soliton.

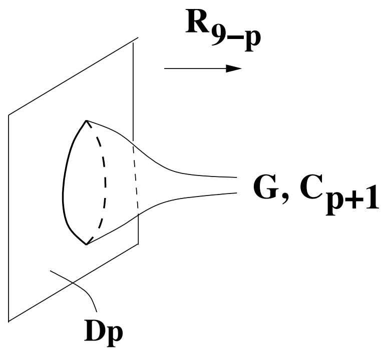

Figure 3: Disk diagram describing the interaction of a Dp brane with closed string modes.

1. Subsequently, it was realized [4] that certain of these objects (known as Dp-branes) admit a fully stringy description, as $(p+1)$-dimensional subspaces on which open strings can end. Notice that these open strings are not present in the vacuum of the underlying string theory, but rather represent the fluctuations of the theory around the topological defect background. Namely, the closed string sector still describes the dynamics of the vacuum (gravitational interactions, etc), while open strings rather describe the dynamics of the object. The situation is shown in figure 2

The basic properties of Dp-branes for our purposes in these lecture are:

- Dp-branes are dynamical, and for instance have non-trivial interactions with closed string modes. Due to these couplings, they carry tension (they interact with the 10d graviton) and charge under a RR $(p+1)$-form potential C_{p+1}, see figure 3. Hence, type IIA (resp. IIB) string theory contains Dp-branes with p even (resp. odd).

- A flat Dp-brane in flat spacetime preserves half the supersymmetries of the theory. Denoting $Q_{\rm L}$, $Q_{\rm R}$ the two 16-component spinor supercharges of type II string theories, arising from the left- or right-moving world-sheet degrees of freedom, a Dp-brane with world-volume spanning the directions $012\ldots p$ preserves the linear combination

$$Q = \epsilon_{\rm R} Q_{\rm R} + \epsilon_{\rm L} Q_{\rm L} \tag{1}$$

  where $\epsilon_{\rm L,R}$ are spinor coefficients satisfying

$$\epsilon_{\rm L} = \Gamma^{01\ldots p} \epsilon_{\rm R} \tag{2}$$

  Thus Dp-branes are BPS states, and their charge and tension are equal.

- Dp-branes may have curved world-volumes. However, they tend to minimize the volume of the submanifold they span, hence in flat space Dp-branes tend to span flat world-volumes. In curved spaces, arising e.g. in compactifications, they may however wrap curved non-trivial homology cycles.

- As mentioned already, open string modes in the presence of D-branes are localized on the world-volume of the latter. This implies that such open strings represent the collective coordinates of the non-perturbative object, and thus their dynamics controls the dynamics of the object. In next section we will center on the zero modes, corresponding to the massless open string sector.

2.2 World-volume fields

The spectrum of fluctuations of the theory in the presence of the Dp-brane is obtained by quantizing closed strings and open strings ending on the Dp-brane. Since the open string endpoints are fixed on the D-brane, the massless modes in the latter sector yield fields propagating on the $(p+1)$-dimensional D-brane world-volume W_{p+1}.

A simplified calculation of the quantization of open strings for a configuration of a single type II Dp-brane in flat 10d is carried out in appendix A.1. The resulting set of massless particles on the Dp-brane world-volume is given by a $U(1)$ gauge boson, $9-p$ real scalars and some fermions (transforming under Lorentz as the decomposition of the 8_C of $SO(8)$ under the $(p+1)$-dimensional little group $SO(p-1)$). The scalars (resp. fermions) can be regarded as Goldstone bosons (resp. Goldstinos) of the translational symmetries (resp. supersymmetries) of the vacuum broken by the presence of the D-brane. The open string sector fills out a $U(1)$ vector multiplet with respect to the 16 supersymmetries unbroken by the D-brane.

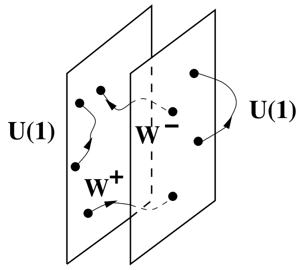

Figure 4: Non-abelian gauge bosons in a configuration of coincident D-branes.

As mentioned above, Dp-branes are charged under the corresponding RR $(p+1)$-form C_{p+1} of type II string theory, via the minimal coupling $\int_{W_{p+1}} C_{p+1}$. Since flat Dp-branes in flat space preserve 1/2 of the 32 supercharges of the type II vacuum, such D-branes are BPS states, and their RR charge is related to their tension. This implies that there is no net force among parallel branes (roughly, gravitational attraction cancels against 'Coulomb' repulsion due to their RR charge). Hence one can consider dynamically stable configurations of several parallel Dp-branes, labeled by a so-called Chan-Paton index a, at locations x_a^i in the transverse coordinates, $i = p+1, \ldots, 9$.

We would like to consider the situation with n coincident Dp-branes, located at the same position in transverse space. In such situation there are n^2 open string sectors, labeled ab for an open string starting at the a^{th} D-brane and ending at the b^{th} D-brane. The computation for each sector ab is similar to the single brane case. Hence the spectrum of physical states contains, at the massless level, n^2 gauge bosons, $n^2 \times (9-p)$ scalars, and n^2 sets of $(p+1)$-dimensional fermions (in representations obtained from decomposing the $\mathbf{8}_{\mathrm{C}}$ of $SO(8)$).

This multiplicity renders interactions between open strings non-abelian. It is possible to see that the gauge bosons in the aa sector correspond to a $U(1)^n$ gauge symmetry, and that states in the ab sector have charges $+1$ and -1 under the a^{th} and b^{th} $U(1)$, respectively. This enhances the gauge symmetry to $U(n)$, and makes the different fields transform in the adjoint representation. The complete massless open string spectrum is given by $U(n)$ gauge bosons, $9-p$ adjoint scalars and adjoint fermions, filling out a $U(n)$ vector multiplet with respect to the 16 unbroken supersymmetries. The structure of gauge bosons for $n=2$ is shown in figure 4.

D-branes provide a nice and simple realization of non-abelian gauge symmetry in string theory. The low-energy effective action for the massless open string modes has several pieces. One of them is the Dirac-Born-Infeld action, which has the form

$$S_{\rm DBI} = -T_p \int_{W_{p+1}} \mathrm{d}^{p+1}\mathrm{x}^\mu \; [-\det(\mathrm{G}+\mathrm{B}+2\pi\alpha'\mathrm{F})]^{1/2} \tag{3}$$

where T_p is the Dp-brane tension, and $G_{\mu\nu} = \partial_\mu\phi^i\partial_\nu\phi^j\,G_{ij}$ is the metric induced on the D-brane worldvolume, and similarly $B_{\mu\nu}$ is the induced 2-form. These terms introduce the dependence of the action on the world-volume scalars $\phi^i(x^\mu)$. Finally $F_{\mu\nu}$ is the field strength of the worldvolume gauge field.

Neglecting the dependence on the field strength, it reduces to the D-brane tension times the D-brane volume $\int(\det G)^{1/2})$. At low energies, i.e. neglecting the α' corrections, it reduces to a kinetic term for the scalars plus the $(p+1)$-dimensional Yang-Mills action for the worldvolume gauge fields, with gauge coupling given by $g^2_{U(n)} = g_s$. Of course the above action should include superpartner fermions, etc, but we skip their discussion.

A second piece of the effective action is the Wess-Zumino terms, of the form

$$S_{\rm WZ} = -Q_p \int_{W_{p+1}} \mathcal{C} \wedge \mathrm{ch}(F)\,\hat{A}(R) \tag{4}$$

where $\mathcal{C} = C_{p+1} + C_{p-1} + C_{p-3} + \ldots$ is a formal sum of the RR forms of the theory, and $\mathrm{ch}(F)$ is the Chern character of the worldvolume gauge bundle on the D-brane volume

$$\mathrm{ch}(F) = \exp(\frac{F}{2\pi}) = 1 + \frac{1}{2\pi}\mathrm{tr}\,F + \frac{1}{8\pi^2}\mathrm{tr}\,F^2 + \ldots \tag{5}$$

and $\hat{A}(R)$ is the A-roof genus, characterizing the tangent bundle of the D-brane worldvolume $\hat{A}(R) = 1 - \mathrm{tr}\,R^2/(2\pi^2) + \ldots$. Integration is implicitly defined to pick up the degree $(p+1)$ pieces in the formal expansion in wedge products. Hence we get terms like

$$\begin{aligned} S_{\rm WZ} = -Q_p \Bigg(& \int_{W_{p+1}} C_{p+1} + \frac{1}{2\pi}\int_{W_{p+1}} C_{p-1}\wedge \mathrm{tr}\,F \\ & + \frac{1}{8\pi^2}\int_{W_{p+1}} C_{p-3}\wedge(\mathrm{tr}\,F^2 - \mathrm{tr}\,R^2) + \ldots \Bigg) \end{aligned} \tag{6}$$

A very important property of this term is that it is topological, independent of the metric or on the particular field representatives in a given topological sector. This is related to the fact that these terms carry the information about the RR charges of the D-brane configuration.

2.3 Chirality and D-branes

We have obtained simple configurations of D-branes leading to non-abelian gauge symmetries on their world-volume. It is interesting to wonder if such configurations could be exploited to reproduce the gauge sector describing high energy particle physics, so as to embed it into a string theory model. Clearly, the main obstruction is that the standard model of particle physics is chiral in four dimensions. This property is incompatible with the large amount of supersymmetry preserved by the D-brane configurations considered.

There is an alternative heuristic way to intuitively understand the lack of chirality in our D-brane configuration. Four-dimensional chirality is a violation of four-dimensional parity. In the spectrum of open strings there is a correlation (implied by the GSO projection) between the 4d chirality and the chirality in the six extra dimensions. Hence to achieve 4d parity violation the configuration must violate 6d parity. However, the above configurations of D-branes do not violate 6d parity, do not introduce a preferred six-dimensional orientation.

The latter remark indeed suggest how to proceed to construct configurations of D-branes leading to four-dimensional chiral fermions. The requirement is that the configuration introduces a preferred orientation in the six transverse dimensions. There are several ways to achieve this, as we discuss now.

• D-branes sitting at *singular* (rather than smooth) points in transverse space can lead to chiral open string spectra. The prototypical example is given by stacks of D3-branes sitting at the singular point of orbifolds of flat space, e.g. orbifold singularities $\mathbf{C^3}/\mathbf{Z_N}$, as studied in [5]. A particularly simple and interesting case is the $\mathbf{C^3}/\mathbf{Z_3}$ orbifold, which will be studied in our second lecture. The key idea is that the discrete rotation implied by the $\mathbf{Z_3}$ action defined a preferred orientation in the 6d space, and allows for chirality on the D-branes.

• Consider a stack of D9-branes in flat 10d spacetime, split as $M_4 \times \mathbf{R^2} \times \mathbf{R^2} \times \mathbf{R^2}$. For simplicity we ignore for the moment the issue of RR tadpole cancellation. Otherwise, to make the configuration consistent it suffices to introduce orientifold 9-planes, namely consider the configuration in type I string theory. Now introduce non-trivial field strength background for the world-volume $U(1)_a$ gauge fields, F_a^i in the i^{th} $\mathbf{R^2}$, with $i = 1, 2, 3$, (see [8, 9] for early discussions, and [10, 11, 12] for more recent ones). The magnetic fields introduce a preferred orientation in the transverse six dimensions (obtained by using $F \wedge F \wedge F$ as the volume form, where F is the 2-form associated to the field strength). Hence the configurations lead naturally to 4d chiral fermions, as we describe in our second lecture.

