Conformal Invariance Workshop Schedule

Monday, April 8

10:00am **Greg Lawler**
On the probability that a loop-erased walk contains a particular edge

11:15am **Brent Werness**
Conformal Uniformization of CLE Carpets

12:15pm **Lunch**

2:15pm **Dmitry Belyaev**
Multifractal spectrum of harmonic measure

2:50pm **Tom Alberts**
Dimension Spectrum of SLE Boundary Collisions

3:30pm **Tea Time**

4:00pm **Eldad Bettelheim**
Conformal restriction in Anderson transitions

4:35pm **Michael Kozdron**
The Green's function for the radial Schramm-Loewner evolution

Tuesday, April 9

10:00am **Bertrand Duplantier**
Multifractality of Whole-Plane SLE

11:15am **Konstantin Izyurov**
SLE variants in the critical Ising model

12:15pm **Lunch**

2:15pm **Kalle Kytölä**
Quantum group solutions to two SLE problems

2:50pm **Benjamin Doyon**
Conformal restriction, conformal geometry and vertex algebras

3:30pm **Tea Time**

4:00pm **Haakan Hedenmalm**
Asymptotic expansion of polyanalytic Bergman kernels

4:35pm **Nam-Gyu Kang**
Distribution of eigenvalues of random normal matrices near the edge of the spectrum

6:00pm **FAX Pattern Exhibition: 3D Printing Demonstration by Phil Webster**
Simons Center Art Gallery

7:00pm **Moroccan Food Tasting with Wine Pairings and Live Music**
Simons Center Cafe

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Wednesday, April 10

10:00am **Richard Kenyon**
Banded states in the Ising model

11:15am **Tony Guttman**
Self-avoiding walks in a rectangular domain

12:15pm **Lunch**

2:15pm **Antti Kemppainen**
Convergence of random curves to Loewner evolutions

3:30pm **Tea Time**

6:00pm **Wine and Cheese Reception With Live Music**
Simons Center Lobby

6:30pm **Staged Readings of Science-Playwright Competition Winners**
Four short plays with a science theme: Simons Center Auditorium, Room 103

Thursday, April 11

10:00am **Scott Sheffield**
TBA

11:15am **Sergei Lukyanov**
ODE/IM correspondence for the Fateev model

12:15pm **Lunch**

2:15pm **Murray Batchelor**
Recent developments on discrete holomorphicity and integrability

2:50pm **Jacopo Viti**
Connectivities of Potts Fortuin-Kasteleyn clusters and Liouville time-like correlator

3:30pm **Tea Time**

4:00pm **Clément Hongler –** Math Colloquim lecture
Planar Ising Model: discrete and continuous structures
this lecture is in Room 102 at the Simons Center

7:15pm **Banquet Dinner**
Complimentary to all participants; Simons Center cafe

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Friday, April 12

10:00am **Fredrik Viklund**
On the continuity of SLE(\kappa) in \kappa and related results

11:15am **Ilia Binder**
Power law convergence of crossing probabilities for critical percolation

12:15pm **Lunch**

2:00pm **Vladimir Bazhanov**
Quasi-classical expansion of the star-triangle relation and integrable systems on quad-graphs

2:35pm **Clément Hongler**
Conformal invariance of Ising Model Fields and Correlations

3:30pm **Tea Time**

Abstracts:

Tom Alberts:
In the range 4 < \kappa < 8, the intersection of the Schramm-Loewner Curve (one of the central objects in the theory of 2-D Conformally Invariant Systems) with the boundary of its domain is a random fractal set. After reviewing some previous results on the dimension and measure of this set, I will describe recent joint work with Ilia Binder and Fredrik Viklund that partitions this set of points according to the generalized "angle" at which the curve hits the boundary, and computes the Hausdorff dimension of each partition set. The Hausdorff dimension as a function of the angle is what we call the dimension spectrum.

Murray Batchelor:
A number of research groups have been exploring the connection between discrete holomorphicity and integrability. In this talk I will review these developments.

Vladimir Bazhanov:
The star-triangle relation is a distinguished form of the quantum Yang-Baxter equation which plays a fundamental role in the theory of two-dimensional integrable models of statistical mechanics and quantum field theory. In this talk I will explain a connection between the star-triangle relation and classical integrable evolution equations on quad-graphs (planar graphs with quadrilateral faces). In particular, I will show how to use a special "master solution" of the star-triangle relation to reproduce the Adler-Bobenko-Suris (ABS) classification of classical integrable equations, originally obtained through the consistency-around-a-cube

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approach. As an example I will consider a lattice model which describes quantum fluctuations of circle patterns and the associated discrete conformal transformations connected with the Thurston's discrete analogue of the Riemann mappings theorem. In the quasi-classical limit the model precisely describe the geometry of integrable circle patterns with prescribed intersection angles on the (hyperbolic) plane.

