Computational Differential Geometry and its Applications in Physics: November 14-18, 2022

Events for:
Monday, November 14th - Friday, November 18th

Monday, November 14th

8:30am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102/ZOOM**

Speaker: Simon Donaldson

Title: Review of some numerical approaches to Kahler-Einstein metrics and other special metrics on complex projective varieties

Abstract: The talk will begin with some general background in complex differential geometry and then move on to discuss notions of “balanced” projective embeddings and their connections to asymptotic analysis. I will present some (rather old) examples of numerical results in the case of toric surfaces.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102**

Speaker: Sven Krippendorf

Title: Calabi-Yau metrics with neural networks

Abstract: I shall give an overview on how to efficiently obtain neural network approximations to Calabi-Yau metrics by optimising appropriate energy functionals (the length shall depend on the previous speakers).

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Michael Douglas

Title: Relations between numerical geometry and machine learning

Abstract: This talk will start out with the formal parallel between machine learning and the numerical geometry methods discussed in several talks at this workshop. Essentially, the dataset for numerical geometry is a point cloud sampled from the manifold together with pointwise values of geometric quantities such as the Kahler potential or metric. Both ML and numerical geometry use general function approximators (neural networks) and optimize an accuracy measure (loss function). From this starting point, we go on to compare the theory on both sides. Much studied issues in ML include approximating power, non-convexity of the loss function and overparameterization. One usually takes a statistical viewpoint and studies generalization and robustness to errors.By contrast our geometric problems give rise to platonic datasets with simple definitions and universal properties. And in geometry we are interested in higher dimensional quantities (p-forms, simplicial complexes, gauge connections etc.) which are not unknown in ML but not well known either. We will explain the points of contact and suggest interesting possibilities for exchange of ideas.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Rafael Oliveira

Title: Non-commutative optimization - geodesic 1st and 2nd order methods for moment maps and polytopes

Abstract: Scaling problems, such as operator scaling and tensor scaling, have recently found several applications in diverse areas of math and CS. They have been used to give efficient algorithms for non-commutative rational identity testing, compute the optimal constant in Brascamp-Lieb inequalities, one-body quantum marginal problem, solution to the Paulsen problem, and the search version of the Horn's problem, to name a few, [GGOW15, GGOW18, BFGOWW18, KLLR18, F18]. Scaling problems are natural geodesic convex optimization problems on Riemannian manifolds that arise from the symmetries of non-commutative groups, and the algorithms (and their analyses) developed in these previous works heavily relied on this general form of convexity. In this talk we discuss our recent systematic development of a theory of non-commutative optimization, which greatly extends ordinary (Euclidean) convex optimization. We will see how NC optimization unify and generalize the algorithms developed for the uniform and non-uniform versions of the operator scaling and tensor scaling problems. More specifically, our algorithms minimize the moment map (a non-commutative notion of the usual gradient), and test membership in moment polytopes (a vast class of polytopes, typically of exponential vertex and facet complexity, which quite magically arise from this a priori non-convex, non-linear setting). In the spirit of standard convex optimization, we develop two general methods in the geodesic setting, a first order and a second order method, which respectively receive first and second order information on the derivatives'' of the function to be optimized. We will also show the key parameters of the underlying group actions which control convergence to the optimum in each of these methods. The non-commutative analogues of smoothness'' (from the commutative case) are far more complex, and require significant algebraic and analytic machinery (much existing and some newly developed in this work). Despite this complexity, we shall see that the way in which these parameters control convergence in both methods is quite simple and elegant.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Toby Wiseman

Title: Computational Riemannian geometry applied to the physics of the AdS-CFT correspondence

Abstract: The AdS-CFT correspondence is a remarkable duality between gravitational systems and certain quantum field theories. One aspect of it is that properties of (Euclidean signature) Einstein metrics, which are perhaps the simplest solutions of these gravitational systems, then become related to properties of strongly coupled field theories, which are interesting and inherently difficult to compute using direct methods. I will review how this correspondence works for these Riemannian geometries. I will then discuss the methods used to explicitly construct Einstein metrics in this context, which involves a 'harmonic' formulation of the Einstein condition. I will discuss how to numerically solve this, using both a direct approach and also geometric flows. If time permits I will give some examples of interesting physics that can be extracted from these calculations.