• Sets of intersecting D-branes can also lead to chiral fermions in the sector of open strings stretched between different kinds of D-brane [13], and are the topic of our

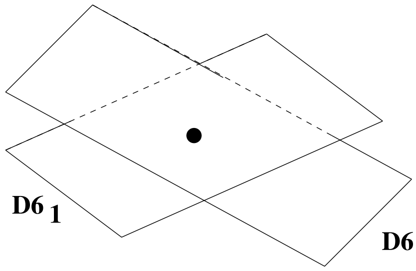

Figure 5: Picture of D6-branes intersecting over a 4d subspace of their volumes.

lecture today.

3 Intersecting D6-branes

3.1 Local geometry and spectrum

The basic configuration of intersecting D-branes leading to chiral 4d fermions at their intersection is two stacks of D6-branes in flat 10d intersecting over a 4d subspace of their volumes. Consider flat 10d space $M_4 \times \mathbf{R^2} \times \mathbf{R^2} \times \mathbf{R^2}$, and two stacks of D6-branes, spanning M_4 times a line in each of the three 2-planes. Figures 5, 6 provide two pictorial representations of the configurations. The local geometry is fully specified by the three angles θ_i which define the rotation between the two stacks of D6-branes. As we discuss below, the chiral fermions are localized at the intersection of the brane volumes.

The appearance of chirality can be understood from the fact that the geometry of the two D-brane introduces a preferred orientation in the transverse 6d space, namely by considering the relative rotation of the second D6-brane with respect to the first. This also explains why one should choose configurations of D6-branes. For example, two sets of D5-branes intersecting over 4d do not lead to 4d chiral fermions, since they do not have enough dimensions to define an orientation in the transverse 6d space.

A more detailed computation of the spectrum of open string models on systems of intersecting branes is provided in appendix A.2. Here it will suffice to mention the results of the spectrum for this configuration. The open string spectrum in a configuration of two stacks of n_1 and n_2 coincident D6-branes in flat 10d intersecting over a 4d subspace of their volumes consists of three open string sectors:

$\mathbf{6}_1\mathbf{6}_1$ Strings stretching between D6_1-branes provide $U(n_1)$ gauge bosons, three real

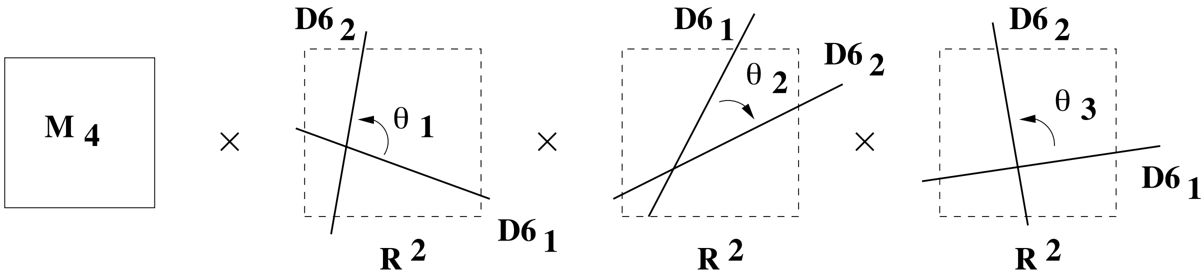

Figure 6: A more concrete picture of the configuration of two D6-branes intersecting over a 4d subspace of their volumes.

adjoint scalars and fermion superpartners, propagating over the 7d world-volume of the $D6_1$-branes.

$\mathbf{6}_2\mathbf{6}_2$ Similarly, strings stretching between $D6_2$-branes provide $U(n_2)$ gauge bosons, three real adjoint scalars and fermion superpartners, propagating over the $D6_2$-brane 7d world-volume.

$\mathbf{6}_1\mathbf{6}_2 + \mathbf{6}_2\mathbf{6}_1$ Strings stretching between both kinds of D6-brane lead to a 4d chiral fermion, transforming in the representation $(n_1, \overline{n}_2)$ of $U(n_1) \times U(n_2)$, and localized at the intersection. The chirality of the fermion is encoded in the orientation defined by the intersection; this will be implicitly taken into account in our discussion.

So we have succeeded in constructing a configuration of D-branes leading to 4d chiral fermions in the open string sector. Again, let us emphasize that the appearance of chiral fermions in the present system is the angles between the branes (technically, leading to the reduction of the Clifford algebra of fermion zero modes in the open strings between branes). Notice that the 4d chiral fermions lead to a localized anomaly at the intersection of the D6-branes. This anomaly is however canceled by the anomaly inflow mechanism, see [14].

In addition to the chiral fermions at intersections, there are several potentially light complex scalars at the intersection, transforming in bifundamental representations, and with masses (in α' units) given by

$$\begin{array}{ll} \tfrac{1}{2\pi}(-\theta_1 + \theta_2 + \theta_3) & \dfrac{1}{2\pi}(\theta_1 - \theta_2 + \theta_3) \\ \tfrac{1}{2\pi}(\theta_1 + \theta_2 - \theta_3) & 1 - \dfrac{1}{2\pi}(-\theta_1 - \theta_2 - \theta_3) \end{array} \qquad (7)$$

These scalars, as we further discuss in section 3.2), can be massless, massive or tachyonic.

3.2 Supersymmetry for intersecting D6-branes

It is interesting to consider if the above configurations preserve some supersymmetry. This can be analyzed following [13]. The condition that there is some supersymmetry preserved by the combined system of two D6-brane stacks is that there exist spinors $\epsilon_{\rm L}$, $\epsilon_{\rm R}$ that satisfy

$$\begin{aligned} \epsilon_{\rm L} = \Gamma_6 \epsilon_{\rm R} \quad &; \quad \Gamma_6 = \Gamma^0 \ldots \Gamma^3 \Gamma^4 \Gamma^6 \Gamma^8 \\ \epsilon_{\rm L} = \Gamma_{6'} \epsilon_{\rm R} \quad &; \quad \Gamma_{6'} = \Gamma^0 \ldots \Gamma^3 \Gamma^{4'} \Gamma^{6'} \Gamma^{8'} \end{aligned} \tag{8}$$

where 468 and $4'6'8'$ denote the directions along the two D6-branes in the six dimensions 456789. The above is simply the condition (2) for each of the branes.

Let R denote the $SO(6)$ rotation that takes the first D6-brane into the second, acting on the spinor representation. Then we have $\Gamma_{6'} = R \Gamma_6 R^{-1}$. A preserved spinor exists if and only if there is a 6d spinor which is invariant under R. This implies that R must belong to an $SU(3)$ subgroup of $SO(6)$. This can be more explicitly stated by rewritting R in the vector representation as

$$R = {\rm diag}\,({\rm e}^{{\rm i}\theta_1}, {\rm e}^{-{\rm i}\theta_1}, {\rm e}^{{\rm i}\theta_2}, {\rm e}^{-{\rm i}\theta_2}, {\rm e}^{{\rm i}\theta_3}, {\rm e}^{-{\rm i}\theta_3}) \tag{9}$$

The condition that the rotation is within $SU(3)$ is

$$\theta_1 \pm \theta_2 \pm \theta_3 = 0 \quad {\rm mod} \quad 2\pi \qquad \text{for some choice of signs} \tag{10}$$

Indeed, one can check that the open string spectrum computed above is boson-fermion degenerate in such cases. In the generic case, there is no supersymmetry invariant under the two stacks of branes, and the open string sector at the intersection is non-supersymmetric. However, if $\theta_1 \pm \theta_2 \pm \theta_3 = 0$ for some choice of signs, one of the scalars becomes massless, reflecting that the configuration is $\mathcal{N} = 1$ supersymmetric. $\mathcal{N} = 2$ supersymmetry arises if e.g. $\theta_3 = 0$ and $\theta_1 \pm \theta_2 = 0$, while $\mathcal{N} = 4$ arises only for parallel stacks $\theta_i = 0$.

As described above, the light scalars at intersections may be massless, massive or tachyonic. The massless case corresponds to a situation with some unbroken supersymmetry. The massless scalar is a modulus, whose vacuum expectation value (vev) parametrizes the possibility of recombining the two intersecting D-branes into a single smooth one, as pictorially shown in figure 7. That is, the intersecting geometry belongs to a one- (complex) parameter family of supersymmetry preserving configurations of D-branes. Mathematically, there is a one-parameter family of supersymmetric 3-cycles,

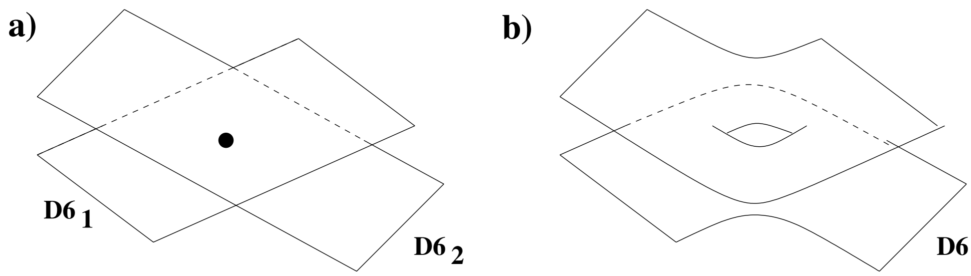

Figure 7: Recombination of two intersecting D6-branes into a single smooth one, corresponding to a vev for an scalar at the intersection.

i.e. special lagrangian submanifolds of $\mathbf{R}^{\mathbf{6}}$, with the same asymptotic behaviour as the intersecting D-brane configuration. In the simpler situation of D-branes intersecting at $SU(2)$ angles (i.e. $\mathcal{N}=2$ supersymmetry), the recombination is very explicit. It is given by deforming two intersecting 2-planes, described by the complex curve $uv=0$, to the smooth 2-cycle $uv=\epsilon$, with ϵ corresponding to the vev of the scalar at the intersection.

The configuration with tachyonic scalars corresponds to situations where this recombination is triggered dynamically. Namely, the recombination process correspond to condensation of the tachyon at the intersection. It is interesting to point out that in the degenerated case where the intersecting brane system becomes a brane-antibrane system (e.g. $\theta_1=\theta_2=0$, $\theta_3=1$), the tachyons are mapped to the well-studied tachyon of brane-antibrane systems. The situation where all light scalars have positive squared masses correspond to a non-supersymmetric intersection, which is nevertheless dynamically stable against recombination. Namely, the recombined 3-cycle has volume larger than the sum of the volumes of the intersecting 3-cycles.

Indeed, the different regimes of dynamics of scalars at intersections have a one-to-one mapping with the different relations between the volumes of intersecting and recombined 3-cycles [15]. Namely, the conditions to have or not tachyons are related to the angle criterion [16] determining which the particular 3-cycle having smaller volume. The supersymmetric situation corresponds to both the intersecting and recombined configurations having the same volume; the tachyonic situation corresponds to the recombined 3-cycle having smaller volume; the massive situation corresponds to the intersecting 3-cycle having smaller volume.

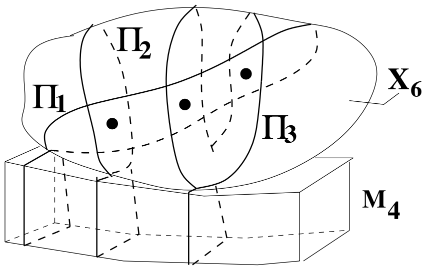

Figure 8: Compactification with intersecting D6-branes wrapped on 3-cycles.