Dmitry Belyaev:
Harmonic measure is one of the basic conformal invariants and its fine structure provides a lot of information about domains. The multifractal spectrum of harmonic measure turns out to be a very useful tool for many problems involving conformal invariance or conformal transformations. In this talk I will give a short survey of the fine structure of harmonic measure, its multifractal formalism and how it could be used.

Eldad Bettelheim:
The theory of conformal restriction as put forward by Lawler Schramm and Werner may be applied to Anderson transitions, namely, quantum critical points of disordered non-interacting systems. The integer quantum Hall transition, in particular, serves as an important example of such a transition, where a full description using conformal field theory is not known. Stochastic geometry thus offers an alternative approach to study such systems. I will review how, starting from a network model describing an Anderson transition, one may obtain statistical clusters, to which classical probabilities are associated but nevertheless determine the quantum properties of the system. These clusters may be described by the theory conformal restriction.

Ilia Binder:
Convergence of the Cardy-Smirnov observables is the crucial element of the famous proof of existence of the scaling limit of critical percolation on hexagonal lattice. I will describe a proof of the power law convergence of Cardy-Smirnov observables on arbitrary simply-connected planar domains. The proof works for the usual critical percolation on hexagonal lattice, as well as for some modified versions. I will also explain the relevance of this result for the investigation of the rate of convergence of the critical percolation to its scaling limit. This is a joint work with L. Chayes and H. K. Lei.

Benjamin Doyon:
Conformal loop ensembles (CLE) provides a measure-theoretic description of the scaling or "continuum" limits of critical statistical models. It describes the scaling limit of cluster boundaries through non-intersecting random loops. One can extract its expected properties through the more general "conformal restriction systems". The scaling limits of critical models are also believed to be described by conformal field theory (CFT), which rather uses algebraic ideas, based on vertex algebras. Relating these two approaches is an important problem, with potential for a deep understanding of criticality. I will overview my work on this. It is based on relating

conformal-symmetry fields to a geometry of conformal maps, in order to construct them in the context of conformal restriction systems. These CFT fields contain the stress-energy tensor, and give rise to the Virasoro vertex operator algebra, the most basic algebraic structure in CFT. I will give a brief introduction on vertex algebras, CLE and conformal restriction systems, then explain my results.

Bertrand Duplantier:
We revisit the Bieberbach conjecture in the framework of the SLE process. The study of its unbounded whole-plane version leads to a discrete series of exact results for the expectations of coefficients and their variances, and, more generally, for the derivative moments of some prescribed order p. These results are generalized to m-fold conformal maps of whole-plane SLE. We study the average integral means multifractal spectra of these unbounded whole-plane SLE curves. We prove the existence of a phase transition at a moment order $p=p^*(\kappa)>0$, at which one goes from the bulk SLE$_\kappa$ average integral means spectrum, valid for $p\leq p^*(\kappa)$, to a new integral means spectrum for $p\geq p^*(\kappa)$. The latter spectrum is furthermore shown to be intimately related, via the associated packing spectrum, to the so-called radial SLE derivative exponents, and to the non-standard, local SLE tip multifractal exponents obtained from quantum gravity. This is generalized to the integral means spectrum of the m-fold transform of the unbounded whole-plane SLE map. Joint work with Nguyen T.P. Chi, Nguyen T.T. Nga and Michel Zinsmeister

Tony Guttmann:
A celebrated problem in numerical analysis was posed in {\em The SIAM 100 digit challenge.} It proposed a particle undergoing Brownian motion, starting at the centre of a rectangle of aspect ratio 10:1, and asked for the probability, to ten significant digits, that it hits the end before hitting a side. In fact that problem can be solved exactly, in terms of radicals. Here we discuss the corresponding problem of a self-avoiding walker, in the scaling limit, assumed to be describable by $SLE_{8/3}.$ Our solution provides arbitrary numerical precision.

