

The Simons Center for Geometry and Physics
Workshop on Moduli Spaces of Pseudo-Holomorphic Curves I
Monday, March 17 - Friday, March 21

Welcome to the first workshop of the Spring 2014 program on *Moduli Spaces of Pseudo-Holomorphic Curves and Their Applications to Symplectic Topology* at the Simons Center for Geometry and Physics. This workshop will study various constructions of the virtual moduli cycle with the aim of clarifying exactly how to count pseudoholomorphic curves. It will be centered around four mini-courses, each devoted to one approach. There will also be 3 introductory talks and 3 supplementary talks on related topics.

Each mini-course will consist of 4 one-hour sessions:

- an introductory lecture outlining the approach and placing it in the general context,
- a more technical lecture focusing on how the analytic and topological problems are solved within the given framework,
- a discussion session on Thursday driven by questions from the participants, and
- a closing lecture on Friday addressing any remaining questions from the discussion session and summing up.

We hope the participants will help make all four sessions as interactive as possible with pointed questions and comments.

In order to stimulate the discussion, each mini-course will have a moderator who has at least some familiarity with the given approach. If for some reason you are hesitant to ask questions directly of the presenters, please feel to bring them up to the attention of the corresponding moderator.

As the primary aim of this workshop is to achieve an understanding regarding the correctness and completeness of the four approaches to VMC under consideration, *please* do bring up any concerns you may have, with either the lectures or the papers.

Since all VMC constructions are rather complicated, there is no way any of them can be described fully, even in 4 hours. We hope that the lectures will provide a good idea of what is going on in the related papers and perhaps inspire a wider audience to study some of these papers more closely. If you would like one or more of the papers printed out during the workshop, please ask the SCGP staff or one of the local participants.

In order to facilitate the interaction between the participants, we are asking everyone to wear the provided name tags throughout the workshop. They are also necessary to pick up your morning coffee and lunch.

We hope you will have a productive and enjoyable visit at the Simons Center for Geometry and Physics.

Schedule

All talks will take place in the Simons Center Lecture Hall, Room 102

Monday	9:00 - 10:00	<i>Breakfast and Registration</i>
	10:00 - 11:00	H. Hofer, <i>The geometry, analysis, and topology of moduli spaces in symplectic geometry</i>
	11:30 - 12:30	Fukaya-Oh-Ohta-Ono I
	12:30 - 2:15	<i>Lunch</i>
	2:15 - 3:15	McDuff-Wehrheim I
	3:15 - 3:45	<i>Tea</i>
	3:45 - 4:45	M. Tehrani, <i>Symplectic sum formulas: an overview</i>
	5:15 - 6:15	P. Seidel, <i>Some applications of pseudo-holomorphic curves</i>
Tuesday	9:00 - 10:00	<i>Breakfast</i>
	10:00 - 11:00	Chen-Li-Wang I
	11:30 - 12:30	Ionel-Parker I
	12:30 - 2:15	<i>Lunch</i>
	2:15 - 3:15	D. Salamon, <i>A construction of the Deligne-Mumford orbifold</i>
	3:15 - 3:45	<i>Tea</i>
	3:45 - 4:45	Fukaya-Oh-Ohta-Ono II
	5:15 - 6:15	McDuff-Wehrheim II
Wednesday	8:00 - 9:00	<i>Breakfast</i>
	9:00 - 10:00	Chen-Li-Wang II
	10:30 - 11:30	Ionel-Parker II
	12:00 - 1:00	A. Gerstenberger, <i>The moduli problems in the approach to transversality via Donaldson hypersurfaces</i>
		<i>Afternoon free</i>
Thursday	9:00 - 10:00	<i>Breakfast</i>
	10:00 - 11:00	Fukaya-Oh-Ohta-Ono D
	11:30 - 12:30	McDuff-Wehrheim D
	12:30 - 1:45	<i>Lunch</i>
	1:45 - 2:45	Chen-Li-Wang D
	2:45 - 3:15	<i>Tea</i>
	3:15 - 4:15	Ionel-Parker D
	4:45 - 5:45	J. Pardon, <i>Implicit atlases and virtual fundamental cycles</i>
6:00 - 8:00	<i>Banquet</i>	
Friday	8:00 - 9:00	<i>Breakfast</i>
	9:00 - 10:00	Fukaya-Oh-Ohta-Ono IV
	10:20 - 11:20	McDuff-Wehrheim IV
	11:40 - 12:40	Chen-Li-Wang IV
	12:40 - 1:30	<i>Lunch</i>
	1:30 - 2:30	Ionel-Parker IV