4 Compact four-dimensional models

Once we have succeeded in describing configurations of D-branes leading to charged chiral fermions, in this section we employ them in building models with 4d gravity and gauge interactions. Although intersecting D6-branes provide 4d chiral fermions already in flat 10d space, gauge interactions remain 7d and gravity interactions remain 10d unless we consider compactification of spacetime.

The general kind of configurations we are to consider (see figure 8) is type IIA string theory on a spacetime of the form $\mathbf{M_4} \times \mathbf{X_6}$ with compact $\mathbf{X_6}$, and with stacks of N_a D6_a-branes with volumes of the form $M_4 \times \Pi_a$, with $\Pi_a \subset \mathbf{X_6}$ a 3-cycles. It is important to realize that generically 3-cycles in a 6d compact space intersect at points, so the corresponding wrapped D6-branes will intersect at M_4 subspaces of their volumes. Hence, compactification reduces the 10d and 7d gravitational and gauge interactions to 4d, and intersections lead to charged 4d chiral fermions. Also, generically two 3-cycles in a 6d space intersect several times, therefore leading to a replicated sector of opens strings at intersections. This is a natural mechanism to explain/reproduce the appearance of replicated families of chiral fermions in Nature!

4.1 Toroidal models

4.1.1 Construction

In this section we mainly follow [12], see also [11]. To start with the simplest configurations, consider compactifying on a six-torus factorized as $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$. Now we consider stacks of D6_a-branes (with a an index labeling the stack), spanning M_4 and wrapping a 1-cycle (n_a^i, m_a^i) in the i^{th} 2-torus. Namely, the a^{th} D6-brane wraps n_a^i, m_a^i times along the horizontal and vertical directions in the i^{th} two-torus, see figure 9

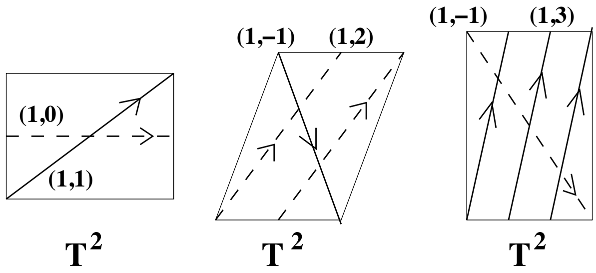

Figure 9: Examples of intersecting 3-cycles in $\mathbf{T^6}$.

for examples [1].

The general kind of configurations we are to consider (see figure 8) is thus type IIA string theory on a spacetime of the form $\mathbf{M_4} \times \mathbf{T^6}$, and with stacks of N_a D6$_a$-branes with volumes of the form $M_4 \times \Pi_a$, with $\Pi_a \subset \mathbf{X_6}$ a 3-cycle as described above. It is important to realize that generically 3-cycles in a 6d compact space intersect at points, so the corresponding wrapped D6-branes will intersect at M_4 subspaces of their volumes. Hence, compactification reduces the 10d and 7d gravitational and gauge interactions to 4d, and intersections lead to charged 4d chiral fermions.

Also, generically two 3-cycles in a 6d space intersect several times. Locally, each intersection is exactly of the form studied in section 3.1, therefore the construction leads to a replicated sector of open strings at intersections. This is a natural mechanism to explain/reproduce the appearance of replicated families of chiral fermions in Nature, as we show below. It is also important to notice that in compactifications, the angles between branes are derived quantities, and depend on the closed string moduli controlling the torus geometry. For instance, for a rectangular torus of radii R_1, R_2 along the horizontal and vertical directions, the angle between the 1-cycle $(1,0)$ and (n,m) is

$$\tan\theta = \frac{mR_2}{nR_1} \tag{11}$$

In this toroidal case, the intersection number is given by the product of the number of intersections in each 2-torus, and reads

$$I_{ab} \,=\, (n_a^1 m_b^1 - m_a^1 n_b^1) \times (n_a^2 m_b^2 - m_a^2 n_b^2) \times (n_a^3 m_b^3 - m_a^3 n_b^3) \tag{12}$$

It is useful to introduce the 3-homology class $[\Pi_a]$ of the 3-cycle Π_a, which can be

[1] These factorizable branes are not the most general possibility. Branes wrapped on non-factorizable cycles exist, and can be obtained e.g. by recombination of factorized branes. For simplicity, we will not use them in these lectures.

thought of as a vector of RR charges of the corresponding D6-brane. The 1-homology class of an (n, m) 1-cycle in a 2-torus is $n[a] + m[b]$, with $[a], [b]$ the basic homology cycles in $\mathbf{T^2}$. For a 3-cycle with wrapping numbers (n_a^i, m_a^i) we have

$$[\Pi_a] \;=\; \otimes_{i=1}^{3} \left(n_a^i\, [a_i] \,+\, m_a^i\, [b_i] \right) \tag{13}$$

The intersection number (12) is intersection number in homology, denoted $I_{ab} = [\Pi_a] \cdot [\Pi_b]$. This is easily shown using $[a_i] \cdot [b_j] = \delta_{ij}$ and linearity and antisymmetry of the intersection pairing.

With the basic data defining the configuration, namely $\mathbf{N_a}$ D6$_a$-branes wrapped on 3-cycles $[\boldsymbol{\Pi}_\mathbf{a}]$, with wrapping numbers $(\mathbf{n_a^i}, \mathbf{m_a^i})$ on each $\mathbf{T^2}$ and intersection numbers $\mathbf{I_{ab}}$, we can compute the spectrum of the model.

The closed string sector produces 4d $\mathcal{N} = 8$ supergravity. There exist different open string sectors:

$\mathbf{6}_a\mathbf{6}_a$ String stretched among D6-branes in the a^{th} stack produce 4d $U(N_a)$ gauge bosons, 6 real adjoint scalars and 4 adjoint Majorana fermions, filling out a vector multiplet of the 4d $\mathcal{N} = 4$ supersymmetry preserved by the corresponding brane.

$\mathbf{6}_a\mathbf{6}_b + \mathbf{6}_b\mathbf{6}_a$ Strings stretched between the a^{th} and b^{th} stack lead to I_{ab} replicated chiral left-handed fermions in the bifundamental representation $(N_a, \overline{N}_b)$. Negative intersection numbers lead to a positive number of chiral fermions with right-handed chirality. Additional light scalars may be present, with masses determined by the wrapping numbers and the $\mathbf{T^2}$ moduli.

Generalization for compact spaces more general than the 6-torus will be discussed in section 4.2. We have therefore obtained a large class of four-dimensional theories with interesting non-abelian gauge symmetries and replicated charged chiral fermions. Hence compactifications with intersecting D6-branes provide a natural setup in which string theory can produce gauge sectors with the same rough features of the Standard Model. In coming sections we explore them further as possible phenomenological models, and construct explicit examples with spectrum as close as possible to the Standard Model.

4.1.2 RR tadpole cancellation

String theories with open string sectors must satisfy a crucial consistency condition, known as cancellation of RR tadpoles. As mentioned above, D-branes act as sources for RR p-forms via the disk coupling $\int_{W_{p+1}} C_p$, see fig 3. The consistency condition amounts to requiring the total RR charge of D-branes to vanish, as implied by Gauss

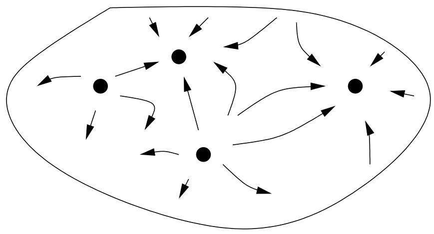

Figure 10: In a compact space, fluxlines cannot escape and the total charge must vanish.

law in a compact space (since RR field fluxlines cannot escape, figure 10). In our setup, the 3-cycle homology classes are vectors of RR charges, hence the condition reads

$$[\Pi_{\rm tot}] \;=\; \sum_a N_a\,[\Pi_a] \;=\; 0 \tag{14}$$

Equivalently, the condition of RR tadpole cancellation can be expressed as the requirement of consistency of the equations of motion for RR fields. In our situation, the terms of the spacetime action depending on the RR 7-form C_7 are

$$\begin{aligned} S_{C7} = & \int_{\mathbf{M_4}\times\mathbf{X_6}} H_8 \wedge *H_8 \,+\, \sum_a N_a \int_{\mathbf{M_4}\times\mathbf{\Pi_a}} C_7 \\ = & \int_{\mathbf{M_4}\times\mathbf{X_6}} C_7\wedge dH_2 \,+\, \sum_a N_a \int_{\mathbf{M_4}\times\mathbf{X_6}} C_7\wedge \delta(\Pi_a) \end{aligned} \tag{15}$$

where H_8 is the 8-form field strength, H_2 its Hodge dual, and $\delta(\Pi_a)$ is a bump 3-form localized on Π_a in $\mathbf{X_6}$. The equations of motion read

$$dH_2 \;=\; \sum_a N_a\,\delta(\Pi_a) \tag{16}$$

The integrability condition (14) is obtained by taking this equation in homology.

In the toroidal setup the RR tadpole conditions provide a set of constraints, given by

$$\begin{aligned} \textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 = &\;\; 0 \\ \textstyle\sum_a N_a n_a^1 n_a^2 m_a^3 = &\;\; 0 \text{ and permutations} \\ \textstyle\sum_a N_a n_a^1 m_a^2 m_a^3 = &\;\; 0 \text{ and permutations} \\ \textstyle\sum_a N_a m_a^1 m_a^2 m_a^3 = &\;\; 0 \end{aligned} \tag{17}$$

4.1.3 Anomaly cancellation

Cancellation of RR tadpoles in the underlying string theory configuration implies cancellation of four-dimensional chiral anomalies in the effective field theory in our con-

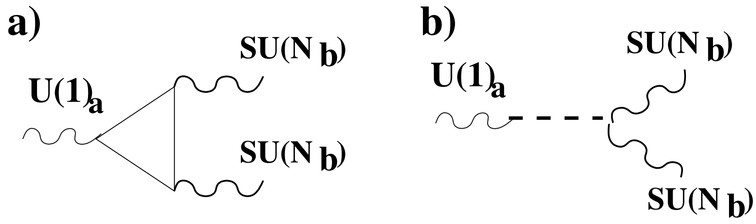

Figure 11: Triangle and Green-Schwarz diagrams contributing to the mixed $U(1)$ - non-abelian anomalies.

figurations. Recall that the chiral piece of the spectrum is given by I_{ab} chiral fermions in the representation $(N_a, \overline{N}_b)$ of the gauge group $\prod_a U(N_a)$.

Cubic non-abelian anomalies

The $SU(N_a)^3$ cubic anomaly is proportional to the number of fundamental minus antifundamental representations of $SU(N_a)$, hence it is proportional to

$$A_a = \sum_b I_{ab} N_b. \tag{18}$$

It is easy to check this vanishes due to RR tadpole cancellation: Starting with (14), we consider the intersection of $[\Pi_{\rm tot}]$ with any $[\Pi]$ to get

$$0 \;=\; [\Pi_a] \cdot \sum_b \, N_b \, [\Pi_b] \;=\; \sum_b N_b I_{ab} \tag{19}$$

as claimed [2].

Cancellation of mixed anomalies

The $U(1)_a$-$SU(N_b)^2$ mixed anomalies also cancel as a consequence of RR tadpole cancellation. They do so in a trickier way, namely the anomaly receives two non-zero contributions which cancel each other, see fig 11. Mixed gravitational triangle anomalies cancel automatically, without Green-Schwarz contributions.

