Facets of Integrability
Workshop Schedule

Monday, January 21

9:30am **Organizers**
Opening and remarks from the organizers

10:00am **Philippe Di Francesco**
Integrable Combinatorics

11:30am **Anton Zabrodin**
Integrability of Laplacian growth in a channel and Hurwitz numbers

12:30pm **Lunch**

2:15pm **Giovanni Felder**
On the universality of random normal matrices

3:45pm **Seung-Yeop Lee**
Topology of quadrature domains: 2D Coulomb gas with algebraic potential

Tuesday, January 22

10:00am **Rick Kenyon**
Integrability in the Ising model

11:30am **Didina Serban**
Algebraic Bethe ansatz for long range spin chains

12:30pm **Lunch**

2:15pm **Alexander Its**
Painlevé Transcendents and their appearance in Physics and Random Matrix Theory

3:45pm **Pavel Bleher**
Orthogonal polynomials in the normal matrix model with a cubic potential

Wednesday, January 23

10:00am **John Harnad**
KP Tau functions, Convolution Flows, and applications

11:30am **Denis Bernard**
Repeated non-demolition measurements and a detour into quantum noises

12:30pm **Lunch**

2:15pm **Nikita Nekrasov**
BPS/CFT correspondence

4:00pm **Ivan Kostov**
On the three-point function of heavy non-BPS operators in N=4 SYM

Thursday, January 24

10:00am **Leon Takhtajan**
Complexified Liouville Equation

11:30am **Sergei Lukyanov**
Classical conformal blocks and Painleve IV

12:30pm **Lunch**

2:15pm **Konstantin Khanin**
On directed polymers and universality of the KPZ class

4:00pm **Andrei Okounkov**
Deep waters of quantum cohomology; SCGP-Math colloquium

6:00pm **Spanish Wine and Tapas Tasting**
SCGP Café. Spanish wine, traditional tapas, and music.

Friday, January 25

10:00am **Alexei Morozov**
On integrability of knots

11:30am **Kazuhiro Hikami**
Cluster algebra and complex volume of 2-bridge knot

12:30pm **Lunch**

2:15pm **Alexei Tsvelik**
Realization of 2-channel Kondo effect in a junction of three quantum Ising chains

4:00pm **Paul Fendley**
Topological Order in Spin Systems

Saturday, January 26

10:00am **Vladimir Kazakov**
Hirota dynamics of quantum integrability

11:30am **Alexander Polyakov**
Unstable and Sensitive Vacua

12:30pm **Lunch**

2:15pm **Igor Krichever**
Universal Whitham heirarchy and its applications

4:00pm **Andrea Cappelli**
Conformal Field Theory of Composite Fermions in the Quantum Hall

6:00pm **Dinner Celebration in honor of Paul Wiegmann's 60th birthday**
SCGP Café. Complimentary for all registered participants

Sunday, January 27

PLEASE NOTE THERE WILL BE AN INFORMAL DISCUSSION FROM 10:00AM – 1:00PM ON SUNDAY, JANUARY 27TH, AS OPPOSED TO SCHEDULED TALKS. LECTURERS AND TOPICS INCLUDE:

Hratchia Babujian
The Off-Shell Bethe Ansatz in 2d CFT and 1+1 Integrable QFT

Abstracts:

Pavel Bleher:

We consider the normal matrix model with a cubic potential. The model is illdefined, and in order to regularize it, Elbau and Felder introduced a model with a cut-off and corresponding system of orthogonal polynomials with respect to a varying exponential weight on the cut-off region on the complex plane. In the present work we show how to define orthogonal polynomials on a specially chosen system of infinite contours on the complex plane, without any cut-off, which satisfy the same recurrence algebraic identity that is asymptotically valid for the orthogonal polynomials of Elbau and Felder. The main goal of our work is to develop the Riemann-Hilbert (RH) approach to the orthogonal polynomials under consideration and to obtain their asymptotic behavior on the complex plane as the degree n of the polynomial goes to infinity. As the first step in the RH approach, we introduce an auxiliary vector equilibrium problem for a pair of measures (μ1, μ2) on the complex plane. We then formulate a 3 × 3 matrix valued RH problem for the orthogonal polynomials in hand, and we apply the nonlinear steepest descent method of Deift-Zhou to the asymptotic analysis of the RH problem. The central steps in our study are a sequence of transformations of the RH problem, based on the equilibrium vector measure (μ1, μ2), and the construction of a global parametrix. The main result of this work is a derivation of the large n asymptotics of the orthogonal polynomials on the whole complex plane. We prove that the distribution of zeros of the orthogonal polynomials converges to the measure μ1, the first component of the equilibrium measure. We also obtain analytical results for the measure μ1 relating it to the distribution of eigenvalues in the normal matrix model which is uniform in a domain bounded by a simple closed curve. This is a joint project with Arno Kuijlaars.

Andrea Cappelli:

The Jain theory of so-called hierarchical Hall states is reconsidered using Hansson et al. exact relations between wavefunctions and conformal field theory correlators. It is shown that the underlying conformal theory is precisely given by the minimal models of W-infinity symmetry introduced earlier. This theory suggests that under certain hypotheses the quasiholes could possess non-Abelian fractional statistics.

Giovanni Felder:

I will report on joint work with Roman Riser on universality of local correlation functions for random normal matrices with quadratic potential. This a very simple but non-trivial special case of a class of models considered by Wiegmann and Zabrodin, in which the eigenvalues, as the size of the matrix tends to infinity, are uniformly distributed on a bounded domain of the complex plane, in our case the interior of an ellipse. Universality means here that the local statistics of eigenvalues at a point, either in the interior or at the boundary of the domain, is essentially

independent of the parameters of the potential and of the point of the domain. We compute the kernel governing the statistics both in the interior and at the boundary.