Next, we turn the problem around and enumerate SAWs in a rectangular domain, and assuming that their scaling limit is given by $SLE_\kappa,$ we estimate κ numerically and we find $\kappa = 2.668 \pm 0.005.$

Haakan Hedenmalm:
We extend the asymptotic analysis of the weighted Bergman kernels due to Tian, Yau, Zelditch, and Catlin, to the setting of polyanalytic Bergman kernels. We focus on the case of the bianalytic Bergman kernel. We base our analysis on the microlocal approach of Berman, Berndtsson, and Sjöstrand. This reports on joint work with A. Haimi.

Clément Hongler: Math Colloquim
The planar Ising model is exactly solvable, as was shown by Onsager. It exhibits many interesting algebraic, probabilistic and analytic discrete structures that have

been investigated for many decades. It has been conjectured that its phase transition point, the Ising model converges to a continuous limit, and acquires conformal symmetry. This leads to continuous theories, such as Conformal Field Theory and Schramm-Loewner Evolutions, that possess beautiful and different structures, and yield spectacular results, in particular exact formulae.

In this talk, I will explain some recent progress in the investigation of the Ising model and the corresponding continuous theories. In particular I will focus on their connections: formulating the lattice integrability in terms of discrete complex analysis, one can develop tools to establish rigorously the existence and conformal symmetry of the continuous limit of the model. This approach also sheds a different light on the discrete and continuous theories, which yields alternative formulations of certain problems and new structures. Based on joint works with S. Benoist, D. Chelkak, H. Duminil-Copin, K. Izyurov, F. Johansson Viklund, A. Kemppainen, K. Kytölä, D.H. Phong and S. Smirnov

Clément Hongler:
The scaling limit of the Ising model is the simplest minimal model of Conformal Field Theory: its various fields can be classified, and their correlation functions computed, on various geometries. The spectacular results and insights of CFT have remained until recently conjectural. It is now possible to analyze the lattice Ising model at critical temperature and to prove rigorously the convergence of its lattice fields to the CFT ones. Based on joint works with D. Chelkak, K. Izyurov and S. Smirnov.

Konstantin Izyurov:
Convergence of interfaces in the critical Ising model to SLE's can be studied far beyond the basic chordal case. We will discuss corresponding results for radial and multiple SLE's, and a general theorem for multiply-connected domains. We will sketch the proof for the radial case which employs a double-cover version of Smirnov's martingale observable, also used in our joint project with D. Chelkak and C. Hongler on spin correlations. Time permitting, we will also discuss a new proof of a theorem by C. Hongler and K. Kytölä on convergence o +/-/free Ising interfaces to dipolar SLE, based on yet another version of Smirnov's observale.

Nam-Gyu Kang:
Microscopic properties of eigenvalues of random normal matrices change drastically in a narrow belt around the edge of the spectrum. I present an elementary method to prove Borodin and Sinclair's theorem on the scaling limit of correlation kernels for the soft-edge Ginibre ensemble. This method gives new result for the hard-edge Ginibre ensemble. After a discussion of the general properties of this scaling limit, I state a universality conjecture and provide arguments to support it. This is a joint work with Y. Ameur and N. Makarov.

Antti Kemppainen:

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In this talk, I review a fairly recent work on quite widely applicable method of showing certain compactness properties for sequences of planar random curves. There is a condition which implies convergence both as curves and as Loewner chains at least along subsequences. I also discuss the convergence of FK Ising model (random-cluster model with q=2) related random curves to Schramm-Loewner evolution type processes. This is a joint work with Stanislav Smirnov (Geneva and St. Petersburg).

Richard Kenyon:
We define a three-parameter family of measures generalizing the standard $q=2$ random cluster model (Ising model) on a periodic planar graph. These are measures on FK configurations with multiple parallel connected domains. They are conformally invariant with a tilted conformal structure. Via a variational principle they give rise to limit shapes in scaling limits with certain boundary connection conditioning, similar to the sense in which the gradient Gibbs measures in the dimer model give rise to limit shapes for the dimer height function.

Michael Kozdron:
The Schramm-Loewner evolution (SLE), a one-parameter family of random two-dimensional growth processes introduced in 1999 by the late Oded Schramm, has proved to be very useful for studying the scaling limits of discrete models from statistical mechanics. One tool for analyzing SLE itself is the Green's function. An exact formula for the Green's function for chordal SLE was used by Rohde and Schramm (2005) and Beffara (2008) for determining the Hausdorff dimension of the SLE trace. In the present talk, we will discuss the Green's function for radial SLE. Unlike the chordal case, an exact formula is known only when the SLE parameter value is 4. For other values, a formula is available in terms of an expectation with respect to SLE conditioned to go through a point. This talk is based on joint work with Tom Alberts and Greg Lawler.