The familiar field theory triangle diagrams give a contribution which, even after using RR tadpole conditions, is non-zero and reads

$$A_{ab} \;\simeq\; N_a \, I_{ab} \tag{20}$$

[2]It is interesting to notice that RR tadpole cancellation is slightly stronger than cancellation of cubic non-abelian anomalies. In fact, the former requires that the number of fundamental minus antifundamentals vanishes even for the cases $N_a = 1, 2$, where no gauge theory anomaly exists. This observation will turn out relevant in phenomenological model building in section 6.

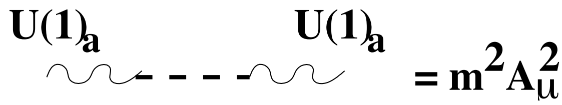

Figure 12: The $B \wedge F$ couplings lead to a $U(1)$ gauge boson mass term.

On the other hand, the theory contains contributions from Green-Schwarz diagrams, where the gauge boson of $U(1)_a$ mixes with a 2-form which subsequently couples to two gauge bosons of $SU(N_b)$, see figure 11. These couplings arise in the KK reduction of the D6-brane world-volume couplings $N_a \int_{\mathrm{D6}_a} C_5 \wedge \mathrm{tr}\, F_a$ and $\int_{\mathrm{D6}_b} C_3 \wedge \mathrm{tr}\, F_b^2$, as follows.

Introducing a basis $[\Lambda_k]$ and its dual $[\Lambda_{\tilde{l}}]$, we can define the KK reduced 4d fields

$$(B_2)_k = \int_{[\Lambda_k]} C_5 \quad , \quad \phi_{\tilde{l}} = \int_{[\Lambda_{\tilde{l}}]} C_3 \qquad \text{with } d\phi_{\tilde{l}} = -\delta_{k\tilde{l}} *_{4d} (B_2)_k \tag{21}$$

The KK reduced 4d couplings read

$$N_a q_{ak} \int_{4d} (B_2)_k \mathrm{tr}\, F_a \quad , \quad q_{b\tilde{l}} \int_{4d} \phi_{\tilde{l}} \mathrm{tr}\, F_b^2 \tag{22}$$

with $q_{ak} = [\Pi_a] \cdot [\Lambda_k]$, and similarly for $q_{b\tilde{l}}$. The total amplitude is proportional to

$$A_{ab}^{\mathrm{GS}} = -N_a \sum_k q_{ak} q_{b\tilde{l}} \delta_{k\tilde{l}} = \ \ldots \ = -N_a I_{ab} \tag{23}$$

leading to a cancellation between both kinds of contributions.

An important observation is that any $U(1)$ gauge boson with $B \wedge F$ couplings gets massive, with mass roughly of the order of the string scale, see fig 12. Such $U(1)$'s disappear as gauge symmetries from the low-energy effective field theory, but remain as global symmetries, unbroken in perturbation theory. Introducing the generators Q_a of the $U(1)$ inside $U(N_a)$, the condition that a $U(1)$ with generator $\sum_a c_a Q_a$ remains massless is

$$\sum_a N_a q_{ak} c_a = 0 \qquad \text{for all } k \tag{24}$$

Such $U(1)$ factors remain as gauge symmmetries of the low energy theory.

4.2 Generalization beyond torus: Model building with A-type branes

Clearly the above setup is not restricted to toroidal compactifications. Indeed one may take take any compact 6-manifold as internal space, for instance a Calabi-Yau threefold, which would lead to 4d $\mathcal{N}=2$ supersymmetry in the closed string sector. In this situation we should pick a set of 3-cycles Π_a on which we wrap N_a D6-branes (for instance special lagrangian 3-cycles of $\mathbf{X_6}$ if we are interested in preserving supersymmetry), making sure they satisfy the RR tadpole cancellation condition $\sum_a N_a[\Pi_a]=0$.

The final open string spectrum (for instance, in the case of supersymmetric wrapped D6-branes) arises in two kinds of sectors

$\mathbf{6}_a$-$\mathbf{6}_a$ Leads to $U(N_a)$ gauge bosons ($\mathcal{N}=1$ vector multiplets in the supersymmetric case) and $b_1(\Pi_a)$ real adjoint scalars (chiral multiplets in susy case).

$\mathbf{6}_a$-$\mathbf{6}_b$+$\mathbf{6}_b$-$\mathbf{6}_a$ We obtain I_{ab} chiral fermions in the representation $(N_a,\overline{N}_b)$ (plus light scalars, massless in supersymmetry preserving intersections). Here $I_{ab}=[\Pi_a]\cdot[\Pi_b]$.

Notice that the chiral spectrum is obtained in terms of purely topological information of the configuration, as should be the case.

Our whole discussion up to this point has simply been a pedagogical way of describing a general class of string compactifications. Namely, compactifications of type IIA string theory on Calabi-Yau threefolds with A-type D-branes. In the geometrical large volume regime, these are described as D-branes wrapped on special lagrangian 3-cycles, and reproduce the structures we have been discussing. A-type branes are extensively studied from the point of view of topological strings, with results of immediate application to our models. To name a few, the fact that such D-brane states do not have lines of marginal stability in Kahler moduli space, that their world-volume superpotential arises exclusively from worldsheet instantons, and their nice relation via mirror symmetry with type IIB compactifications with B-type D-branes, see next lecture. It is very satisfactory that a phenomenological motivation has driven us to consider a kind of configurations so interesting from the theoretical viewpoint as well.

The phenomenology of toroidal and non-toroidal models is quite similar to that of toroidal compactifications with D-branes, see next subsection. Thus, the later are in any event good toy model for many features of general compactifications with intersecting branes. This is particularly interesting since it is relatively difficult to construct explicit configurations of intersecting D6-branes in Calabi-Yau models (although some explicit examples have been discussed in [17, 18]).

4.3 Phenomenological features

We now turn to a brief discussion of the phenomenological properties natural in this setup [12].

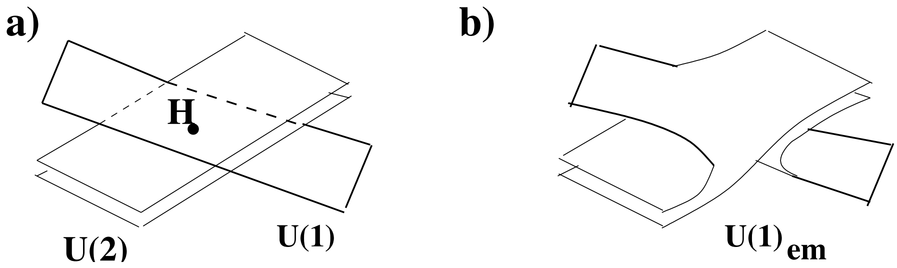

Figure 13: .

• Most models constructed in the literature are non-supersymmmetric. It is however possible to construct fully $\mathcal{N}=1$ supersymmetric models, see section 7. For non-supersymmetric models, unless alternative solutions to the hierarchy model are provided, the best proposal low string scale $M_s \simeq$ TeV to avoid hierarchy, along the lines of [19].

• The proton is stable in these models, since the $U(1)$ within the $U(3)$ color factor plays the role of baryon number, and is preserved as a global symmetry, exactly unbroken in perturbation theory. Non-perturbative effects breaking it arise from euclidean D2-branes wrapped on 3-cycles, and have the interpretation of spacetime gauge theory instantons, hence reproducing the non-perturbative breaking of baryon number in the Standard Model.

• These models do *not* have a natural gauge coupling unification, even at the string scale. Each gauge factor has a gauge coupling controlled by the volume of the wrapped 3-cycle. Gauge couplings are related to geometric volumes, hence their experimental values can be adjusted/reproduced in concrete models, rather than predicted by the general setup.

• There exists a geometric interpretation for the spontaneous electroweak symmetry breaking. In explicit models, the Higgs scalar multiplet arises from the light scalars at intersections, and parametrizes the possibility of recombining two intersecting cycles into a single smooth one, as shown in figure 13. In the process, the gauge symmetry is reduced, corresponding to a Higgs mechanism in the effective field theory. See [20] for further discussion.

• There is a natural exponential hierarchy of the Yukawa couplings. Yukawa couplings among the scalar Higgs and chiral fermions at intersections arise at tree level in the string coupling from open string worldsheet instantons; namely from string worldsheets spanning the triangle with vertices at the intersections and sides on the

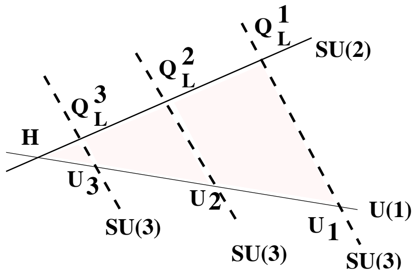

Figure 14: Geometric origin of the hierarchy of Yukawa couplings for different generations.

D-branes. Their value is roughly given by e^{-A}, with A the triangle area in string units. Since different families are located at different intersections, their triangles have areas increasing linearly with the family index, leading to an exponential Yukawa hierarchy, see fig 14. See e.g. [21] for further analysis of yukawa couplings in explicit models.

5 Orientifold models

The above constructed models are non-supersymmetric. One simple way to see it is that we start with type IIA string theory compactified on $\mathbf{X_6}$, and introduce D6-branes. Since RR tadpole cancellation requires that the total RR charge vanishes, we are forced to introduce objects with opposite RR charges, in a sense branes and antibranes, a notoriously non-supersymmetric combination.

An equivalent derivation of the result is as follows: If we would succeed in constructing a supersymmetric configuration of D6-branes, the system as a whole would be a supersymmetric BPS state of type IIA on $\mathbf{X_6}$. Since for a BPS state the tension is proportional to the RR charge, and the latter vanishes due to RR tadpole cancellation, the tension of the state must vanish. The only D6-brane configuration with zero tension is having no D6-brane at all. Hence the only supersymmetric configuration would be just type IIA on $\mathbf{X_6}$, with no brane at all.

These arguments suggest a way out of the impasse. In order to obtain $\mathcal{N} = 1$ supersymmetric compactifications we need to introduce objects with negative tension and negative RR charge, and which preserve the same supersymmetry as the D6-branes. Such objects exist in string theory and are orientifold 6-planes, O6-planes. Introduction of these objects leads to an interesting extension of the configurations above constructed, and will be studied in section 5.3. In particular we will use them to construct supersymmetric compactifications with intersecting D6-branes.

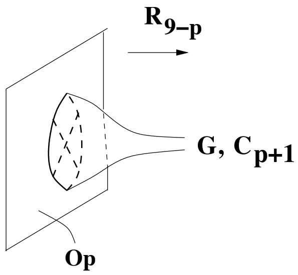

Figure 15: Diagram describing the interaction of an Op brane with closed string modes. The dashed cross denotes a crosscap, namely a disk with an identification of antipodal points in the boundary, so that the world-sheet is closed and unoriented.

5.1 Properties of O6-planes

To start, consider type IIA string theory on 10d flat space M_{10}, and mod it out by the so-called orientifold action $\Omega R(-)^{F_L}$. Here Ω is world-sheet parity, which flips the orientation of the fundamental strings; R is a $\mathbf{Z_2}$ geometric action, acting locally as $(x^5, x^7, x^9) \to (-x^5, -x^7, -x^9)$; finally $(-)^{F_L}$ is left-moving world-sheet fermion number, introduced for technical reasons.