John Harnad:

The idea of "convolution symmetries" and "convolution flows" in the context of integrable hierarchies is very natural when the dynamics are viewed, as in the Sato-Segal-Wilson approach to the KP hierarchy, as commuting flows on an infinite Hilbert space Grassmannian. Since convolution products on the Hilbert space of square integrable functions, say, on the unit circle are represented within the natural orthonormal basis by a diagonal operator, such flows are, in some sense, more natural than the "shift" flows generating the KP dynamics in the more usual approach. (This was pointed out, in particular, by Paul Wiegman, who emphasized that it is natural to choose flow generators that are self-adjoint, while the "shift flow" generators are not.) It turns out that it is possible, however, to find intertwining operators between these two inequivalent abelian group actions. In fact, there are infinitely many of them, expressed, within the natural orthonormal basis, by doubly infinite VanderMonde matrices, with different possible choices of parameters. The cost of introducing such an intertwining operator, within the fermi Fock space setting, is that one (or two) of the vacuum vectors appearing in the expression of the tau function as an expectation value of a suitably defined flow operator, is replaced by a "dressed boundary state". Such states have appeared, in particular, in the works of Bettelheim-Abanov-Wiegmann, and also in a completely different setting in the work of Okounkov et al on crystal growth and on generating functions for Gromov-Witten invariants. A finite dimensional analog of such "dressed boundary states" can moreover be identified in the construction of muiltsolition solutions of the KP hierarchy. Convolution symmetries can also be used to relate the partition functions of externally coupled matrix models of the Brezin-Hikami type, as well as two-matrix models, to the more standard exponential deformations of self-coupled ones. This can be extended to more general, non-exponential couplings as well. (This talk is based on joint work with Alexander Orlov.)

Alexander Its:

The classical Painlevé equations have been playing an increasingly important role in physics since 1970s-1980s works of Barouch, McCoy, Tracy, and Wu, and of Jimbo, Miwa, Mori and Sato devoted to the quantum correlation functions. Since the early nineties, the Painlevé transcendents have become a major player in the theory and applications of Random Matrices as well (the pioneering works of Brézin and Kazakov, Douglas and Shenker, Gross and Migdal, Mehta and Mahoux, and Tracy and Widom). In this talk we will try to review these, and also some of the more recent results concerning Painlevé transcendents and their appearance in random matrices, statistical mechanics and quantum field theory. We will present a unified point view on the subject based on the Riemann-Hilbert method. The emphasis will be made on the various double scaling limits related to the universal properties of random matrices for which Painlevé functions provide an adequate “special function environment”

Rick Kenyon:

This is joint work with Robin Pemantle. We define an integrable recurrence relation, a cousin of the octahedron recurrence, which we call the hexahedron recurrence. We show how the Kashaev difference equation arising in the Ising model Yang-Baxter equation is a specialization of the hexahedron recurrence. This allows us to illustrate in particular a cluster structure underlying the planar Ising model.

Ivan Kostov:

We derive an analytic expression for the correlation function of three heavy (classical) non-BPS operators in N = 4 super-Yang– Mills theory at weak coupling. The three operators belong to three different su(2) sectors and are dual to three classical strings moving on the sphere. The computation is based on the reformulation of the problem in terms of the Bethe ansatz for periodic XXX spin-1/2 chains. In these terms, the three operators are described by long-wavelength excitations over the ferromagnetic vacuum, for which the number of the overturned spins is a finite fraction of the length of the chain.

Seong-Yeop Lee:

A large number of particles on the plane interacting by 2 dimensional Coulomb repulsion tend to condense over the region that we call droplet. Such droplets appear in random normal matrix theory and Hele-Shaw flows, and is closely related to quadrature domains. We study the topology of such droplets when the external potential is algebraic. The results have several applications, e.g. in the 2 dimensional inverse moment problem and in gravitational lensing. A joint work with Nikolai Makarov.

Sergei Lukyanov:

Conformal blocks are basic functions through which correlation functions of 2D CFT are constructed.
In the limit when the central charge and all conformal dimensions go to infinity, conformal blocks exponentiate in terms of the so called classical conformal blocks. The latter are of much interest, in particular they provide solution to the monodromy problem of
ordinary linear differential equations with regular singularities.
We will discuss how the classical conformal blocks can be expressed through certain solutions of the Painleve VI equation.

Didina Serban:

Motivated by the computation of the correlation functions in N=4 gauge theory in the planar limit we are studying the possibility of applying the algebraic Bethe ansatz technology to long-range interacting spin chains which are deformations of the XXX spin chain. Modulo some subtleties with the periodic boundary conditions, which can be neglected on long chains, we are able to construct the eigenvectors and the inner products associated with these spin chains.

Alexei Tsvelik:

It is shown that a junction of three quantum Ising chains (Y-junction) can be described as the 2-channel Kondo model in a box which size is of the order of the Ising model correlation length with spin S=1/2 localized at the junction. The local spin is composed of the zero energy boundary Majorana modes of the Ising models.

Anton Zabrodin:

We report on the integrable structure of the 2D Laplacian growth problem with zero surface tension in an infinite channel with periodic boundary conditions in the transverse direction. Similar to the Laplacian growth in the plane with a point-like source or sink, this problem can be embedded into the 2D Toda lattice hierarchy in the zero dispersion limit. However, the relevant solution to the hierarchy is different. We characterize this solution by the string equations and construct the corresponding dispersionless tau-function. The latter is shown to coincide with the genus-zero part of the generating function for double Hurwitz numbers which count ramified coverings of the 2D sphere of a certain ramification type.