Tuesday, November 15th

8:30am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: James Halverson

Title: Ricci Flow and Neural Network Gradient Descent

Abstract: In this talk I will present a framework for studying metric flows induced by neural network gradient descent, which has recently been utilized to construct approximate Ricci-flat metrics. Adapting neural tangent kernel (NTK) theory to the case of metric flows, I will demonstrate that metric flows simplify in certain infinite width limits and have dynamics given by a non-local update equation governed by the NTK. For appropriately chosen architecture and loss function, this infinite neural network metric flow reduces to Perelman's formulation of Ricci flow.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102**

Speaker: Dalimil Mazac

Title: New upper bounds on the spectral gap of hyperbolic manifolds

Abstract: I will describe a method for constraining Laplacian spectra of hyperbolic surfaces and d-manifolds. The main ingredient is consistency of the spectral decomposition of integrals of products of four automorphic forms. Using a combination of representation theory of SO(1,d) and linear programming, the method yields rigorous upper bounds on the spectral gap. In several examples, the bound is nearly sharp. The bounds also allow us to determine the set of spectral gaps attained by all hyperbolic 2-orbifolds. The ideas were inspired by recent developments in the conformal bootstrap. The linear program is similar to the Cohn+Elkies linear program for bounding sphere packing density. Based on https://arxiv.org/abs/2111.12716 with P. Kravchuk and S. Pal and work in progress with the same collaborators and J. Bonifacio.

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Dieter Luest

Title: Distances Conjectures and Primordial Black Holes as Dark Matter

Abstract: Distance conjectures predict a light tower of states near the boundary of moduli space in string compactifications. It was argues that this should be also the case in the limit of a small cosmological constant in quantum gravity and in string theory. In my talk, I will introduce the distance conjectures and I will further discuss that within the recently introduced dark universe scenario, where a micron-size extra dimension is related to the size of the cosmological, primordial black holes are viable dark matter candidates.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Daniel Platt

Title: An application of numerical techniques to rigorous proof in special holonomy

Abstract: Approximations of Calabi-Yau metric are a popular tool to produce heuristics, but so far have not been leveraged to rigorously prove theorems in geometry. I present one work in progress, in which we prove that the real loci of certain Calabi-Yau manifolds admit harmonic nowhere vanishing 1-forms, which are needed for an application in G2-geometry. I will explain the proof strategy, which consists of two parts: first, I formulate an estimate for the difference between approximate metric and true Calabi-Yau metric in terms of the Ricci curvature of the approximate metric which is of independent interest. Second, I explain the connection between nowhere vanishing 1-forms with respect to the two different metrics. This is joint work with Rodrigo Barbosa, Michael Douglas, and Yidi Qi.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Matthew Headrick

Title: Numerical solution of systolic minimal-area problems

Abstract: Zwiebach's covariant closed string field theory requires knowledge of the minimal-area metric with unit systole on a given punctured Riemann surface (the systole being the length of the shortest non-contractible loop). Exact solutions are known only in genus 0 with any number of punctures, and genus 1 with zero punctures. I'll explain how, using calibrations, the systolic condition can be imposed as a local constraint, allowing the minimal-area problem to be formulated as a convex program that is amenable to numerical solution. This program admits a dual program that is also numerically solvable, and that reveals interesting qualitative features of the solutions. I will exhibit some solutions obtained by this method, and comment on applications and generalizations of the method. (Based on arXiv:1806.00449 and arXiv:1806.00450 with B. Zwiebach.)