The quotient theory contains a special subspace in spacetime, fixed under the geometric part R of the above action. Namely, it is a 7d plane defined by $x^5 = x^7 = x^9 = 0$, and spanned by the coordinates 0123468. This set of points fixed under the orientifold action is called an orientifold 6-plane (O6-plane), since it has six spatial dimensions (in general one can define other orientifold quotients of type II string theories, containing Op-planes of p spatial dimensions). Physically, it corresponds to a region of spacetime where the orientation of a string can flip (since a string at the O6-plane is identified, by the orientifold action, with itself with the opposite orientation). The description of string theory is the presence of orientifold planes is modified only by the inclusion of unoriented world-sheets, for instance with the topology of the Klein bottle.

Orientifold planes have some features similar to D-branes of the same dimension. For instance, Op-planes carry tension and are are charged under the RR $(p+1)$-form C_{p+1}. The diagram responsible for these couplings is shown in figure 15. For instance, and O6-plane is charged under the RR 7-form, and its charge is given by $Q_{\rm O6} = \pm 4$, in units where the D6-brane charge is $+1$. Here the two possible signs correspond to two different kinds of O6-planes; we will center on the negatively charged O6-plane in what follows. Also, O-planes preserve the same supersymmetry as a D-brane. This implies that there is a relation between the tension and charge of O-planes.

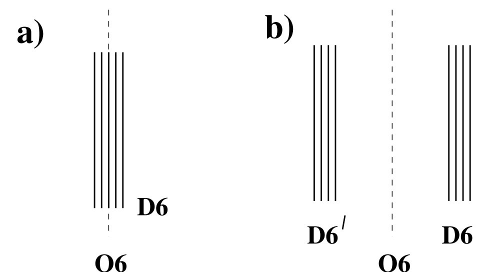

Figure 16: Configurations of D6-brane stacks parallel to an O6-plane. Figure a) show the situation where the branes are on top of the O6-plane, while figure b) corresponds to branes separated from it. Although the branes within a stack are coincident, they are shown slightly separated, for clarity.

There are however some important differences between O-planes and D-branes, the main one being that O-planes do not carry world-volume degrees of freedom. Hence, they are better regarded as part of the spacetime geometrical data, rather than dynamical objects.

5.2 O6-planes and D6-branes

It is interesting to include orientifold planes in compactifications or configurations with D-branes. These configurations are most simply described in the covering space of the orientifold quotient. Here we must include the images of the D-branes under the orientifold action, denoted by primed indices. The spectrum of open strings in the orientifold quotient theory is obtained by simply computing the spectrum in the covering space, and then imposing the identifications implied by the orientifold action (taking into account the flip in the open string orientation implied by the latter).

• Let us start by considering the simple situation of configurations of parallel D6-branes and O6-planes. Consider first a stack of n D6-branes on top of an O6-plane, see figure 16a. The open string spectrum before the orientifold action is given by the n^2 open string sectors, giving rise to an $U(n)$ vector multiplet. The orientifold action implies the following identification among the ab open strings

$$|ab\rangle \leftrightarrow \pm|ba\rangle \tag{25}$$

with the negative (positive) sign corresponding to the choice of negatively (positively) charged O6-plane. Centering on the former case, the physical states in the quotient correspond to the $n(n-1)/2$ antisymmetric linear combinations $(|ab\rangle - |ba\rangle)/2$. The

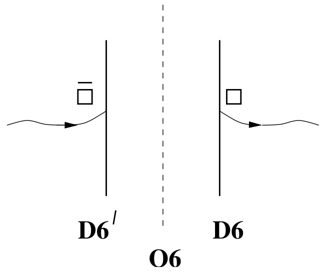

Figure 17: The orientifold projection relates the gauge groups on D-branes and images such that open string endpoint in the fundamental representation of one map to endpoints in the antifundamental of the other.

massless modes correspond to an $SO(n)$ vector multiplet with respect to the 16 supercharges unbroken by the O6/D6 configuration.

• Consider now a configuration of n coincident D6-branes, parallel but separated from the O6-plane. The configuration must include an orientifold image of the D-brane stack, namely a set of n D6'-branes, see figure 16b. The massless open string spectrum before the orientifold projection is given by a $U(n) \times U(n)'$ gauge group plus superpartners. The orientifold action implies and identification of the degrees of freedom in both $U(n)$ factors, so that only a linear combination survives. In the quotient, we just obtain an $U(n)$ vector multiplet (which agrees with the intuition that massless modes on D-branes are not sensitive to distant objects, hence the n D6-branes in the quotient do not notice, at the level of the massless spectrum, the distant O6-plane).

An important observation in the identification of the $U(n)$ factors is that, due to the orientation reversal, and open string starting on the D6-brane stack is mapped to an open string ending on the D6'-brane stack, and viceversa, see figure 17. This implies that the $U(n)$ is identified with $U(n')$ with the fundamental $\square$ mapping to the anti-fundamental $\overline{\square}'$, and viceversa. This will be important in the computation of open string massless spectra in more involved configurations (or in these simple ones, if one is interested in computing the massive spectrum).

Let us consider another local geometry similar to the above. Let us consider configurations of D6-branes orthogonal to the O6-plane in some of the directions, so that the D6-brane stack is still mapped to itself under the orientifold action. A configuration which appears often (since it preserves 8 supercharges) is when there are four dimensions not commonly along or commonly transverse to the objects. For instance,

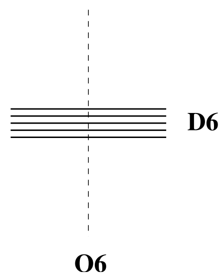

Figure 18: Configurations of D6-brane stacks with some directions orthogonal to an O6-plane.

consider an O6-plane along 0123456 and a D6-brane along 0123478, see figure 18. For one such stack of n D6-branes, the final gauge group (for a negatively charged O6-plane) is $USp(n)$ (hence n must be even) and fills out a vector multiplet with respect to the eight unbroken supersymmetries. In addition there is a hypermultiplet in the two-index symmetric (reducible) representation. The change of gauge group with respect to the case of the parallel O6/D6 system is due to an additional sign in the orientifold action [23].

Let us now consider situations with intersecting D6-branes (and their images) in the presence of O6-planes. All the D6-branes and O6-planes are taken to be parallel in four of their common dimensions, so that the intersections are geometrically of the kind studied above. There are several different situations to be considered, depending on the relative geometry of the intersection and the O6-plane. To simplify the discussion, we center on describing the gauge group and chiral fermions at intersections.

• Consider two stacks of D6-branes, labeled a and b, intersecting away from the O6-plane. The configuration also includes the image D6'-branes, labeled a', b', see figure 19a. Before the orientifold projection, the gauge group is $U(N_a) \times U(N_b) \times U(N_a)' \times U(N_b)'$. Also, the intersections in the figure (ignoring other possible intersections of the branes) provide 4d chiral fermions in the representation $(\square_a, \overline{\square}_b)$ and $(\square'_b, \overline{\square}'_a)$, due to the different relative orientation of the branes and their images. After the identification implied by the orientifold action (recalling the effect on fundamental representations and their images), we are left with a gauge group $U(N_a) \times U(N_b)$ and a 4d chiral fermion in the $(\square_a, \overline{\square}_b)$.

• Consider now the intersection of a stack of D6$_a$-branes with D6$'_b$-branes, namely the orientifold image of a stack of D6$_b$-branes, see figure 19b. Before the orientifold projection, the gauge group is $U(N_a) \times U(N_b)' \times U(N_a)' \times U(N_b)$, with 4d chiral fermions

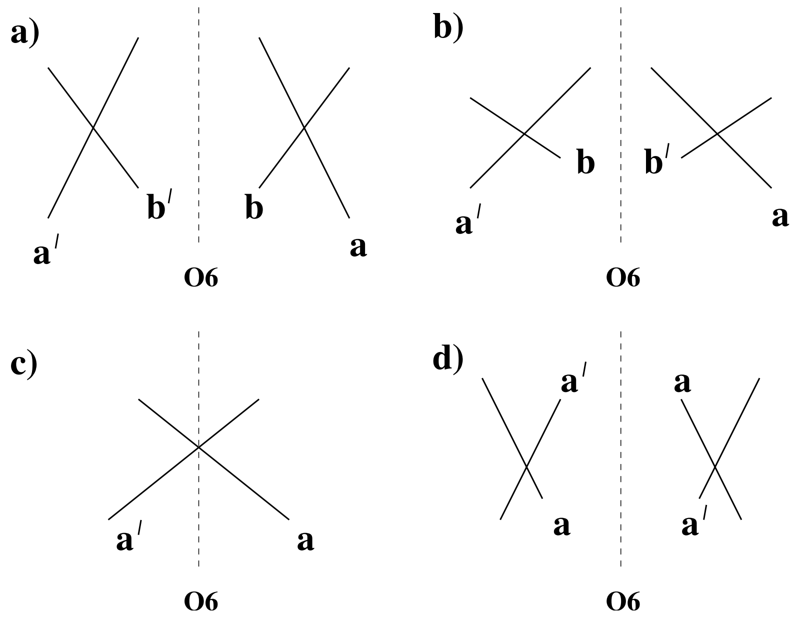

Figure 19: Configurations of intersecting D6-brane stacks in the presence of an O6-plane. Figure a) shows the intersections of two stacks a and b, away from the O6-plane. Figure b) shows the intersection of a stack a with the image of another b'. Figures c) and d) show the intersection of the stacks a and its image a' on top of the O6-plane and away from it, respectively.

in the representation $(\Box_a, \overline{\Box}'_b)$ and $(\Box_b, \overline{\Box}'_a)$. After the orientifold action, we have a gauge group $U(N_a) \times U(N_b)$ and a 4d chiral fermion in the $(\Box_a, \Box_b)$.

• Consider the intersection of a stack of D6$_a$-branes, with its own image, on top of the O6-plane, see figure 19c. Before the orientifold action, the gauge group is $U(N_a) \times U(N_a)'$ and there is a 4d chiral fermion in the $(\Box_a, \overline{\Box}'_a)$. The orientifold action reduces the gauge group to $U(N_a)$. The initial 4d chiral fermions thus transform under this as the tensor product of $\Box_a$ and $\overline{\Box}'_a = \Box_a$, namely $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a + \Box\Box_a$. After the orientifold projection (for a negatively charged O6-plane), however, only 4d fermions in the $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$ component survive.

• Consider finally the intersection of a stack of D6$_a$-branes, with D6$'_a$-branes, away from the O6-plane, see figure 19d. Before the orientifold action, the gauge group is $U(N_a) \times U(N_a)'$ and there are 4d chiral fermions in the $2(\Box_a, \overline{\Box}'_a)$, due to the two intersections. The orientifold action reduces the gauge group to $U(N_a)$, and identifies both intersections. Thus, the 4d chiral fermions in the quotient transform in the representation $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a + \Box\Box_a$.

It is easy to derive the spectra for intersections of generic D6-brane stack with stacks overlapping or orthogonal to the O6-plane. With these ingredients, we have enough information to describe compactifications with O6-planes and intersecting D6-branes.

5.3 Orientifold compactifications with intersecting D6-branes

5.3.1 Construction

Consider type IIA theory on e.g. a Calabi-Yau $\mathbf{X_6}$, and mod out the configuration by $\Omega R(-)^{F_L}$, where R is an antiholomorphic $\mathbf{Z_2}$ symmetry of $\mathbf{X_6}$. Hence it locally acts as $(z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3)$ on the CY complex coordinates, or as $(x^5, x^7, x^9) \to (-x^5, -x^7, -x^9)$ in suitable real ones. The set of fixed points of R are O6-planes, similar to those introduced above, with the difference that they are not flat in general, but rather wrap a (special lagrangian) 3-cycle in $\mathbf{X_6}$. Let us denote $\Pi_{\rm O6}$ the total 3-cycle spanned by the set of O6-planes in the configuration.

