Workshop Schedule

Events for:
Monday, June 6th - Friday, June 10th

Monday, June 6th

9:00am **Pavel Exner - SCGP 102**

Title: Vertex coupling effects in quantum graph spectra

Abstract: The main topic of this talk are quantum graphs with the vertex coupling violating the time-reversal invariance. As a case study we analyze a simple example in which the asymmetry is maximal at a fixed energy. This has an interesting consequence, namely that high-energy scattering dependscrucially on the vertex parity; we will demonstrate implications of this fact for spectral and transport properties in several classes of graphs, both finite and infinite periodic ones. We will also discuss other time-asymmetric graphs and identify a class of such couplings which exhibits a nontrivial PT-symmetry despite being self-adjoint. The results come from a common work with Marzieh Baradaran, Ji?r ?? Lipovsk ?y, and Milo?s Tater.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Wencai Liu - SCGP 102**

Title: Fermi isospectrality for discrete periodic Schrödinger operators

Abstract: Let $\Delta+V$ be the discrete Schrödinger operator, where Δ is the discrete Laplacian on $\Z^d$ and the potential $V:\Z^d\to \R$ is Γ-periodic. We prove three rigidity theorems for discrete periodic Schrödinger operators in any dimension $d\geq 3$:1) if V and Y are Fermi isospectral (that is, at some energy level, Fermi varieties of the Γ-periodic potential V and the Γ-periodic potential Y are the same), and $Y $ is a separable function, then V is separable as well;2) if potentials V and Y are Fermi isospectral and both $V=\bigoplus_{j=1}^rV_j$ and $Y=\bigoplus_{j=1}^r Y_j$ are separable functions, then, up to a constant, lower dimensional decompositions V_j and Y_j are Floquet isospectral, $j=1,2,\cdots,r$;3) if a potential V and the zero potential are Fermi isospectral, then V is zero.

11:30am **Break - SCGP Cafe**

11:45am **Rui Han (remote) - Zoom**

Title: Localization for anisotropic XY spin chain in quasi-periodic magnetic field

Abstract: Quantum spin models are a paradigm for the study of many-body effects. In this talk, we will discuss some recent results on the XY spin chain in quasi-periodic exterior magnetic field. In particular, we show dynamical localization for a family of anisotropic XY chains for all Diophantine magnetic fluxes, in the sense that the time evolution of local observables satisfies a zero-velocity Lieb-Robinson bound.

12:45pm **Lunch - SCGP Cafe**

2:15pm **Jonathan Rohleder - Zoom**

Title: On the hot spots of quantum graphs

Abstract: It is a conjecture going back to J. Rauch (1974) that the hottest and coldest spots in an insulated homogeneous medium such as an insulated plate of metal should converge to the boundary, for ”most” initial heat distributions, as time tends to infinity. This so-called hot spots conjecture can be phrased alternatively as follows: the eigenfunction(s) corresponding to the first non-zero eigenvalue of the Neumann Laplacian on a Euclidean domain should take its maximum and minimum on the boundary only. In this talk we present results on the conjecture for both metric graphs (joint work with James B. Kennedy) and Euclidean domains.

3:15pm **Tea Time - SCGP Cafe**

4:00pm **Gilad Sofer - SCGP 102**

Title: Time dependent quantum graphs and the geometric phase

Abstract: Consider a metric graph whose edge lengths are functions of time. While this systemis relatively simple to describe (at least naively), it turns out that presenting it as awell posed boundary value problem holds several diculties. For instance, the changein time of the domain which supports the eigenfunctions gives rise to non-unitary timeevolution. This problem has been discussed by Duca and Joly in [1] in the context ofthe Schrödinger equation on moving domains.We attempt to solve this problem by suggesting an alternative description of thesystem. We show that the original problem can be translated into the time independentproblem of a harmonic oscillator on a non-homogeneous quantum graph, along with amagnetic potential.One can then derive an expression for the geometric phase (dened in [2]) accu-mulated by the wave function as the edge lengths complete a cycle in the parameterspace.The talk is based on a work in progress with Uzy Smilansky.