Constructing the Virtual Moduli Cycle

Mini-Course I: Kuranishi Structures

By K. Fukaya (SCGP), Y.-G. Oh (IBS CGP), H. Ohta (Nagoya), and K. Ono (RIMS Koyto)
Moderator: H. Hofer (IAS)

In these talks we explain the story of Kuranishi structure, its virtual fundamental cycle, and how we can apply it to the study of the moduli space of pseudo-holomorphic curves. Within a limited time, we focus our attention on the study of a space with a Kuranishi structure (not a system of Kuranishi structures) and do not discuss chain level arguments. This is good enough for GW-invariant, but not enough in particular for the study of Lagrangian Floer theory. The talks are divided into two parts.

In one part we describe the construction of Kuranishi structure on the moduli space of pseudo-holomorphic curves. In this part we focus on the proof of smoothness of the coordinate change. (This is a point we are asked from several people.) The proof is based on the exponential decay estimate of the gluing map and its derivatives (with respect to the gluing parameter).

The other part is on abstract story of Kuranishi structure. We explain how we construct virtual fundamental class. We plan to discuss it based on de Rham theory. We will define integration of the differential forms on the space with Kuranishi structure and use to define virtual fundamental class. This is different from the way taken in some of our previous references where singular homology theory was used. However, this is the way we used in various papers to apply our theory, especially in the application to toric manifolds.

Related Readings:

K. Fukaya and K. Ono, *Arnold conjecture and Gromov-Witten invariant*, Topology 38 (1999), no. 5, 933-1048

K. Fukaya, Y.-G. Oh, H. Ohta, and K. Ono, *Lagrangian Intersection Floer Theory: Anomaly and Obstruction*, Studies in Advanced Mathematics 46, AMS, 2009

K. Fukaya, *Kuranishi structure on moduli space of J -holomorphic curves*, notes taken by M. Tehrani, available at <http://mysbfiles.stonybrook.edu/~mfarajzadeht/Fukaya.html>

Videos of lectures available at <http://mysbfiles.stonybrook.edu/~mfarajzadeht/Fukaya.html>

Mini-Course III: Virtual Manifolds

By B. Chen (Sichuan), A.-M. Li (Sichuan), and B.-L. Wang (Australian National University)
Moderator: C.-C. Liu (Columbia)

In these lectures, we explain how to construct a finite-dimensional virtual system for the moduli space of stable maps directly from the infinite-dimensional system. Then we introduce the gluing theory for the virtual manifolds to get a smooth structure for this virtual system. The Gromov-Witten invariants and their K-theory analogs follow from the canonical orientations in cohomology and K-theory on the underlying virtual system.

Related Readings:

B. Chen, A.-M. Li, and B.-L. Wang, *Virtual neighborhood technique for pseudo-holomorphic spheres*, [math/1306.3276](https://arxiv.org/abs/math/1306.3276)

B. Chen and G. Tian, *Virtual manifolds and localization*, Acta Math. Sin. 26 (2010), no. 1, 1-24

Constructing the Virtual Moduli Cycle

Mini-Course II: Kuranishi Atlases

By D. McDuff (Barnard College) and K. Wehrheim (Berkeley)
Moderator: T. Ekholm (Uppsala)

The first lecture will provide a quick overview of Kuranishi atlases, explaining the similarities to and differences from the Kuranishi structure approach. The second lecture will probably discuss reductions and the construction of perturbation sections, which leads to the construction of the VMC. The topic of the final lecture will depend on the discussion. The lectures will be given by Dusa McDuff.