We now introduce stacks of N_a D6$_a$-branes, and their image D6$'_a$-branes, in the above orientifold quotient, see figure 20. They are wrapped on 3-cycles, denoted Π_a and Π'_a, respectively. The model is $\mathcal{N} = 1$ supersymmetric if all the D6-branes are wrapped on special lagrangian 3-cycles, see section 7 for concrete examples.

Taking into account the different sources of RR 7-form in the configuration, the RR

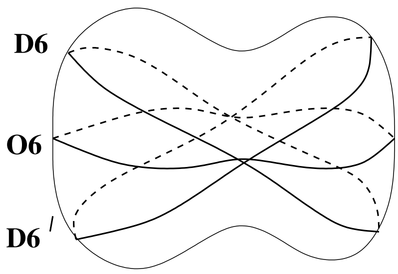

Figure 20: D6-branes and their images in an orientifold compactification.

tadpole cancellation conditions read [3]

$$\sum_a N_a\,[\Pi_a] \,+\, \sum_a N_a\,[\Pi_{a'}] \,-\, 4\times[\Pi_{\rm O6}] \,=\, 0 \tag{26}$$

The open string spectrum in orientifolded models can be easily computed. It only requires computing the relevant numbers of intersections, and if required how many lie on top of the orientifold planes. The results for the different sectors and the corresponding chiral spectra, assuming that no D6-branes are mapped to themselves under the orientifold action, are as follows

aa+a'a' Contains $U(N_a)$ gauge bosons and superpartners

ab+ba+b'a'+a'b' Contains I_{ab} chiral fermions in the representation $(N_a,\overline{N}_b)$, plus light scalars.

ab'+b'a+ba'+a'b Contains $I_{ab'}$ chiral fermions in the representation (N_a,N_b), plus light scalars.

aa'+a'a Contains $n_{\Box\!\Box_a}$ 4d chiral fermions in the representation $\Box\!\Box_a$ and $n_{\genfrac{}{}{0pt}{}{\Box}{\Box}_a}$ in the $\genfrac{}{}{0pt}{}{\Box}{\Box}_a$, with

$$n_{\Box\!\Box_a} = \frac{1}{2}(I_{aa'} - I_{a,{\rm O6}}) \quad , \quad n_{\genfrac{}{}{0pt}{}{\Box}{\Box}_a} = \frac{1}{2}(I_{aa'} + I_{a,{\rm O6}}) \tag{27}$$

where $I_{a,{\rm O6}} = [\Pi_a]\cdot[\Pi_{\rm O6}]$ is the number of aa' intersections on top of O6-planes.

As expected, the new RR tadpole conditions in the presence of O6-planes guarantee the cancellation of 4d anomalies of the new chiral spectrum, in analogy with the toroidal case. (In the orientifold case, mixed gravitational anomalies may receive Green-Schwarz contributions, see appendix in the first reference in [24]). The condition that a $U(1)$ remains massless is given by the orientifold version of (24)

$$\sum_a N_a(q_{ak} - q_{a'k})c_a = 0 \qquad \text{for all } k \tag{28}$$

[3]There are additional discrete constraints arising from cancellation of $\mathbf{Z_2}$-valued K-theory charges. We skip their discussion for the moment.

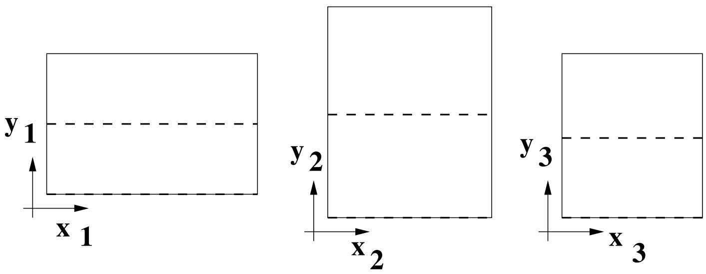

Figure 21: Orientifold 6-planes in the orientifold quotient of IIA on $\mathbf{T^6}$ by $\Omega R(-)^{F_L}$, with $R: y_i \to -y_i$.

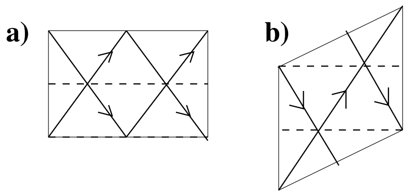

Figure 22: Cycles and their orientifold images in a rectangular and tilted 2-tori.

5.3.2 Toroidal orientifold models

A simple class of examples is provided by compactifications on $\mathbf{X_6} = \mathbf{T^6}$, with factorized $\mathbf{T^6}$, and with R given by the action $y_i \to -y_i$, where y_i are the vertical direction on each $\mathbf{T^2}$. This is a symmetry for rectangular two-tori or for two-tori tilted by a specific angle [25], see figure 22. Let us introduce a quantity $\beta = 0, \frac{1}{2}$, corresponding to the rectangular and tilted cases.

For a geometry with rectangular two-tori, as in figure 21, the set of fixed points is given by x_i arbitrary, $y_i = 0, R_{y_i}/2$, hence it has 8 components. They correspond to O6-planes wrapped on the 3-cycle with wrapping numbers $(n_i, m_i) = (1, 0)$, so that $[\Pi_{\rm O6}] = 8[a_1][a_2][a_3]$.

We now introduce D6$_a$-branes, with multiplicities N_a and wrapping numbers (n_a^i, m_a^i) of the D6-brane stacks. We also introduce their orientifold images, with wrapping numbers $(n_a^i, -m_a^i)$ for rectangular 2-tori, or $(n_a^i, -n_a^i - m_a^i)$ for tilted tori, see fig 22. To unify their description, we introduce $\tilde{m}_a = m_a + \beta n_a$, so that branes and images have wrapping numbers $(n_a, \tilde{m}_a)$ and $(n_a, -\tilde{m}_a)$ respectively.

The RR tadpole conditions are simple to obtain. In the case of rectangular two-tori, they are explicitly given by

$$\begin{array}{rl} \sum_a N_a n_a^1 n_a^2 n_a^3 = & 16 \\ \sum_a N_a n_a^1 m_a^2 m_a^3 = & 0 \text{ and permutations} \end{array} \tag{29}$$

The spectrum is as discussed above, with the specific intersection numbers computed using (12). Namely

Explicit examples are discussed in further sections.

6 Getting just the standard model

In this section we consider some of the phenomenologically most interesting constructions, where the chiral part of the low-energy spectrum is given by that of the Standard Model (SM). The models are based on brane configurations first discussed in [26, 21].

As we have discussed above, models without orientifold planes do not lead to the chiral spectrum of the Standard Model. In fact, there is a general argument [26] showing that any such construction always contains additional chiral fermions in $SU(2)$ doublets, beyond those in the SM, as follows. First notice that in such models, the gauge group is a product of unitary factors, so the electroweak $SU(2)$ must belong to a $U(2)$ factor in the gauge group. As mentioned in section 4.1.3, the RR tadpole cancellation conditions imply that the number of fundamentals and antifundamentals for each $U(N)$ factor must be equal, even for $U(2)$ (where the 2 and the $\overline{2}$ are distinguished by their $U(1)$ charge). Now in any such model with SM gauge group containing $SU(3) \times SU(2)$, the left-handed quarks must belong to a representation $3(3,\overline{2})$, contributing nine antifundamentals of $SU(2)$. The complete spectrum must necessarily contain nine fundamentals of $SU(2)$, three of which may be interpreted as left-handed leptons; the remaining six doublets are however exotic chiral fermions, beyond the spectrum of the SM.

The introduction of orientifold planes in the construction allows to avoid this issue in several ways, as we describe in this section. In fact, as a consequence, they allow to construct string compactifications with the chiral spectrum of just the SM. This is a remarkable achievement.

6.1 The $U(2)$ class

One possibility [26] is to exploit the fact that in orientifold models there are two different kinds of bifundamental representations that arise in the spectrum, namely

$(\Box,\overline{\Box})$ and $(\Box,\Box)$. This allows an alternative construction of the SM chiral fermion spectrum, satisfying the RR tadpole constraint on the spectrum without exotics, as follows. Consider realizing the three families of left-handed quarks as $(3,\overline{2})+2(3,2)$. This contributes three net $SU(2)$ doublets, hence the three $SU(2)$ doublets required in the model correspond simply to the three left-handed leptons.

Indeed, it is possible to propose a set of intersection numbers, such that any configuration of D6-branes wrapped on 3-cycles with those intersections numbers reproduces the chiral spectrum of the SM. Consider [26] four stacks of D6-branes, denoted a, b, c, d (and their images), giving rise to a gauge group $U(3)_a \times U(2)_b \times U(2)_c \times U(1)_d$. If the intersections numbers of the corresponding 3-cycles are given by

$$\begin{array}{llll} I_{ab}=1 & I_{ab'}=2 & I_{ac}=-3 & I_{ac'}=-3 \\ I_{bd}=0 & I_{bd'}=-3 & I_{cd}=-3 & I_{cd'}=3 \end{array} \tag{30}$$

then the chiral spectrum of the model has the non-abelian quantum numbers of the chiral fermions in the SM (plus right-handed neutrinos). In order to reproduce exactly the SM spectrum, one also needs to require that the linear combination of $U(1)$'s

$$Q_y = \frac{1}{6}Q_a - \frac{1}{2}Q_c + \frac{1}{2}Q_d \tag{31}$$

which reproduces the hypercharge quantum numbers, remains as the only massless $U(1)$ in the model.

It is important to emphasize that at this level, we have not constructed any explicit model. Rather, we have made a general proposal of what kind of structure one must implement in concrete examples to lead to the SM chiral spectrum. This is however a very useful step.

In [26] there is a large class of examples of models of this kind, constructed explicitly in terms of D6-branes on factorized 3-cycles in the orientifold of $\mathbf{T^6}$ discussed in section 5.3.2. To illustrate the discussion with an example, consider the model in [26] corresponding to the parameters

$$\beta^1=\beta^2=1 \quad ; \quad \epsilon=\rho=1 \quad ; \quad n_a^2=4 \quad ; \quad n_b^1=1 \quad ; \quad n_c^1=5 \quad ; \quad n_d^2=2 \tag{32}$$

The D6-brane configuration (without specifying the images) is given by

	N	(n^1,m^1)	(n^2,m^2)	$(n^3,\tilde{m}^3)$
a	3	$(1,0)$	$(4,1)$	$(1,\frac{1}{2})$
b	2	$(1,1)$	$(1,0)$	$(1,\frac{3}{2})$
c	1	$(5,3)$	$(1,0)$	$(0,1)$
d	1	$(1,0)$	$(2,-1)$	$(1,\frac{3}{2})$

Let us emphasize again that the proposal to obtain the SM from models with the intersection numbers above is not restricted to the toroidal orientifold setup. Indeed, they have been discussed in [17, 18] in the large volume regime of geometric compactifications, and in [27] a large class of models has been constructed in Gepner constructions. The latter models are fully supersymmetric, leading to almost MSSM spectra (differing from it in the structure of the non-chiral Higgs sector), showing that the proposal can be exploited to construct supersymmetric models as well.

