Combinatorics and Geometry of Convex Polyhedra

Events for:
Monday, March 20th - Friday, March 24th

Monday, March 20th

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Alexander Barvinok - SCGP 102**

Speaker: Alexander Barvinok

Title: A quick estimate for the volume of a polyhedron

Abstract: Abstract: Let P be a bounded polyhedron defined as the intersectionof the non-negative orthant in R^n and an affine subspace of codimension m. I present a simple and computationally efficient formula that approximates the volume of P within a factor of c^m, where c > 0 is an absolute constant. I also plan to discuss ramifications, based on the maximum entropy approach, for counting integer points in polytopes. The first part of the talk is based on a recent joint work with Mark Rudelson, whereas the second part is based on an earlier joint work with John Hartigan.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Arnau Padrol - SCGP 102**

Speaker: Arnau Padrol

Title: Acyclonestohedra

Abstract: Motivated by Galashin's poset associahedra, we introduceoriented building sets defined on the ground set of an oriented matroid, and their associated acyclic nested complexes, which are nested complexes fulfilling an additional acyclicity condition. Feichtner and Kozlov defined building sets and nested complexes for arbitrary meet semi-lattices, and acyclic nested complexes are those associated to the Las Vergnas face lattices of acyclic oriented matroids. They can be realized via stellar subdivisions, which provides a polytopal realization whenever the original oriented matroid is realizable. When the original oriented matroid is the graphical matroid of a Hasse diagram, we recover this way Galashin's realization of poset associahedra as an iterated stellar subdivision of an order polytope. Since the Las Vergnas lattices are atomic, acyclic nested complexes can be embedded into (boolean) nested complexes. For realizable oriented matroids, we provide a novel polytopal construction that realizes this embedding geometrically by exhibiting acyclonestohedra as linear sections of certain (boolean) nestohedra. This is joint work with Chiara Mantovani and Vincent Pilaud.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

2:30pm **Workshop: Pavel Galashin - SCGP 102**

Speaker: Pavel Galashin

Title: Parity duality for the amplituhedron

Abstract: I will give some background on the (tree) amplituhedron A_{n,k,m}(Z) and its "triangulations". Then I will explain a duality operation on such triangulations that interchanges the parameters k and n-m-k. Joint work with Thomas Lam.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Gaku Liu - SCGP 102**

Speaker: Gaku Liu

Title: Unimodular triangulations of sufficiently large dilations

Abstract: An integral polytope is a polytope whose vertices haveinteger coordinates. A unimodular triangulation of an integral polytope in R^d is a triangulation in which all simplices are integral with volume 1/d!. A classic result of Kempf, Mumford, and Waterman states that for every integral polytope P, there exists a positive integer c such that cP has a unimodular triangulation. We strengthen this result by showing that for every integral polytope P, there exists c such that for every positive integer c'> c, polytope c'P also admits a unimodular triangulation.

Tuesday, March 21st

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Katharina Jochemko - SCGP 102**

Speaker: Katharina Jochemko

Title: Generalized permutahedra: Minkowski linear functionals andEhrhart positivity

Abstract: Generalized permutahedra form a combinatorially richclass of polytopes that appear naturally in various areas of mathematics. They include many interesting and significant classes of polytopes, in particular, matroid polytopes. We study functions on generalized permutahedra that behave linearly with respect to dilation and taking Minkowski sums. We present classification results and discuss how these can be applied to prove positivity of the linear coefficient of the Ehrhart polynomial of generalized permutahedra. This is joint work with Mohan Ravichandran.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Luis Ferroni - SCGP 102**

Speaker: Luis Ferroni

Title: Valuative invariants for large classes of matroids

Abstract: Matroid invariants are ubiquitous, and it is surprising that many of them behave valuatively under base polytope subdivisions, even when they are defined in totally unrelated matroidal frameworks. We will give a brief explanation of why certain invariants, such as the Tutte polynomial, the Kazhdan-Lusztig polynomial or the Hilbert series of the Chow ring of a matroid are valuations. Also, we will explain how one can use this property in practice to obtain explicit formulas of this invariants on "large" classes of matroids. This framework is particularly useful to find counterexamples to conjectures in matroid theory, or to support conjectures. Based on joint work with Benjamin Schroter.