Related Readings:

D. McDuff and K. Wehrheim, *Smooth Kuranishi atlases with trivial isotopy*, [math/1208.1340](#)

D. McDuff, *Smooth Kuranishi atlases*, slides for Special Session on Advances in Symplectic Geometry and Topology, Guanajuato, revised February 2014, available at <http://scgp.stonybrook.edu/wp-content/uploads/2014/02/mexsmoothlectv3.pdf>

D. McDuff, *Notes on Kuranishi atlases*, available at <http://scgp.stonybrook.edu/wp-content/uploads/2014/02/kurnotesfeb20.pdf>

Videos of lectures available at <http://scgp.stonybrook.edu/scientific/mini-courses/mini-course-by-dusa-mcduff-kuranishi-atlases>

Mini-Course IV: Construction of the VMC

By E. Ionel (Stanford) and T. Parker (Michigan State)
Moderator: A. Zinger (Stony Brook)

These lectures will describe the key ideas in our proof that the Gromov-Witten moduli space of a compact symplectic manifold carries a unique virtual fundamental class that satisfies certain naturality conditions. The virtual fundamental class is constructed using only Gromov-type perturbations by introducing stabilizing divisors and systematically applying the naturality conditions.

Related Readings:

E. Ionel and T. Parker, *A natural Gromov-Witten virtual fundamental class*, [math/1302.3472](#).

E. Ionel, *GW invariants relative normal crossings divisors*, [math/1103.3977](#).

E. Ionel and T. Parker, *Relative Gromov-Witten invariants*, Ann. of Math. 157 (2003), no. 1, 45–96.

E. Ionel and T. Parker, *The symplectic sum formula for Gromov-Witten invariants*, Ann. of Math. 159 (2004), no. 3, 935–1025.

Introductory Talks

H. Hofer (IAS), *The geometry, analysis, and topology of moduli spaces in symplectic geometry*

The talk will focus on the problems which have to be addressed when studying pseudoholomorphic curves in symplectic geometry.

D. Salamon (ETH), *A construction of the Deligne-Mumford orbifold*

The Deligne-Mumford moduli space is the space of isomorphism classes of stable nodal Riemann surfaces of arithmetic genus g with n marked points. A marked nodal Riemann surface is stable if and only if its isomorphism group is finite, and it admits a universal unfolding if and only if it is stable. A natural construction based on the existence of universal unfoldings endows the Deligne-Mumford moduli space with an orbifold structure. The proofs discussed in this lecture use the methods of differential geometry rather than algebraic geometry. This is joint work with Joel Robbin.

P. Seidel (MIT), *Some applications of pseudo-holomorphic curves*

This talk is intended as a “light interlude” in the workshop. There will be no new results, but some survey of questions to which pseudo-holomorphic curve methods have contributed (or can contribute).

Supplementary Talks

A. Gerstenberger (Hamburg), *The moduli problems in the approach to transversality via Donaldson hypersurfaces*

This talk will aim to provide an overview of the construction of the Gromov-Witten pseudocycle as introduced by K. Cieliebak and K. Mohnke. The focus of the presentation will be on a coherent picture of the moduli spaces (as orbit spaces of groupoids) appearing along the way and on how the problems are addressed that one has to deal with when trying to equip them with smooth structures.

J. Pardon (Stanford), *Implicit atlases and virtual fundamental cycles*

I will define implicit atlases and show how they may be constructed on various moduli spaces of holomorphic curves. I will also show how they may be used to define virtual fundamental cycles and in particular to construct Hamiltonian Floer homology.

M. Tehrani (SCGP), *Symplectic sum formulas: an overview*

This talk provides some background for the Donaldson hypersurface approach to the construction of Gromov-Witten invariants. We recall the notions of stable relative maps and relative GW-invariants. We then review the symplectic sum construction and the symplectic sum formula for GW-invariants.