6.2 The $USp(2)$ class

Another possible way to avoid the problem of the extra $SU(2)$ doublets, is to exploit the fact that D6-branes in the presence of orientifold planes may contain $USp(N)$ gauge factors (see section 5.2). For the latter, all representations are real, and RR tadpole conditions do not impose any constraint on the matter content. Since $USp(2) \equiv SU(2)$, it is possible to realize the electroweak $SU(2)$ in terms of such D6-brane with $USp(2)$ gauge group, and thus circumvent the constraints on the number of doublets.

Indeed such a construction is proposed in [21]. The SM spectrum would arise in terms of a configurations of four stacks of D6-branes, leading to a gauge group $U(3)_a \times USp(2)_b \times U(1)_c \times U(1)_d$, with intersection numbers

$$\begin{array}{cccccc} & I_{ab} = 3 & I_{ab'} = 3 & I_{ac} = -3 & I_{ac'} = -3 & \\ I_{db} = 3 & I_{db'} = 3 & I_{dc} = -3 & I_{dc'} = 3 & I_{bc} = -1 & I_{bc'} = 1 \end{array} \tag{33}$$

the $U(1)$ that needs to be massless in order to reproduce the SM hypercharge is

$$Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d \tag{34}$$

Moreover, explicit realizations of D6-branes on 3-cycles with those intersection numbers (and with massless hypercharge) have been constructed in toroidal orientifolds [21]. Let us consider an illustrative example, corresponding to $\rho = 1$ in that reference. The set of D6-branes (to which we should add the images) is specified by

	N	(n^1, m^1)	(n^2, m^2)	(n^3, m^3)
a	3	$(1, 0)$	$(1, 3)$	$(1, -3)$
b	1	$(0, 1)$	$(1, 0)$	$(0, -1)$
c	1	$(0, 1)$	$(0, -1)$	$(1, 0)$
d	1	$(1, 0)$	$(1, 3)$	$(1, -3)$

One needs to add additional branes to satisfy the RR tadpole condition, but this may be done with the latter having no intersection with the above one. Hence the additional D6-branes are decoupled, and we do not discuss them for simplicity. The above D6-brane configuration can preserve supersymmetry locally, but with supersymmetry is eventually broken by the additional decoupled D6-brane sector.

Notice that in this realization, the $USp(2)$ factor arises from the D6-brane b and its image, when they are coincident. Notice also that the D6-brane d and its image, and the a and c stacks, can be taken to coincide. Thus the above standard model configuration can be considered a spontaneously broken Pati-Salam theory, with original gauge group $U(4) \times USp(2)_L \times USp(2)_R$.

As emphasized above, the proposed intersection numbers may be realized in other contexts, also with or without supersymmetry. Explicit constructions of models with those intersection numbers, with supersymmetry will be studied in next section (see also [28], and [27] for Gepner model constructions).

7 Supersymmetric models

In this section we review some simple supersymmetric 4d chiral models of intersecting D6-branes, in [24] to which we refer the reader for additional details (see e.g. [29, 30, 31] for additional models in other orbifolds, see also [32] for early work on diverse non-chiral supersymmetric orbifolds with intersecting branes). For a more geometric description of the model, adapting the recipe in section 4.2, see [17].

7.1 Orientifold of $T^6/Z_2 \times Z_2$

In order to obtain supersymmetric models, one needs a sufficient number of O6-planes in the construction. One of the simplest possibilities is the $\Omega R(-)^{F_L}$ orientifold of the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ orbifold.

We consider type IIA theory on $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$, with generators θ, ω associated to the twists $v = (\frac{1}{2}, -\frac{1}{2}, 0)$ and $w = (0, \frac{1}{2}, -\frac{1}{2})$, hence acting as

$$\begin{aligned} \theta : &\quad (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega : &\quad (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{35}$$

where z_i are complex coordinates in the $\mathbf{T^6}$. The action projects out some of the moduli, in particular implies the $\mathbf{T^6}$ is factorizable. We mod out this theory by $\Omega R(-)^{F_L}$, where

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \tag{36}$$

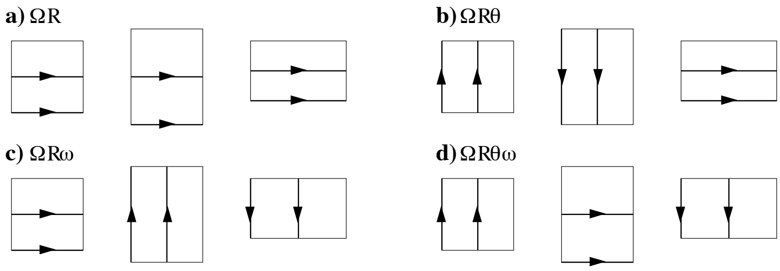

Figure 23: O6-planes in the orientifold of $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$.

The model contains four kinds of O6-planes, associated to the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, $\Omega R\theta\omega$, as shown in Figure 23 (for rectangular 2-tori). For simplicity we henceforth center on rectangular two-tori.

In order to cancel the corresponding RR tadpoles, we introduce D6-branes wrapped on three-cycles as in previous discussions. Also for simplicity we assume that each stack of D6-branes is passing through $\mathbf{Z_2} \times \mathbf{Z_2}$ fixed points. These extra projections are responsible for the fact that N D6-branes lead to an $U(N/2)$ gauge symmetry. The RR tadpole conditions have the familiar form

$$\sum_a N_a\,[\Pi_a] + \sum_a N_a\,[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{37}$$

where $[\Pi_{O6}]$ is the homology charge of the complete set of O6-planes. More explicitly, for instance for rectangular tori we have

$$\begin{aligned} \sum_a N_a n_a^1 n_a^2 n_a^3 - 16 &= 0 \\ \sum_a N_a n_a^1 m_a^2 m_a^3 + 16 &= 0 \\ \sum_a N_a m_a^1 n_a^2 m_a^3 + 16 &= 0 \\ \sum_a N_a m_a^1 m_a^2 n_a^3 + 16 &= 0 \end{aligned} \tag{38}$$

Skipping the details, the chiral spectrum is

Sector	Representation
aa	$U(N_a/2)$ vector multiplet
	3 Adj. chiral multiplets
$ab + ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab' + b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa' + a'a$	$-\frac{1}{2}(I_{aa'} - 4I_{a,O6})$ $\square\!\square$ fermions
	$-\frac{1}{2}(I_{aa'} + 4I_{a,O6})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 6$	$(1,0)$	$(3,1)$	$(3,-1)$
$N_b = 2$	$(0,1)$	$(1,0)$	$(0,-1)$
$N_c = 2$	$(0,1)$	$(0,-1)$	$(1,0)$
$N_d = 2$	$(1,0)$	$(3,1)$	$(3,-1)$
$N_{h_1} = 2$	$(-2,1)$	$(-3,1)$	$(-4,1)$
$N_{h_2} = 2$	$(-2,1)$	$(-4,1)$	$(-3,1)$
40	$(1,0)$	$(1,0)$	$(1,0)$

Table 1: D-brane magnetic numbers giving rise to an $\mathcal{N} = 1$ MSSM like model, in the $\mathbf{T^6}/(\mathbf{Z_2} \times \mathbf{Z_2})$ orientifold.

The condition that the system of branes preserves $N = 1$ supersymmetry is that each stack of D6-branes is related to the O6-planes by a rotation in $SU(3)$, see section 3.2. More specifically, denoting by θ_i the angles the D6-brane forms with the horizontal direction in the i^{th}two-torus, supersymmetry preserving configurations must satisfy

$$\theta_1 + \theta_2 + \theta_3 = 0 \tag{39}$$

For fixed wrapping numbers (n^i, m^i), the condition translates into a constraint on the ratio of the two radii on each torus. For rectangular tori, denoting $\chi_i = (R_2/R_1)_i$, with R_2, R_1 the vertical resp. horizontal directions, the constraint is

$$\arctan(\chi_1 \frac{m_1}{n_1}) + \arctan(\chi_2 \frac{m_2}{n_2}) + \arctan(\chi_3 \frac{m_3}{n_3}) = 0 \tag{40}$$

To provide an illustrative example, we consider a model [30] containing a sector of branes leading to the SM fields (belonging to the $USp(2)$ class in section 6.2, plus an additional set of branes required for RR tadpole cancellation (and contributing vector-like exotic matter in the spectrum). The set of branes is given in table 7.1

The spectrum is fairly complicated. However, under recombination of the branes h_1, h_2 and images, it has the simpler form given in table 7.1.

We hope that these examples suffice to illustrate the flexibility of the techniques we have discussed and allows the reader to safely jump into the literature for further details.

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times [USp(40)]$	Q_a	Q_h	Q'
(ab)	F_L	$3(4,2,1)$	1	0	1/3
(ac)	F_R	$3(\bar{4},1,2)$	-1	0	$-1/3$
(bc)	H	$(1,2,2)$	0	0	0
(bh)		$2(1,2,1)$	0	-1	2
(ch)		$2(1,1,2)$	0	+1	-2

Table 2: $\mathcal{N} = 1$ spectrum derived from the D-brane content of table 7.1 after D-brane recombination. There is no chiral matter arising from ah, ah', hh' or charged under $USp(40)$. The generator of $U(1)'$ is now given by $Q' = \frac{1}{3}Q_a - 2Q_h$.

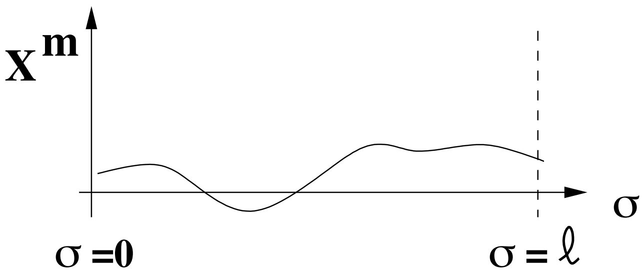

Figure 24: A string configuration is specified (in the light-cone gauge) by the position $X^i(\sigma)$ in transverse space for the point at the coordinate value σ along the string.

A Spectrum of open strings

A.1 Single D-brane in flat 10d space

In this appendix we describe a simplified calculation of the spectrum of open strings for a configuration of a single type II Dp-brane in flat 10d space.

In string theory the physical degrees of freedom for the string oscillation (in the light-cone gauge) are described by a set of functions $X^m(\sigma,t)$, which, at each (world-sheet) time t, define the graph of the string oscillation in the $i^{\rm th}$ transverse dimension, with $m = 2,\ldots,9$. The coordinate σ parametrizes the length of the string, and runs from 0 to $\ell = 4\pi\alpha' p^+$, where $\alpha' = M_s^{-2}$ is the inverse of the string tension, and p^+ is the light cone momentum. This is shown in figure 24. For each such transverse direction, denoted generically $X(\sigma,t)$, one can perform a general mode expansion

$$X(\sigma,t) = x + \frac{p}{p^+}t + i\sqrt{\frac{\alpha'}{2}}\sum_\nu \frac{\alpha_\nu}{\nu}\,\exp[-\pi i\,\nu\,(\sigma+t)/\ell] + i\sqrt{\frac{\alpha'}{2}}\sum_{\tilde\nu}\frac{\tilde\alpha^i_{\tilde\nu}}{\tilde\nu}\,\exp[-\pi i\,\tilde\nu\,(\sigma-t)/\ell]$$

We need to impose that the open string endpoints can move freely along the coordinates spanned by the D-brane world-volume, denoted X^μ, with $\mu = 2,\ldots,p$, but are fixed at the D-brane location in the transverse coordinates, denoted X^i, with $i = p+1,\ldots,9$. This is implemented by the boundary conditions (of Neumann and Dirichlet type)

$$\begin{array}{cccc} \mu = 2,\ldots,p & \partial_\sigma X^\mu(\sigma,t) = 0 & \text{at } \sigma = 0,\ell & NN \\ i = p+1,\ldots,9 & \partial_t X^i(\sigma,t) = 0 & \text{at } \sigma = 0,\ell & DD \end{array} \qquad (41)$$

Imposing these constraints, the expansions become

$$\begin{aligned} X^\mu(\sigma,t) &= x^\mu + \frac{p^\mu}{p^+}t + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha^\mu_n}{n}\cos[\pi n\sigma/\ell]\exp[-\pi int/\ell] \\ X^i(\sigma,t) &= x^i + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha^i_n}{n}\sin[\pi n\sigma/\ell]\exp[-\pi int/\ell] \end{aligned} \qquad (42)$$

The parameters x^μ are arbitrary, while x^i must correspond to the coordinates of the D-brane in the corresponding dimension. Hence the string centre of mass is localized on the D-brane world-volume, as announced.