Kalle Kytölä:
In this talk we consider two questions related to Schramm-Loewner evolutions. The first question is about the "boundary zig-zags", i.e. the probabilities for a chordal SLE to pass through small neighborhoods of given boundary points in a given order. The second question is that of obtaining explicit descriptions of "multiple SLE pure geometries", i.e. those extremal multiple SLE probability measures which can not be expressed as non-trivial convex combinations of other multiple SLEs. For both problems one needs to find solutions of a system of partial differential equations with asymptotics conditions written recursively in terms of solution of the same problem with a smaller number of variables. We present a general correspondence, which translates these problems to linear systems of equations in finite dimensional representations of the quantum group U_q(sl_2), and we then explicitly solve these systems. The talk is based on joint works with Eveliina Peltola (Helsinki), and with Niko Jokela (Santiago de Compostela) and Matti Järvinen (Crete).

Greg Lawler:

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We give a new proof of a result of Rick Kenyon that shows that the probability that a loop-erased walk crossing a square of side length n decays like n^{-5/4}. In fact, we improve by showing that that estimates holds up to constants (no logarithmic terms).

The proof uses nothing about spanning trees, dimers, or the Schramm-Loewner evolution but does use the random walk and Brownian motion loop measures. I will discuss implications of this to natural parametrization of the Schramm-Loewner evolution(kappa = 2).

Sergei Lukyanov:
The Fateev model is somewhat special among two-dimensional quantum field theories. For different values of the parameters,it can be reduced to a variety of integrable systems. An incomplete list of the reductions includes O(3) and O(4) non-linear sigma models and their continuous deformations (2D and 3D sausages, anisotropic principal chiral field), the Bukhvostov-Lipatov model, the N=2 supersymmetric sine-Gordon model, as well as the integrable perturbed SU_2(n)\otimes SU_2(p-2)/SU_2(n+p-2) coset CFT. The model possesses a mysterious symmetry structure of the exceptional quantum superalgebras U_q{\widehat{\big(D(2|1;\alpha)}}\big).

In the talk, I'll discuss the ODE/IM correspondence between the Fateev model and a certain generalization of the classical problem of constant mean curvature embedding of a thrice-punctured sphere in AdS_3.

Fredrik Viklund:
SLE(\kappa) is a family of random fractal curves constructed by solving the Loewner equation with a standard Brownian motion times the square-root of \kappa as driving term. A natural question that has been asked is whether almost surely the SLE(\kappa) curves simultaneously exist and change continuously as the parameter \kappa is varied in an interval; there exist examples of deterministic Loewner chains with driving terms more regular than Brownian motion for which the corresponding statement is false. We will discuss a result giving a positive answer to this question and also indicate how these ideas combined with certain geometric estimates can be used to obtain power-law convergence rate results for, e.g., the loop-erased random walk path, when a rate for the driving term is known. The talk is in part based on joint work with Rohde and Wong.

Jacopo Viti:
I will consider the problem of computing the probability that three points are in the same Fortuin-Kasteleyn cluster at the critical point in the Q-state Potts model. Such probability is related to a conformal invariant three-point function and I will conjecture that this three-point function is given by a suitable analytic continuation of the structure constants of the conformal minimal model. The analytic continuation is the same obtained by Al. Zamolodchikov and Kostov-Petkova in the

study of the so-called time-like Liouville field theory. Very accurate numerical simulations which confirm the conjecture are discussed at the end of the talk.

Brent Werness:
The development of Schramm-Loewner Evolutions, and other related objects, have lead to an explosion in the understanding of various conformally invariant scaling limits of discrete models from statistical physics. Of particular interest for this talk is the development of the conformal loop ensembles (CLE) gaskets which are designed to serve as the scaling limits of the outer cluster boundaries of various spin models. In parallel, there has been much development in the theory of quasi-conformal unifomization, which aims to give meaning to questions like, "what is a version of the Riemann mapping theorem for the Sierpinski carpet?"

In this talk, I will discuss such uniformization type results for the CLE gasket. In a suitable sense, I will show there exists a random circle domain (a region whose complementary components are disks) which should be thought of as a conformal image of the CLE carpet for kappa less than four. The work presented in this talk is a current work-in-progress with Steffen Rohde.

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