11:30am **Workshop: Jose Samper - SCGP 102**

Speaker: Jose Samper

Title: Unbounded matroids

Abstract: An unbounded matroid on a finite set E is a (possibly) unbounded generalized permutohedron in R^E such that all its vertices have 0-1 coordinates. Associated to each such polyhedron Q is a poset S on E coming from the recession cone of Q. We will explain how the theory of these polyhedra admits several cryptomorphic descriptions that specialize to classical matroid theory if we take S to be an antichain and to polymatroid theory if we allow S to be a disjoint union of chains. We show that each such polyhedron can be constructed (non-uniquely) from a matroid and a poset on E. Then we use this to produce several matroid constructions that seem to be interesting on their own. This talk is based on joint work with Jonah Berggren and Jeremy Martin.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **SCGP Weekly Talk & Workshop Speaker: Jesus De Loera - SCGP 102**

Speaker: Jesus De Loera

Title: A path through mathematics the polyhedral way

Abstract: Convex Polyhedra are among some of the oldest mathematical objects (e.g., platonic solids) and captivate mathematicians and non-mathematicians alike with their beauty. But many mathematicians do not know the surprising fact they naturally appear in many mathematical areas from the pure (representation theory, number theory, combinatorics), to the applied (optimization, statistics). I will talk about several open problems on convex polyhedra that prove their strong centrality and depth in mathematics and that are the subject of active research, including some of what is presented in the Simons Institute event.

2:30pm **Workshop: Federico Castillo - SCGP 102**

Speaker: Federico Castillo

Title: Lineup polytopes

Abstract: Motivated by an instance of the quantum marginal problem in physics, we define the r-lineup polytope of P as a polytope parametrizing all possible linear orders on the vertices of P. We focus on the combinatorial cases such as the hypersimplex and products of simplices. These examples lie in between the sweep polytopes of Padrol and Philippe and the theory of symmetric polytopes.This is based on joint work with JP. Labbe, J. Liebert, A. Padrol, E. Philippe and C. Schilling.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Open Problem Session - SCGP 102**

Title: Open Problem Session

Wednesday, March 22nd

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Vincent Pilaud - SCGP 102**

Speaker: Vincent Pilaud

Title: Unexpected diagonals

Abstract: Cellular approximations of diagonals of polytopes are important tools homotopy theory. Geometrically, they can be constructed via the theory of fiber polytopes, and their projections produce interesting polyhedral subdivisions. This talk will present some unexpected enumerative properties of the diagonals of the permutahedron and the associahedron. It is based on ongoing joint work with Alin Bostan and Frederic Chyzak (diagonal of the associahedron) and with Berenice Delcroix-Oger, Matthieu Josuat-Verges, Guillaume Laplante-Anfossi and Kurt Stoeckl (diagonal of the permutahedron).

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Geva Yashfe - SCGP102/ZOOM**

Speaker: Geva Yashfe

Title: Sphere triangulations, Dual Graphs, and Expanders

Abstract: Kalai asked whether the dual graphs of a family of simplicial 4-polytopes can be an expander family; Gromov asked the analogous question for the dual graphs of simplicial 3-spheres. A construction due to Loiskekoski and Ziegler shows that there exists a family of simplicial 4-polytopes whose dual graphs do not have small separators, but their examples do not form an expander family. We construct a family of (possibly non-polytopal) simplicial 3-spheres whose dual graphs do expand. More generally, we describe a flexible method to construct simplicial homology spheres from a certain class of bounded-degree graphs. Based on joint work with Karim Adiprasito.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

2:30pm **Workshop: Hugh Thomas - SCGP 102**

Speaker: Hugh Thomas

Title: Harder-Narasimhan polytopes

Abstract: Over the past five years, a number of classes of lattice polytopes have been realized as Harder-Narasimhan polytopes of a representation of a quiver. I will explain this idea. No familiarity with quiver representations will be assumed. One of the motivations driving this work is from the physics of scattering amplitudes, and if time permits, I will say something about this.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Alexey Garber - SCGP 102**

Speaker: Alexey Garber

Title: Flips in two-dimensional hypertriangulations

Abstract: The notion of hypertriangulations was coined by Olarte and Santos as the finest subdivisions induced by projections of hypersimplices. The notion builds on the definition of (partial) triangulations of a point set A as subdivisions induced by a projection of a simplex on the convex hull of A. In the talk I will give a geometric interpretation of flips coming from the Baues poset of all subdivisions in the planar case and prove that in that situation the level-2 hypertriangulations of generic point set are flip-connected. The talk is based on a joint work with Herbert Edelsbrunner, Mohadese Ghafari, Teresa Heiss, and Morteza Saghafian.