Wednesday, November 16th

8:30am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: Mark Haskins

Title: A working differential geometer's experiences with computational differential geometry

Abstract: In this talk I will explain how computational differential geometry has influenced some of my own theoretical work in differential geometry and, in particular, led to proving theorems that I would not have even guessed at without computational input. The geometry in question relates to exceptional holonomy metrics. Time permitting, I will discuss other related problems where I expect that useful insights could be gleaned from further computations and what possible roadblocks there might be.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102**

Speaker: Tommy Murphy

Title: The geometry of molecular surfaces

Abstract: The similarity principle can be phrased as saying two molecules which have similar shape will have a similar effect when used in a drug, meaning that finding a way to quantify how similar two given molecules are is a major problem in chemistry. I will give an overview on some applications of differential geometry to this topic, mostly focusing on describing the surface as a complex manifold and employing Kaehler quantization to provide a new answer to this question. This is joint work with D.Cole, R. Pirie and S.J. Hall.

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Andre Lukas

Title: Yukawa Couplings from String Theory

Abstract: Computing the masses of matter fields is one of the major open problems in the quest for realistic physics from string theory. I discuss the status, recent progress and outstanding issues in this area, focusing on compactifications of the heterotic string on Calabi-Yau three-folds. In particular, I will explain how recent computational advances in finding Ricci-flat Calabi-Yau metrics and related quantities can help with computing Yukawa couplings and the resulting masses of matter fields.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Fabian Ruehle

Title: Numerical metrics for Calabi-Yau and other reduced holonomy manifolds

Abstract: I will explain how to leverage neural networks to approximate Calabi-Yau or SU(3) structure metrics. We vastly extend previous work in this area to provide approximations for manifolds that can be described as hypersurfaces in toric varieties or complete intersections in projective spaces. While extensions to the latter are rather straight-forward, the former require more work. I will first explain how to obtain a metric that servers as a starting point, and then how to sample points uniformly from these spaces with respect to this metric.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Maksymilian Manko

Title: Critical Volumes of Toric Sasaki-Einstein Manifolds and Neural Networks Explainability

Abstract: The problem of Z-minimisation was introduced and well-studied early this century. For a Kaehler cone over a (compact) Sasaki manifold base it requires finding such a contact structure that minimises the base's volume, which is equivalent to the Ricci flatness condition on the cone and Einstein condition on the base. The advent of datasets of reflexive polytopes allowed computational studies of the problem in a large class of toric Calabi-Yau varieties leading to new observations, such as the upper bound on the said volumes we proved. After briefly recounting the problem, we will show that the critical volumes and other related quantities can be effectively machine-learned using Neural Networks and easy-to-compute geometric features of the varieties. Furthermore, we will discuss the gradient saliency method, which allows to intuit the relative importance of particular features for the NN. It has recently proved successful in structurally similar problems in knot theory and we will show it can be similarly applied in this toric-geometric case.

6:00pm **Workshop Banquet - SCGP Cafe**

Title: Banquet Dinner

Thursday, November 17th

8:30am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: Christopher Bishop

Title: Weil-Petersson curves, knot energies, traveling salesman theorems, and minimal surfaces

Abstract: Weil-Petersson curves are a class of rectifiable closed Jordan curves in the plane, defined as the closure of the smooth curves with respect to the Weil-Petersson metric defined by Takhtajan and Teo in 2009. Their work was motivated by string theory and makes the space of closed loops into a Hilbert manifold, but the same class of curves also arises naturally in complex analysis, geometric measure theory, probability theory, knot theory, computer vision, and other areas. No geometric description of Weil-Petersson curves was known until 2019, but there are now more than twenty equivalent conditions, many of which extend to curves in higher dimensions and remain equivalent there. One involves inscribed polygons and can be explained to a calculus student. Another is a strengthening of Peter Jones's traveling salesman theorem characterizing rectifiable curves. A third says a curve is Weil-Petersson iff arclength has finite renormalized electrostatic energy for an inverse cube law, and yet another says a curve if Weil-Petersson iff it bounds a minimal surface in hyperbolic 3-space that has finite total curvature, or equivalently, finite renormalized area. I will discuss these and several other characterizations and sketch why they are all equivalent to each other. The lecture will contain many pictures, several definitions, but not too many proofs or technical details.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102**

Speaker: Javier Gomez Serrano

Title: Self-similar blow up profiles for fluids via physics-informed neural networks

Abstract: In this talk I will explain a new numerical framework, employing physics-informed neural networks, to find a smooth self-similar solution for different equations in fluid dynamics. The new numerical framework is shown to be both robust and readily adaptable to several situations.

