SCGP Workshop

Novel Vistas on Vortices

November 11–15, 2019

Titles and Abstracts

SimonsCenter
FOR GEOMETRY AND PHYSICS

Tudor Dimofte (University of California, Davis)

Vortices and Categories of Line Operators

Friday 15, 10:00

Three-dimensional $N = 4$ gauge theories admit BPS vortex-line operators. I shall discuss how, in the A-type topological twist, these vortex lines have the structure of a category, whose morphism spaces may be realized as the cohomology of generalized vortex moduli spaces. Moreover, 3d mirror symmetry predicts remarkable equivalences between these categories of vortex lines and certain categories of coherent sheaves.
Based on work with N. Garner, M. Geracie, and J. Hilburn.

Aleksander Doan (Columbia University)

The ADHM Vortex Equations on Riemann Surfaces

Tuesday 12, 16:00

The ADHM Seiberg–Witten equations were introduced by Haydys and Walpuski in relation to Yang–Mills theory on higher-dimensional manifolds with special holonomy. I will discuss a dimensional reduction of these equations to Riemann surfaces and their potential applications to enumerative theories of symplectic six-manifolds.
Part of the talk is based on joint work with Thomas Walpuski.

Óscar García-Prada (ICMAT-CSIC, Madrid)

SCGP Colloquium: Older Vistas on Vortices

Tuesday 12, 13:00

Starting with the seminal result of Taubes on the existence of solutions to the vortex equations on the plane, we go on to describe the theory of vortices on compact Riemann surfaces, and its relation to other themes, including the theorem of Donaldson–Uhlenbeck–Yau, Hitchin's Higgs bundles, and the Seiberg–Witten equations.

Óscar García-Prada (ICMAT-CSIC, Madrid)

Vortices, Variations of Hodge Structure, and Arakelov–Milnor Inequalities

Wednesday 13, 11:30

As shown by Hitchin, abelian vortices on a compact Riemann surface appear as fixed points in the moduli space of rank 2 Higgs bundles under the $\mathbb{C}^*$-action obtained by rescaling the Higgs field. In this talk we consider the $\mathbb{C}^*$-action on the moduli space of G-Higgs bundles over a compact Riemann surface X, where G is a semisimple Lie group (real or complex). By the non-abelian Hodge correspondence, this moduli space is homeomorphic to the moduli space of representations of the fundamental group of X in G. The fixed points (generalizations of vortices) correspond to variations of Hodge structure, and coincide with the critical points of a Morse function on the moduli space of Higgs bundles, known as the Hitchin functional. We show that one can define in this context an invariant that generalizes the Toledo invariant in the case where G is a real group of Hermitian type. Moreover, there are bounds on this invariant similar to the Milnor–Wood inequalities of the Hermitian case. These bounds also generalize the Arakelov inequalities of

classical Hodge bundles arising from families of varieties over a compact Riemann surface. We study the case where this invariant is maximal, and show that there is a rigidity phenomenon, relating to Fuchsian representations and higher Teichmüller spaces.
Joint work with O. Biquard, B. Collier and D. Toledo.

Eduardo González (University of Massachusetts, Boston)

Stratifications for Gauged Maps

Friday 15, 15:45

We will report on joint work with Solis and Halpern-Leistner on the Θ-stratification for gauged maps. We show that one can extend the usual Harder–Narasimhan theory for principal bundles over a curve to gauged maps, that is, a gauged map is either semistable, or it admits a canonical filtration whose associated graded bundle is semistable in a graded sense.

Justin Hilburn (University of Pennsylvannia)

3d Mirror Symmetry and Vortex Invariants

Tuesday 12, 10:00

In this talk I will give a short survey of 3d $N = 4$ theories and their relation to 2d $N = (2, 2)$ theories. In particular, I will explain work with Bullimore, Dimofte, Gaiotto, and Kim generalizing the classical computation of the equivariant J-function of flag varieties in terms of the Toda lattice and the solution of the Toda lattice using Whittaker vectors in Verma modules. I will also explain the relationship of this work to some proposals of Teleman.

Hans Jockers (University of Bonn)

Aspects of 3d $N = 2$ Gauge Theories and of Quantum K-Theory

Thursday 14, 10:00

Three-dimensional supersymmetric gauge theories compactified on a circle are fundamentally tied to quantum K-theory. In this talk I will describe a particular correspondence between 3d $N = 2$ gauge theories compactified on a circle and quantum K-theory, which connects suitable partition functions of such three-dimensional gauge theories to Givental's formulation of quantum K-theory. I will discuss the parameters that appear on both sides of this gauge theory/quantum K-theory correspondence.

