SPRING SCHOOL IN DISCRETE AND COMPUTATIONAL GEOMETRY

Simons Center for Geometry and Physics
Stony Brook University, April 17-21, 2017

Monday, April 17

12-12:45	SCGP lobby	Registration
12:45-1	SCGP 102	Welcoming remarks
1-2	SCGP 102	Mini-course Scott Sheffield 1
2:30-3:30	SCGP 102	Mini-course Scott Sheffield 2

Tuesday, April 18

9-10	SCGP 102	Mini-course Scott Sheffield 3
10:30-11:30	SCGP 102	Mini-course Esther Ezra 1
1-2	SCGP 102	SCGP Weekly Talk, Chee Yap
2:30-3:30	SCGP 102	Mini-course Esther Ezra 2
4-5	SCGP 102	Open Problem Session
7-9	Math S-240	TBA

Wednesday, April 19

9-10	SCGP 102	Mini-course Esther Ezra 3
11-12	SCGP 102	Mini-course Yusu Wang 1
1-2	Math S-240	Math Club, Joe Malkevitch
2:30-3:30	SCGP 102	Mini-course Yusu Wang 2
4-5	Math S-240	Student Presentations

Thursday, April 20

9-10	SCGP 102	Mini-course Yusu Wang 3
10:30-11:30	SCGP 102	Mini-course David Mount 1
1-2	SCGP 102	Invited lecture, Hu Ding
2:30-3:30	SCGP 102	Mini-course David Mount 2
4-5	SCGP 102	Math Department Colloquium, Jie Gao

Friday, April 21

9-10	SCGP 102	Mini-course David Mount 3
10:30-11:30	SCGP 102	Invited lecture, Boris Aronov

THE MINICOURSES

Esther Ezra, Dept of Mathematics, Georgia Institute of Technology

TITLE: Geometric Set Systems: Structures and Applications

ABSTRACT: An abstract set system (or a hypergraph) is a collection of subsets defined over a given set of objects.

In a typical geometric set system the objects are points in some low dimensional space, and the subsets are simply-shaped, regions (each of which is realized by the subset of points captured by the underlying region), for example, points and halfspaces, balls, or orthants in d-dimensions, to name a few.

I will present three fundamental structures on geometric set systems, each of which has algorithmic applications to geometric computing. The first talk of this minicourse will be devoted to the structure of "Epsilon-nets", roughly speaking, in geometric set systems these are hitting sets for the large regions, I will discuss the sample complexity of Epsilon nets and show improved bounds in some natural geometric scenarios. On the algorithmic front, I will discuss the relation between Epsilon nets and improved approximation factors for the smallest hitting set and set cover in geometric set systems. In the second talk, I will discuss "Relative Approximations", in principle, they generalize the structure of Epsilon nets, but they are mostly useful in approximate range counting, where one wishes to efficiently answe rrange counting queries (without looking at the entire input). I will show how to reduce the sample complexity on relative approximations in several geometric scenarios, using the Lovasz Local Lemma.

The third talk of this minicourse will be devoted to Geometric Discrepancy. Given a geometric set system, the goal is to color its point set in two colors, such that in each region the coloring is as even as possible. I will present combinatorial bounds on the maximum possible deviation from an even split in several scenarios, and, on the more applied front, I will present the connection between discrepancy and relative approximations.

David Mount, Dept. of Computer Science and Institute for Advanced Computer Studies, University of Maryland

TITLE: Approximation Algorithms for Multidimensional Proximity Problems

ABSTRACT: While the field of computational geometry has been very successful in the development of efficient algorithms for computational problems in spaces of dimension two and three, many of these algorithms either fail to generalize to higher dimensions or suffer from running times that grow very rapidly as the dimension increases. This has led to interest in approximation algorithms, where an approximation parameter $\epsilon > 0$ is given, and the algorithm provides a solution that is within a factor of $(1 + \epsilon)$ of the optimum solution.

In this series of lectures, I will present a number of computational methods and structures that are widely used in the design of geometric approximation algorithms. This includes the well-separated pair decomposition, John's Theorem, coresets, convex approximation, and Macbeath regions. I will also present recent applications of these methods to problems such as approximate nearest-neighbor searching, computing approximate Euclidean minimum spanning trees, computing coresets, and approximate polytope membership queries.

Scott Sheffield, Department of Mathematics, Massachusetts Institute of Technology

TITLE: Random geometric structures: a three-lecture overview

ABSTRACT: I will describe several special probability measures on spaces of geometric objects: random trees, random paths, random surfaces, and so forth. All of these objects have discrete and continuous analogs, and the interplay between the discrete and continuous has played an important role in statistical and particle physics in recent decades.

Over the course of a three-lecture series, I will describe the combinatorial ideas behind these various constructions, explain on a discrete level how these objects are related to one another, and discuss a few of the scaling limit results that connect the discrete theories to their continuous analogs.

Yusu Wang, Computer Science and Engineering, The Ohio State University

TITLE: Mini-course on computational topology

ABSTRACT: Topology aims at studying intrinsic structures of a given object or space. It is a powerful ool for identifying, describing, and characterizing essential features of functions and spaces. Indeed, in recent years topological methods have been availed as new promising tools for analyzing complex and diverse data. Fundamental progress has been made in both the theoretical and algorithmic development in computational topology. The resulting methods have been successfully applied in a broad range of application fields. This minicourse consists of three lectures, and aims to introduce some of the basic concepts and topological structures behind these recent developments computational topology, as well as algorithms to compute them. Specifically, we plan to focus on (i) persistent homology and its applications, and (ii) the mapper / multiscale mapper methodology for analyzing complex maps, as well as related structures (Reeb graphs/contour trees). Time permitting, we may also introduce discrete Morse theory.