4:30pm **Xingya Wang - SCGP 102**

Title: 1-Dim Half-line Schrödinger Operators with H^{-1} Potentials

Abstract: In this talk, I will present some spectral results of Schrödinger operators with locally H^{-1} potentials. In the first part, we will recover some general spectral theoretical results in the current setting, including a Last-Simon-type criteria for the presence and absence of the absolutely continuous spectrum on the open half-line. In the second part, we will focus on potentials which are decaying in a locally H^{-1} sense and present an analogue of short-range decay in the distributional setting. In particular, we will examine a class of Pearson-type distributional potentials and establish a spectral transition between short-range and long-range decay.

Tuesday, June 7th

9:00am **Pavel Kurasov (remote) - Zoom**

Title: M-functions and metric graphs:hierarchy, inverse problems and magnetic fluxes

Abstract: ?dinger operators on metric graphs, also known as quantum graphs, are determined by the underlying metric graph, the electric and magnetic potentials and the vertex conditions. If the underlying graph is a tree, then the M-function associated with the degree one vertices determines the operator under certain mild assumptions on the vertex conditions. This talk is devoted to the inverse problem for graphs with cycles. To this end we shall analyse the **hierarchy** of M-functions appeared when graphs are glued together as well as the dependence of M-functions on the magnetic fluxes through the cycles. Two approaches leading to unique solution of the inverseproblem will be presented:
• **dismantling graphs**: the original graph has sufficiently many contact points that dismantles it into a set of trees;
• **Magnetic Boundary Control**: dependence of the spectral data on the magnetic fluxes through the cycles is used to dissolve vertices and thus reconstruct so-called **infiltration domains**.Optimal solution of the inverse problem is obtained by combining these two methods.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Uzy Smilansky - SCGP 102**

Title: The Kronig-Pennet model in a quadratic channel with delta interactions

Abstract: Returning to and old problem -- the Schr\"odinger operator

\[
H(x,y)=-\frac{\partial^2}{\partial x^2}+\frac 12 \left(-\frac{\partial^2}{\partial y^2}+y^2 \right)+\lambda y\delta(x);\ x,y\in\mathbb R^2
\]
a paradigm model for multi-mode quantum graphs, I shell review some of its surprising spectral features by addressing it using a scattering approach. Once familiar with this approach, it will be applied in order to investigate a variation on the same theme -- where the delta potential is periodic along the x axis.
\[
H(x,y)=-\frac{\partial^2}{\partial x^2}+\frac 12 \left(-\frac{\partial^2}{\partial y^2}+y^2 \right)+\lambda y\sum_{n\in\mathbb Z}\delta(x-n L);\ x,y\in\mathbb R^2
\]

11:30am **Lunch - SCGP Cafe**

1:00pm **SCGP Weekly Talk: Pavel Exner**

Title: Quantum mechanics on graphs:why is it interesting?

Abstract: The concept of quantum graph was born twice, and since its rebirth in the1980s it enjoyed a permanent attention. It comes from the fact that suchmodels are not only useful to describe properties of nanostructures but theyalso provide a test field where many features of quantum mechanics can beexamined, sometimes viewed in an unusual light. We review examples showing relations between spectral properties of such graphs and their geometryand topology. We also mention an alternative model of the so-called leakygraphs in which the quantum tunneling is taken into account.