Heeyeon Kim (University of Oxford)

Twisted Indices of 3d Supersymmetric Gauge Theories and Moduli Space of Vortices

Wednesday 13, 10:00

I discuss the geometric interpretation of the twisted index of 3d supersymmetric gauge theories on a closed Riemann surface. I show that the twisted index reproduces the virtual Euler characteristic of the moduli space of solutions to vortex equations on the Riemann surface. I also discuss 3d $N = 4$ mirror symmetry in this context, which implies non-trivial relations between enumerative invariants associated to the moduli space of vortices. Finally, I discuss a wall-crossing formula of the twisted indices derived from the gauge theory point of view.

Andrew Lee (St. Thomas Aquinas College NY)

Mapping Tori and Stable Pairs

Wednesday 13, 15:45

In this talk we first recall a construction over a Riemann surface of a certain moduli space of vortices, also called stable pairs, which carries a symplectic structure. Symplectic geometry in this space allows us to produce a Floer-theoretic invariant of a particular class of 3-manifolds called mapping tori (surface bundles over the circle). This invariant can in fact be calculated for a certain subset of these mapping tori. Time permitting, we also describe a related construction of a more general invariant using the same spaces.
This is joint work with Tim Perutz.

Chih-Chung Liu (National Cheng-Kung University)

Gromov Compactification of Vortex Moduli Spaces and Applications to L^2 Geometry

Monday 11, 15:45

The moduli spaces of abelian vortices are known to be generically in correspondence to spaces of holomorphic maps to projective spaces. Both spaces fail to be C^∞ compact, and a suitable "Gromov compactification" of both sides will be described in this talk. Moreover, I will describe how these compactifications might be applied to prove a conjectured formula for the L^2 volume of the space of maps.

Erica Minuz (Aarhus University)

Cohomology of Generalized Configuration Spaces and Graph Complexes

Tuesday 12, 11:00

We describe a type of configuration space where some points are allowed to coincide. This information can be encoded in a graph whose vertices correspond to the points in the configuration space, and whose edges record which of them are not allowed to overlap. In particular, we study its cohomology through graph complexes. In the case of points in $\mathbb{R}^r$, an explicit formula for the cohomology can be given. This kind of configuration space is a particular case of a more general one, depending on a graph and a function on the vertices, that arises in the calculation of the fundamental groups of the moduli spaces of vortices on a surface with toric target.

Ákos Nagy (Duke University)

Vortices on Noncompact Surfaces and Black Holes

Monday 11, 11:30

In this talk, I will be concerned with vortex equations on noncompact surfaces with certain geometric and topological conditions on their ends. I will show how to extended previously known results about the existence of vortices over such surfaces, and I will compute the corresponding vortex moduli spaces. As an application, I will construct novel solutions of the 4-dimensional SU(2) instanton equations on the Euclidean Schwarzschild manifold.
This is joint work with Gonçalo Oliveira.

Nikita Nekrasov (SCGP, Stony Brook University)

Quantum Wavefunctions and Vortex Counting

Monday 11, 10:00

I will review the progress in extracting eigenfunctions of quantum integrable systems from exact computations in supersymmetric gauge theories.
Partly based on joint work with Norton Lee.

Jeongseok Oh (KIAS Seoul)

Localization by 2-Periodic Complexes and Virtual Structure Sheaves

Thursday 14, 15:45

We discuss a K-theoretic analogue of a localized Chern character map and its properties. A K-theoretic localized Chern character map for a Koszul 2-periodic complex coincides with the co-section localized Gysin map defined by Y.-H. Kiem and J. Li. As an application, we compare the virtual structure sheaves of the moduli space of stable quasimaps and stable Landau–Ginzburg quasimaps.

Petr Pushkar (IST Austria)

Quantum K-theory for Nakajima Quiver Varieties and Quantum Integrable Systems

Thursday 14, 14:00

In this talk, I will define the quantum K-theory of Nakajima quiver varieties via quasimaps and show its connection to representation theory of quantum groups and quantum integrable systems on the examples of the Grassmannian and the flag variety. In particular, the Baxter operator will be identified with operators of quantum multiplication by quantum tautological classes via Bethe equations.
Based on joint works with P. Koroteev, A. Smirnov and A. Zeitlin.