4:30pm **Workshop: Hailun Zheng - SCGP 102**

Speaker: Hailun Zheng
Asbtract: A d-polytope is called (d-i)-simplicial if all of its (d-i)-faces are simplices. It is i-simple if every (d-i-1)-face belongs to exactly i+1 facets. A few low-dimensional examples of (d-i)-simplicial i-simple polytopes arising from regular polytopes are known. For d>3, a d-dimensional demicube and its dual are 3-simplicial (d-3)-simple and (d-3)-simplicial 3-simple, respectively. In addition to these finitely many examples, Paffenholz and Ziegler proved the existence of infinite families of (d-2)-simplicial 2-simple d-polytopes for all d>3. However, for general i>4 and d>2i-1, it is not known whether non-simplex (d-i)-simplicial i-simple d-polytopes exist. Given two d-polytopes P and Q, where P has a simplex facet F and Q has a simple vertex, we define an operation called the merge of P and Q along F and v. We show that if for some 0<i<d, both P and Q are (d-i)-simplicial i-simple, then the merge of P and Q is also (d-i)-simplicial i-simple. We then use this operation to construct infinite families of i-simplicial i-simple 2i-polytopes for all
i<5. Furthermore, infinitely many of these polytopes have another nice property: they are self-dual. Joint work with Isabella Novik.

Title: The merging operation and (d-i)-simplicial i-simple d-polytopes

5:30pm **Workshop: Banquet - SCGP Cafe**

Title: Banquet Dinner

Thursday, March 23rd

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Alejandro Morales - SCGP 102**

Speaker: Alejandro Morales

Title: Combinatorial and metric properties of flow polytopes

Abstract: Flow polytopes of graphs are a rich family of polytopes of interest in probability, optimization, representation theory, and algebraic combinatorics. Computing their volumes and enumerating lattice points of some particular flow polytopes turn out to be combinatorially interesting problems that involve beautiful enumeration formulas and many familiar combinatorial objects like permutahedra and associahedra. Baldoni and Vergne found a series of formulas for both of these purposes, which they call Lidskii formulas, that are combinatorially powerful and pleasant and have log-concavity properties. Flow polytopes also have interesting subdivisions by Postnikov and Stanley that are recursive and triangulations by Danilov-Karzanov-Koshevoy related to cluster algebras. I will give an overview of recent work on these polytopes including formulas that relate their volume to the number of lattice points, the geometry and combinatorics of their triangulations, and some open questions.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Theo Douvropoulos - SCGP 102**

Speaker: Theo Douvropoulos

Title: Almost colored f-vectors for generalized Associahedra

Abstract: After seminal works of Fomin, Reading, Zelevinsky, and Chapoton, the (generalized) W-Associahedra D(W) have become prominent members in the world of Coxeter-Catalan combinatorics. Fomin and Reading computed their f-vectors in a case-by-case way for all finite reflection groups W and observed that they are often given in terms of product formulas generalizing Kirkman-Cayley-type numbers. During the last twenty years the community has developed this theory extensively, building further realizations for the dual cluster complexes, enhancing connections to the noncrossing partition lattice NC(W), the representation theory of W and more. Still however we were missing a uniform understanding of the f-vectors of D(W) and an explanation for their structure. We present recent work, in part joint with Matthieu Josuat-Verges, where we give case-free proofs of product formulas for an almost colored refinement of the f-vectors of D(W). This refinement counts faces with respect to parabolic type, gives a justification for the Fomin-Reading counts, and in type A reveals connections with formal power series inversions and the nabla operator from symmetric functions.