2:15pm **Lior Alon - SCGP 102**

Title: Generic Laplace eigenfunctions of finite metric graphs

Abstract: Up to certain pathologies, a compact metric graph with standard vertex conditions has a Baire-generic set of choices of edge lengths such that all Laplacian eigenvalues are simple (Friedlander 05') with eigenfunctions that do not vanish at any vertex (Berkolaiko-Liu 17'). However, this set of edge lengths is implicit. An explicit condition can be made, at the expense of a density zero subsequence of eigenvalues. For any choice of rationally-independent edge lengths, almost every eigenvalue is simple and has an eigenfunction that does not vanish on vertices (Alon-Band-Berkolaiko 18'). I will refer to the latter type of genericity as "ergodic genericity." To explain what is the ergodic system in the background, I will introduce a moduli space of solutions (eigenpairs) to all possible choices of edge lengths. I will explain how the eigenpairs of a graph with rationally-independent lengths equidistribute in this moduli space. Recently, Kurasov and Sarnak proved a conjecture of Colin de Verdiere regarding the irreducible algebraic structure of this moduli space. I will show how this irreducibility can be applied to prove the previous genericity results and many more. I will show that, generically, an eigenfunction fails to satisfy any additional vertex condition. Moreover, I will show that given two different metric graphs with the same edge lengths, generically, they do not share any non-zero eigenvalue.

3:15pm **Break - SCGP Cafe**

3:30pm **Gregory Berkolaiko - SCGP 102**

Title: Universality of nodal count statistics in large graphs

Abstract: TBA

Wednesday, June 8th

9:00am **Simon Becker (remote) - Zoom**

Title: Magnetic oscillations in a model of (twisted bilayer) graphene

Abstract: We consider the simplest model for graphene in a magnetic field given by a hexagonal quantum graphs. Using semiclassical methods (with the strength of the magnetic field as the small parameter) we obtain a geometric description of the density of states showing asymmetry seen in physical experiments but not in commonly used perfect cone approximations. That density of states can then be used to see magnetic oscillations such as the de Haas--van Alphen effect. We then discuss extensions of this result close to every rational flux which implies results on the Hausdorff dimension of the spectrum and recent results for twisted bilayer graphene. Joint work with M Zworski, R Han, R Kim and X Zhu.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Yuri Latushkin - SCGP 102**

Title: The Maslov index and the spectrum of differential operators

Abstract: We will review some recent results on connections between the Maslov and the Morse indices for differential operators. The Morse index is a spectral quantity defined as the number of negative eigenvalues counting multiplicities while the Maslov index is a geometric characteristic defined as the signed number of intersections of a path in the space of Lagrangian planes with the train of a given plane. The problem of relating these two quantities is rooted in Sturm's Theory and has a long history going back to the classical work by Arnold, Bott and Smale, and has attracted recent attention of several groups of mathematicians. We will show how the relation between the two indices helps to prove the fact that a generic pulse in a gradient system of reaction diffusion equations is unstable.We will also discuss a fairly general theorem relating the indices for a broad class of multidimensional elliptic selfadjoint operators. Connections of the Maslov index and Hadamard-type formulas for the derivative of eigenvalues will be also discussed. This talk is based on joint work with M. Beck, G. Cox, C. Jones, P. Howard, R. Marangell, K. McQuighan, A. Sukhtayev, and S. Sukhtaiev.

11:30am **Break - SCGP Cafe**

11:45am **Ram Band - SCGP 102**

Title: Spectral flows and the Robin count deficiency for metric graphs

Abstract: We introduce the Robin count, a generalization of the nodal and Neumann counts of eigenfunctions. Specifically, we count the number of points with a particular value of f'/f (which is known as the Robin parameter or delta coupling or cotangent of Prufer angle). Correspondingly, we introduce the Robin map (a generalization of the Dirichlet to Neumann map) and prove an index theorem - relating the Morse index of the Robin map with the Robin count deficiency. In this context, we discuss the relations between spectral flows, the Morse index and the Maslov index. It is shown that spectral flows may be used to express geometric properties of a graph, such as the number of interaction vertices and the first Betti number.This is a joint work with Marina Prokhorova and Gilad Sofer.