Artan Sheshmani (Harvard University CMSA; QGM Aarhus)

Stable Higher Rank Flag Sheaves on Surfaces

Friday 15, 14:00

We study moduli spaces of holomorphic triples $f : E_1 \to E_2$, composed of (possibly rank > 1) torsion-free sheaves (E_1, E_2) and a holomorphic map between them, over a smooth complex projective surface S. The triples are equipped with a Schmitt stability condition. We prove that when the Schmitt stability parameter becomes sufficiently large, the moduli space of triples benefits from having a perfect relative and absolute obstruction theory in some cases (depending on the rank of E_1). We further generalize our construction to higher-length flags of higher rank sheaves by gluing triple moduli spaces, and extend earlier work, with Gholampur and Yau, where the obstruction theory of nested Hilbert schemes over the surface was studied. Here we extend the earlier results to the moduli space of flags $E_1 \to E_2 \to \cdots \to E_n$, where the maps are injective (by stability). There is a connection developed by Mochizuki, by wall-crossing in the master space, between the theory of such higher rank flags and the theory of Higgs pairs on the surface, which

provides the means to relate the flag invariants to the local Donaldson–Thomas invariants of any threefold given by a line bundle over the surface, $X := \mathrm{Tot}(L \to S)$. The latter DT invariants, when L is the canonical bundle of S, contribute to the Vafa–Witten invariants.
Joint work with Shing-Tung Yau.

Martin Speight (University of Leeds)

The L^2 Geometry of Moduli Spaces of $\mathbb{P}^1$ Vortices

Monday 11, 14:00

The gauged sigma model with target $\mathbb{P}^1$ supports two distinct species of vortex, one for each fixed point of the action of the structure group. Vortices of one species may coexist in stable equilibrium with antivortices of the other. The moduli space of such static solutions is noncompact, even on compact domains. It has a natural Riemannian metric which is of intrinsic geometric interest and encodes much useful information about the low energy dynamics of (anti)vortices. I will review what is known about this metric, concentrating on the moduli space of vortex-antivortex pairs, and describe some conjectures motivated by a formal compactification of the moduli space constructed using an auxiliary gauged linear sigma model.
Joint work with Nuno Romão and René García Lara.

Donghao Wang (Massachusetts Institute of Technology)

Finite-Energy SW Monopoles on $\mathbb{C} \times \Sigma$

Tuesday 12, 14:00

The Seiberg–Witten (monopole) equations and the monopole invariants introduced by Witten have greatly influenced the study of smooth 4-manifolds since 1994. By studying their dimensional reduction in dimension 3, Kronheimer–Mrowka defined monopole Floer homology for any closed 3-manifold. In this talk, we continue this reduction process and consider the moduli space of solutions on $X = \mathbb{C} \times \Sigma$, where Σ is a compact Riemann surface. We will classify solutions to the Seiberg–Witten equations on X with finite analytic energy and estimate their decay rates at infinity according to the algebraic input. The motivation is to extend the construction of Kronheimer–Mrowka to compact 3-manifolds with boundary, and this work can be viewed as the first step towards this goal.

Graeme Wilkin (University of York)

Vortices and the Morse Theory of the Yang–Mills–Higgs Functional

Wednesday 13, 14:00

Vortices appear as energy minimisers of the Yang–Mills–Higgs functional. In previous work (joint with Richard Wentworth), we used the Morse theory of this functional to compute topological invariants of moduli spaces of rank 2 vortices on Riemann surfaces. After discussing this, I will then describe some more recent work in which the vortex equations appear as linearised solutions of a reverse heat flow on the space of Higgs bundles (in the sense studied by Hitchin and Simpson). Spaces of flow lines for this energy functional have an interesting algebro-geometric structure, and I will describe some recent results and conjectures in this direction.

Chris Woodward (Rutgers University)

Vortices and Quantum K-Theory

Thursday 14, 11:30

I will talk about joint work with E. González on the quantum Kirwan map in K-theory $QK_G(X) \to QK(X/\!\!/G)$ defined by K-theoretic integration over the moduli spaces of vortices on the plane. As an application, we give a presentation for the quantum K-theory of toric varieties (at a bulk deformation).

Guangbo Xu (Texas A&M University)

Adiabatic Limit of the Gauged Witten Equation

Friday 15, 11:30

Tian and I constructed correlation functions associated to a gauged linear sigma model space in the geometric phase. The gauged Witten equation used in the construction is a generalization of the vortex equation in two dimensions. These correlators are expected to agree with Gromov–Witten invariants up to a "coordinate change". I will explain how to prove this correspondence using the adiabatic limit of the gauged Witten equation.
Joint work with Gang Tian.