11:30am **Workshop: Eran Nevo - SCGP 102**

Speaker: Eran Nevo

Title: Rigidity expanders

Abstract: The d-dimensional algebraic connectivity a_d(G) of a graph G=(V,E) is a quantitative measure of d-dimensional rigidity introduced by Jordan and Tanigawa, in particular a_d(G)>0 if and only if G is generically d-rigid. It extends Fiedler's notion of algebraic connectivity, corresponding to the d=1 case. We prove the existence of (2d+1)-regular "d-rigidity spectral expanders". That is, we show that there exists a constant c(d)>0 and a family of (2d+1)-regular graphs G_n with increasing number of vertices satisfying a_d(G_n) > c(d). We conjecture that no such positive constant exists for 2d-regular graphs. All the relevant background will be given during the talk. Based on joint work in progress with Alan Lew, Yuval Peled and Orit Raz.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Daria Poliakova - SCGP 102**

Speaker: Daria Poliakova

Title: Polyhedral diagonals and beyond

Abstract: Algebraists (or at least some of them) are interested in cellular diagonals of operadically meaningful polyhedra. I will remind some known formulas - "magic formula" for associahedra, Saneblidze-Umble/Laplante-Anfossi formulas for permutahedra. I will explain how a slight generalization of these ideas from polyhedra to subdivisions should lie behind homotopy monoidality.

2:30pm **Workshop: Raman Sanyal - SCGP 102**

Speaker: Raman Sanyal

Title: Pivots, polytopes, and combinatorics

Abstract: A pivot rule is the mechanism that tells the simplex algorithm which path to take on a linear program from a given vertex to an optimal one. We introduced pivot polytopes as a mean to capture the behaviour of certain classes of pivot rules on a given linear program. While this gives a new perspective on pivot rules, it turns out that pivot polytopes are also of interest to combinatorialists. I this talk I will discuss - how pivot polytopes relate flag matroid polytopes to nestohedra, - how pivot polytopes encode associative structures on 2-dimensional noncrossing structures (subsuming permutahedra, associahedra, multiplihedra), and - how pivot polytopes give a new perspective on fiber polytopes of cyclic polytopes. The talk is based on joint work with Benjes, Black, De Loera, Lutjeharms, and Poullot.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

Friday, March 24th

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Zuzana Patakova - SCGP 102**

Speaker: Zuzana Patakova

Title: Radon and fractional Helly type theorems

Abstract: Radon theorem plays a basic role in many results of combinatorial convexity. It says that any set of d+2 points in R^d can be split into two parts so that their convex hulls intersect. It implies Helly theorem and as shown recently also its more robust version, so-called fractional Helly theorem. By standard techniques this consequently yields an existence of weak epsilon nets and a (p,q)-theorem. We will show that we can obtain these results even without assuming convexity, replacing it with very weak topological conditions. More precisely, given an intersection-closed family F of subsets of R^d, we will measure the complexity of F by the supremum of the first d/2 Betti numbers over all elements of F. We show that constant complexity of F guarantees versions of all the results mentioned above. Partially based on joint work with Xavier Goaoc and Andreas Holmsen.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Lei Xue - SCGP 102**

Speaker: Lei Xue

Title: A Proof of Grunbaum's Lower Bound Conjecture for polytopes, lattices, and strongly regular pseudomanifolds.

Abstract: If we fix the dimension and the number of vertices of a polytope, what is the smallest number of faces of each dimension? In 1967, Grunbaum made a conjecture on this lower bound problem for d-dimensional polytopes with at most 2d vertices. In the talk, we will discuss the proof of this conjecture and the equality cases. We will then extend our results to lattices with diamond property (the inequality part) and to strongly regular normal pseudomanifolds (the equality part). We will also talk about recent results on d-dimensional polytopes with 2d+1 or 2d+2 vertices.

11:30am **Workshop: Joshua Hinman - SCGP 102**

Speaker: Joshua Hinman

Title: A Positive Answer to Barany's Question on Face Numbers ofPolytopes

Abstract: Although the face numbers of simple and simplicial polytopes are well understood, we still know frustratingly little about the face numbers of polytopes in general. In the late nineties, Imre Barany asked a fascinating question: for every convex polytope, does the number of k-dimensional faces is no less than the minimum of the numbers of vertices and the number of facets? In practice, the answer always seemed to be "yes", but for no obvious reason. In this talk, we will add a piece to the puzzle of understanding face numbers by answering Barany's question in the affirmative. We will also prove a stronger statement in the form of linear inequalities on the face numbers of P.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time