12:45pm **Conference Photo - Front Stairs**

12:50pm **Lunch - SCGP Cafe**

2:15pm **Rodrigo Matos - SCGP 102**

Title: Localization and eigenvalue statistics within Hartree-Fock theory

Abstract: TBA

3:15pm **Tea Time - SCGP Cafe**

4:00pm **Noema Nicolussi - SCGP 102**

Title: Laplacians on Infinite Graphs

Abstract: There are two different notions of a Laplacian operator associated with infinitegraphs: discrete Laplacians and quantum graphs. Both objects have a venerable history and their spectral theory relates to several diverse branches ofmathematics (random walks, combinatorics, geometric group theory, ...).In our talk we explore connections between these two types of operators (spectral, parabolic and geometric properties), and exploit these relations to prove anumber of new results in spectral theory for both settings.Based on joint work with Aleksey Kostenko (Ljubljana&Vienna) and MarkMalamud (Moscow).

6:00pm **Banquet - SCGP Cafe**

Thursday, June 9th

9:00am **Selim Sukhtaiev (remote) - Zoom**

Title: Two classes of ergodic operators on quantum graphs

Abstract: is talk, we will discuss two classes of dynamically
defined quantum graphs exhibiting interesting spectral behavior. The
first part of the talk concerns spectral and dynamical localization for
Anderson-type models on trees and random Hamiltonians with general point
interactions. In the second part, we will discuss a class of dynamically
defined anti-trees exhibiting singular continuous spectrum. This talk is
based on several joint projects with D. Damanik, L. Fang, J. Fillman, M. Helman, J. Kesten.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Burak Hatinoglu - SCGP 103**

Title: Spectral Properties of Periodic Elastic Beam Hamiltonians on Hexagonal Lattices

Abstract: Elastic beam Hamiltonians are constructed out of Euler-Bernoulli beams, each governed by a scalar valued fourth-order Schrödinger operator equipped with a real symmetric potential. Unlike the second-order Schrödinger operator commonly applied in quantum graph literature, here the self-adjoint vertex conditions encode geometry of the graph by their dependence on angles at which edges are met. In this talk, I will firstly consider spectral properties of this Hamiltonian with periodic potentials on a special equal-angle lattice, known as graphene or honeycomb lattice. I will also discuss spectral properties for the same operator on lattices in the geometric neighborhood of graphene. This talk is based on a recent joint work with Mahmood Ettehad (University of Minnesota), https://arxiv.org/pdf/2110.05466.pdf.

11:45am **Problem Session - SCGP 102**

Title: Problem Session

12:45pm **Lunch - SCGP Cafe**

2:15pm **Siegfried Beckus - SCGP 102**

Title: Spectral approximations beyond dimension one

Abstract: The table tiling is two aperiodic tilings of the plane that is a typical example of a two-dimensional quasicrystals. One way to treat such systems in dimension one, is to approximate these systems by suitable (periodic) approximations. Based on this, we raise the following questions: Is there a general method to approximate spectral properties of a given operator by the underlying geometry or dynamics? If so, can we control the approximations and which spectral properties are preserved?
During the talk, we mainly focus on this specific example and generalize along the lines.
These results are joint works with Ram Band, Jean Bellissard, Horia Cornean, Giusseppe De Nittis, Felix Pogorzelski, Alberto Takase and Lior Tenenbaum.

3:15pm **Tea Time - SCGP Cafe**

4:00pm **Laura Shou - SCGP 102**

Title: Eigenvector behavior for graphs from quantized interval maps

Abstract: TBA

Friday, June 10th

9:00am **Stephen Shipman - SCGP 102**

Title: Reducibility of the Bloch and Fermi Varieties

Abstract: TBA

10:00am **Coffee Break - SCGP Cafe**

10:30am **Yulia Karpeshina - SCGP 102**

Title: Multiscale Analysis in Momentum Space for Multi-Dimensional Quasi-Periodic Schrodinger Operators

Abstract: We consider Schrödinger operators H = ??+V (x) in Rd, d ? 2, withquasi-periodic potentials V (x). We prove that the absolutely continuous spectrumof a generic H contains a semi-axis [??, ?). We also construct a family of eigenfunctions of the absolutely continuous spectrum; these eigenfunctions are smallperturbations of the exponentials. The proof is based on the multi-scale analysisin the momentum space.

11:30am **Lunch - SCGP Cafe**