INVITED LECTURES

Chee Yap , Courant Institute, NYU

SCGP weekly talk, Tuesday, April 18, 1-2pm

TITLE: On Soft Foundations for Geometric Computation

ABSTRACT: For over two decades, Exact Geometric Computation (EGC) has provided a paradigm in Computational Geometry for the correct implementation of geometric algorithms. It is the most successful approach to numerical nonrobustness issues, leading to software libraries and practical algorithms. We review some pressing reasons to extend this paradigm:

- EGC algorithms may not be Turing computable (e.g., transcendental functions)
- EGC may be too inefficient (e.g., shortest path problems)
- EGC entails numerous/difficult algebraic analysis (e.g., Vor diagram of polyhedra)
- Exact computation is inappropriate for the physical world (e.g., robot motion planning)

This talk describes a research program to develop "soft" (i.e., purely numerical) approaches for addressing these issues. We illustrate these ideas through recent work in several areas:

- root isolation and clustering (ISSAC'09,'11,'12,'16, SNC'11, CiE'13, JSC'17)
- isotopic approximation of curves and surfaces (ISSAC'08, SoCG'09, SPM'12, ICMS'14)
- Morse-Smale complex (SoCG'12)
- Voronoi diagrams (ISVD'13, SGP'16)
- robot motion planning (SoCG'13, WAFR'14, FAW'15, WAFR'16)

Some common themes in this list: We replace the Real RAM model by one based on numerical interval approximations. The main algorithmic paradigms are subdivision and iteration. We introduce an input resolution parameter (epsilon), but use it in a novel "soft way". We introduce soft versions of classical ("hard") geometric predicates.

What are the consequences of such a computational paradigm ?

- scope of computational geometry is vastly broadened
- unsolvable/hard problems in the Real RAM model becomes feasible
- soft algorithms are implementable and practical

One challenge is to revisit other classical problems of computational geometry with this view point. Another is to produce complexity analysis of such algorithms.

Joe Malkevitch, York College, CUNY ,

Math Club lecture, Math S-240, Wednesday, April 19, 1-2pm

TITLE: Euler's Polyhedral Formula Still Inspires New Geometrical Results

ABSTRACT: If P is a convex 3-dimensional polyhedron then V (vertices) + F (faces) - E (edges) = 2 is satisfied by P. New geometrical questions inspired by and related to this formula and its extension to plane graphs will be discussed.

Hu Ding, Michigan State ,

Invited lecture, Thursday, April 20, 1-2pm

TITLE: Simplex Lemma And The Applications in Data Analysis

ABSTRACT: Given a set of points in Euclidean space, one could easily compute their mean or median point. However, in many scenarios we only have some partial knowledge about the point set and hence cannot directly compute the mean or median point. Simplex lemma is a tool for approximately estimating the mean and median point of a hidden point set based on several novel geometric insights. In this talk, we will introduce this simplex lemma and two important applications in data analysis.

(1) Constrained clustering in high dimension. It is well known that the ordinary clustering problems, e.g., k-means/median, satisfy locality property; that is, each resulting cluster always locates inside an individual Voronoi cell induced by the k cluster centers. However, when adding some constraint, e.g., cluster size bound or color conflict, the locality property could be lost and therefore the existing clustering algorithms would no longer work.

(2) Truth discovery is an important problem arising in data analytics related fields such as data mining, database, and big data. It concerns about finding the most trustworthy information from a dataset acquired from a number of unreliable sources. Actually, it can be naturally modeled as a geometric optimization problem in high dimension. Due to its importance, the problem has been extensively studied in recent years and a number techniques have already been proposed. However, all of them are of heuristic nature and do not have any quality guarantee.

We will show that simplex lemma can be applied to efficiently solve the above two problems with theoretical quality guarantees. We will also show some interesting future work in theory and practice.

Jia Gao, CS Department, Stony Brook University

Mathematics Colloquium, Math P-131, Thursday, April 20, 4-5pm

TITLE: Networking Applications of Curvature and Ricci Flow

ABSTRACT: In this talk I will review the work we have done in the past few years using continuous maps for problems in networking applications. The talk will cover three topics:

(1) using conforrmal maps for efficient greedy routing with quality guarantee;

(2) using area preserving maps for load balanced routing and

(3) discrete graph curvature for complex networks. All three topics are centered around the theme of connecting intrinsic network geometry to networks operations.

Boris Aronov, Courant Institute, NYU

Invited Lecture, SCGP 102, Friday, April 21, 10:30-11am

TITLE: Breaking depth-order cycles and other adventures in 3D

ABSTRACT: Given a collection of non-vertical lines in general position in 3D, there is a natural above/below relation defined on the lines. One line is “above” another if the unique vertical line that meets both meets the former at a point above the point where it meets the latter. One can similarly define the (partial) above/below relation on any set of reasonably well-behaved pairwise disjoint objects; a pair of objects is not related at all, if no vertical line meets both.

Motivated by a computer graphics problem, the following question was asked more than 35 years ago: What is the minimum number of pieces one must cut N lines into, in the worst case, to make sure that the resulting pieces have no cycles in their above/below relation? An N^2 upper bound is easy, but is the answer sub-quadratic? An approximately $N^{3/2}$ lower bound was known, but there were no non-trivial upper bounds, in the general case. Restricted versions of the problem have been studied and will be briefly discussed. We present a near-optimal near-$N^{3/2}$ upper bound.

We also sketch how to extend this to the original motivating question for computer graphics, which until now was unreachable: How many pieces does one have to cut N triangles into, to eliminate all cycles in the above/below relationship, as above? We obtain a near-$N^{3/2}$ bound in this case as well, though slightly weaker.

Joint work with Micha Sharir (Tel Aviv U), and also with Edward Y. Miller (NYU), for the extension to triangles.