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102/ZOOM**

Speaker: Andrei Constantin

Title: Line bundle cohomology formulae on complex projective varieties

Abstract: Vector bundle cohomology represents a key ingredient for string phenomenology, being associated with the massless spectrum arising in string compactifications on smooth compact manifolds. Although standard algorithmic techniques exist for performing cohomology calculations, they are laborious and ill-suited for scanning over large sets of string compactifications to find those most relevant to particle physics. In this talk I will review some recent progress in deriving closed-form expressions for line bundle cohomology and discuss applications to string phenomenology.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Burt Ovrut

Title: Realistic Heterotic M-Theory Vacua and Stabilizing Kahler Moduli

Abstract: Within the context of Heterotic M-Theory, we analyze a class ofvacua that are phenomenologically realistic at low energy; that is, they have the correct gauge group, particle spectrum, W/Z gauge boson masses, the experimentally observed Higgs mass, all superpartner masses exceed their present lower bounds and so on. However, it is demonstrated in detail that these realistic properties only occur if the K ?ahler moduli and dilaton vacuum expectation values all lie in a specific region of moduli space. Within the context of a simple model, it is shown that these moduli can indeed be stabilized in the physically acceptable regime. The importance of extending these results to a general theory of stabilizing all moduli is emphasized.

3:30pm **- SCGP 102**

Speaker: Warner Miller

Title: Insights into classical and quantum gravity from Regge Calculus

Abstract: A brief introduction to Regge Calculus where curved spaces are approximated by higher-dimensional polyhedra will be provided. The importance of the hybrid cells coupling the Voronoi and Delaunay lattices will be emphasized in developing the underlying structure of this discrete geometry. The orthogonality in these cells is used to analyze the discrete versions of the equations of general relativity and to better understand the nature of its approximate diffeomorphism structure and to reveal the gravitational degrees of freedom in the theory. The structure of these hybrid cells yields equations that appear to be less complex than their continuum counterparts. We will then develop the initial-value data and evolution for Regge Calculus and briefly highlight some numerical applications, including Ricci flow in 3D. We introduce a novel path integral for quantum gravity following motivation from A. D. Alexandrov. Following the words of J. A. Wheeler that "a theory in and of itself should in principle reveal what questions can and cannot be asked," we seek a deeper interpretation of the propagator in quantum gravity.

Friday, November 18th

8:30am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: Daniel Waldram

Title: Generalising G2 geometry: involutivity and moment maps

Abstract: Seven-dimensional Riemannian manifolds M with holonomy given by the exceptional Lie group G2 play a special role in geometry and compactifications of string theory. In the latter context, they are part of a larger class of supersymmetric geometries that include "flux" degrees of freedom. In this talk I'll review how this more general class can be described using a version of Hitchin's generalised geometry based on the exceptional group E7. The geometries appear formally as a Kahler quotient of an infinite-dimensional space of generalised structures, analogous to the description of, for example, Kahler-Einstein metrics. One finds an analogue of Kahler-Ricci flow, an extension of Hitchin's G2 functional and possible analogues of Futaki invariants. Crucially, the only solutions for compact M have vanishing fluxes, and so this framework may suggest new ways of understanding the existence of conventional G2 manifolds.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102/ZOOM**

Speaker: Yang-Hui He

Title: Machine-Learning Mathematical Structures

Abstract: We present a number of recent experiments on how various standard machine-learning algorithms can help with pattern detection across disciplines ranging from algebraic geometry, to representation theory, to combinatorics, and to number theory. We speculate on whether there is an inherent hierarchy of "difficulty" in mathematics reflected by data. At the heart of the programme is the question how does AI help with mathematical discovery.

12:00pm **Lunch - SCGP Cafe**

3:30pm **Tea Time - SCGP Cafe**