The oscillation modes α^μ_n, α^i_n satisfy the commutation relations

$$[\alpha^\mu_n, \alpha^\nu_{-m}] = n\delta_{n,m}\delta^{\mu\nu} \quad ; \quad [\alpha^i_n, \alpha^j_{-m}] = n\delta_{n,m}\delta^{ij} \qquad (43)$$

corresponding to one infinite set of decoupled harmonic oscillators, for each spacetime dimension.

Recall that type II superstrings also have fermionic oscillation degrees of freedom. They can be similarly described in terms of an infinite set of decoupled fermionic harmonic oscillators, for each spacetime dimension. Hence we have an additional set of operators, $\Psi^\mu_{n+\nu}$, Ψ^i_{n+r}, with $r = \frac{1}{2}, 0$ for fermions with NS or R boundary (or periodicity) conditions, obeying the anticommutation relations

$$\{\Psi^\mu_{n+r}, \alpha^\nu_{-(m+r)}\} = \delta_{n,m}\delta^{\mu\nu} \quad ; \quad \{\Psi^i_{n+r}, \Psi^j_{-(m+r)}\} = \delta_{n,m}\delta^{ij} \qquad (44)$$

To construct the Hilbert space of string oscillation states, one first defines a vacuum state given by the product of groundstates of the infinite (bosonic and fermionic) harmonic oscillators, namely annihilated by all positive modding oscillators. Next one builds physical states by applying raising operators, corresponding to negative modding oscillators. In string theory, each string oscillation quantum state corresponds to a particle in spacetime. Its spacetime mass is given by

$$\alpha' M^2 = N_B + N_F - \frac{1}{2}r(1-r) \tag{45}$$

where N_B and N_F are the bosonic and fermionic oscillator number operators

$$N_B = \sum_{n=1}^{\infty}\sum_{\mu,i} \alpha_{-n}\alpha_n \quad ; \quad N_F = \sum_{n=0}^{\infty}\sum_{\mu,i}(n+r)\Psi_{-(n+r)}\Psi_{n+r}. \tag{46}$$

We will be interested in the lightest (in fact, massless, states).

In the NS sector (where world-sheet fermions satisfy NS periodicity conditions) there are no fermion zero modes and the vacuum is non-degenerate. The lightest states, along with their mass and their interpretation as particles is the $(p+1)$-dimensional D-brane world-volume, are

State	$\alpha' M^2$	GSO proj.	$(p+1)$-dim.field
$\vert 0\rangle$	$-\frac{1}{2}$	Out	-
$\psi^{\mu}_{-\frac{1}{2}}\vert 0\rangle$	0	OK	A_μ
$\psi^{i}_{-\frac{1}{2}}\vert 0\rangle$	0	OK	ϕ^i

We have also indicated the effect of the GSO projection (required from open-closed duality and the GSO projection for closed strings) on these fields. Notice that the tachyonic mode in the first line is projected out and removed from the physical spectrum, while the massless modes survive.

In the R sector, there are fermion zero modes Ψ_0^μ, Ψ_0^i. Since their application has zero cost in energy, the vacuum state is degenerate. Denoting zero modes collectively by Ψ_0^k, they satisfy the algebra $\{\Psi_0^k, \Psi_0^l\} = \delta^{kl}$. This implies that the degenerated vacuum states form a representation of this Clifford algebra, namely they transform as spinors of its $SO(8)$ invariance group. Hence the groundstates can be labeled by the two $SO(8)$ spinor representations of opposite chiralities, denoted $\mathbf{8}_{\rm S}$, $\mathbf{8}_{\rm C}$. The corresponding particles in the $(p+1)$-dimensional world-volume transform in the representations of the Lorentz group obtained by decomposing the representations of $SO(8)$ under $SO(p-1)$ (the subgroup of $SO(p+1)$ manifest in light-cone gauge). This corresponds to a set of $(p+1)$-dimensional fermions, whose detailed structure is dimension-dependent, but straightforward to determine in each specific case. The set of light states, along with their masses, behaviour under GSO, and $(p+1)$-dimensional interpretation, are

State	$\alpha' M^2$	GSO proj.	$(p+1)$-dim.field
$\mathbf{8}_{\rm S}$	0	Out	-
$\mathbf{8}_{\rm C}$	0	OK	λ_α

The final result is that the set of massless particles on the Dp-brane world-volume is given by a $U(1)$ gauge boson, $9-p$ real scalars and some fermions. The scalars (resp. fermions) can be regarded as Goldstone bosons (resp. Goldstinos) of the translational symmetries (resp. supersymmetries) of the vacuum broken by the presence of the D-brane. The open string sector fills out a $U(1)$ vector multiplet with respect to the 16 supersymmetries unbroken by the D-brane.

As described in the main text, when n parallel D-branes overlap, the world-volume gauge theory is enhanced to an $U(n)$ gauge group, and matter fields transform in the adjoint representation.

A.2 Open string spectrum for intersecting D6-branes

In this section we carry out the computation of the spectrum of open strings in the configuration of two stacks of intersecting D6-branes [13]. In particular, we explicitly show the appearance of 4d chiral fermions from the sector of open strings stretching between the different D6-brane stacks. The key point in getting chiral fermions is that the non-trivial angles between the branes removes fermion zero modes in the R sector, and leads to a smaller Clifford algebra.

As discussed above, the spectrum of states for open strings stretched between branes in the same stack is exactly as in section 2.2. It yields an $U(n_a)$ vector multiplet in the 7d world-volume of the $a^{\rm th}$ D6-brane stack.

We thus center in the computation of the spectrum of states for open strings stretched between two stacks a, b. The open string boundary conditions for the coordinates along M_4 are of the NN kind, and lead to the oscillators α_n^μ, Ψ_{n+r}^μ. For the directions where the branes form non-trivial angles, for instance in the 45 2-plane, we have boundary conditions

$$\begin{aligned} \partial_\sigma X^4|_{\sigma=0} &= 0 \\ \partial_t X^5|_{\sigma=0} &= 0 \\ \left(\cos\pi\theta^1\,\partial_\sigma X^4 + \sin\pi\theta^1\,\partial_\sigma X^5\right)|_{\sigma=\ell} &= 0 \\ \left(-\sin\pi\theta^1\,\partial_t X^4 + \cos\pi\theta^1\,\partial_t X^5\right)|_{\sigma=\ell} &= 0 \end{aligned} \qquad (47)$$

where $\pi(\theta_1)_{ab}$ is the angle from the $a^{\rm th}$ to the $b^{\rm th}$ D6-brane, written $\pi\theta_1$ for short. One has similar expression for the coordinates associated to the remaining two-planes.

It is convenient to define complex coordinates $Z^i = X^{2i+2} + iX^{2i+3}$, $i = 1,2,3$. The boundary conditions for ab open strings thus read

$$\partial_\sigma(\mathrm{Re}\,Z^i)|_{\sigma=0} = 0 \qquad ; \quad \partial_t(\mathrm{Im}\,Z^i)|_{\sigma=0} = 0$$

$$\partial_\sigma[\mathrm{Re}\,(e^{i\theta_i}Z^i)]|_{\sigma=0} = \ell \qquad ; \quad \partial_t[\mathrm{Im}\,(e^{i\theta_i}Z^i)]|_{\sigma=\ell} = 0 \tag{48}$$

These boundary conditions shift the oscillator moddings by an amount $\pm\theta^i$. The oscillator operators (which are now associated to complex coordinates) are $\alpha^i_{n+\theta_i}$, $\alpha^{\bar{i}}_{n-\theta_i}$, $\Psi^i_{n+r+\theta_i}$, $\Psi^{\bar{i}}_{n+r-\theta_i}$. It is important to point out that the centre of mass degrees of freedom are frozen in these directions, so that the open strings are localized at the intersection between the D6-branes.

All these operators satisfy decoupled harmonic oscillator (anti)commutation relations. As before, the Hilbert space of string oscillation modes is obtained by first constructing a vacuum (annihilated by all positive modding oscillators) and then applying creation operators to it (corresponding to negative modding oscillators). Each oscillation state corresponds to a particle that propagates on the 4d intersection of the D6-brane world-volumes. Its spacetime mass is given by

$$\alpha' M^2 = N_B + N_F + E_0 \tag{49}$$

where N_B and N_F are the oscillator numbers, and $E_0 = -\frac{1}{2}(1+\sum_i \theta_i)$ in the NS sector and $E_0 = 0$ in the R sector (in the normal ordering here and in what follows, we have assumed that $0 \leq \theta_i \leq 1$).

In the NS sector, the groundstate is non-degenerate. The lightest states surviving the GSO projection are (for the above range of θ_i)

State	$\alpha' M^2$	4d field
$\Psi^1_{-\frac{1}{2}+\theta_1}\|0\rangle$	$\frac{1}{2}(-\theta_1+\theta_2+\theta_3)$	Scalar
$\Psi^2_{-\frac{1}{2}+\theta_2}\|0\rangle$	$\frac{1}{2}(\theta_1-\theta_2+\theta_3)$	Scalar
$\Psi^3_{-\frac{1}{2}+\theta_3}\|0\rangle$	$\frac{1}{2}(\theta_1+\theta_2-\theta_3)$	Scalar
$\Psi^1_{-\frac{1}{2}+\theta_1}\Psi^2_{-\frac{1}{2}+\theta_2}\Psi^3_{-\frac{1}{2}+\theta_3}\|0\rangle$	$1-\frac{1}{2}(-\theta_1-\theta_2-\theta_3)$	Scalar

These scalars are complexified by similar scalars in the *ba* open string sector. Hence we obtain complex scalars, with masses given above, in the bi-fundamental representation $(n_a, \overline{n}_b)$ of the $U(n_a)\times U(n_b)$ gauge factor.

In the R sector, there are two fermion zero modes associated to the M_4 directions, hence the vacuum is degenerate. They satisfy a Clifford algebra, so the vacuum fills out two opposite-chirality spinor representations of $SO(2)$. Denoting them by $\pm\frac{1}{2}$, where the label corresponds to the 4d chirality, the $+\frac{1}{2}$ state is projected out by the GSO projection, while the $-\frac{1}{2}$ state survives. Taking into account a similar state surviving in the *ba* sector, in total we have a 4d left-handed chiral fermion in the bi-fundamental representation $(n_a, \overline{n}_b)$. The chirality of the fermion is determined by the orientation of the intersection.

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