Mathematical Billiards: At the Crossroads of Dynamics, Geometry, Analysis, and Mathematical Physics: November 13 – 17, 2023

Events for:
Monday, November 13th - Friday, November 17th

Monday, November 13th

8:30am   **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am   **Workshop: Vered Rom-Kedar - SCGP 102**

Speaker: Vered Rom-Kedar

Title: On some Impact-like systems

Abstract: Near integrable Hamiltonian flows with impacts that respect the symmetries of the integrable structure, either globally or locally, provide classes of non-smooth near integrable, or, respectively, near Quasi-Integrable (QI), systems [1-3]. Their extension to smooth systems with large gradients at the domains' boundary, namely to impact-like systems [4], allows, in the near-integrable case, away from grazing tori, despite the non-smooth limit of such systems, to utilize KAM theory [1]. Near grazing tori, the return map of the impact system has a piecewise-smooth rotation with a square-root singularity. Under perturbation, the corresponding tangency-standard-map is derived and is shown to exhibit long transients that are yet to be explained [5]. While the level sets of the integrable impact systems are tori, QI systems also include level sets of genus 2 and higher [3]. Ergodicity on levels sets is proved for some classes of QI Hamiltonian impact systems for a full measure of iso-energy level sets [6] (yet, quantization of such systems suggests that their wavefunctions do not equidistribute in the configuration space in the large energy limit [7]). Return maps for the near-QI systems are shown to be piecewise smooth symplectic maps which are close to families of interval exchange maps. Ongoing projects (e.g. [8]) include the development and study of simplified models for these return maps: regular and singular resonances emerge, as well as transient behavior, leading to conjectures regarding the non-existence of dividing circles in the corresponding singularity band.

10:30am   **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee

11:00am   **Workshop: Dmitry Jakobson - SCGP 102**

Speaker: Dmitry Jakobson

Title: Quantum ergodicity for manifolds and billiards."

Abstract: We shall survey some classical and more recent results on quantum ergodicity for manifolds and billiards.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

2:30pm **Workshop: Amir Vig - SCGP 102**

Speaker: Amir Vig

Title: Compactness of the Marked Length Isospectral Set

Abstract: For strictly convex billiard tables, the marked length spectrum encodes the lengths of action minimizing orbits having a given rational rotation number. A renormalization of these lengths extends to a continuous function known as Mather's beta function or the mean minimal action. We show that using the algebraic structure of its Taylor coefficients, one can prove C infinity compactness of marked length isospectral sets, giving a dynamical analogue of the Laplace spectral results of Melrose, Osgood, Phillips and Sarnak.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Martin Leguil - SCGP 102**

Speaker: Martin Leguil

Title: Birkhoff attractors of dissipative billiards

Abstract: We consider a particle moving within some convex planar billiard according to a modified reflection law, where collisions become inelastic; more precisely, at each (non-orthogonal) collision with the boundary, the (unoriented) outgoing angle of reflection is strictly smaller than the incoming angle of incidence, both being measured with respect to the normal. The resulting dissipative billiard map has a global attractor. In a joint work with A. Florio and O. Bernardi, we study the topological and dynamical complexity of an invariant subset of this attractor, the so-called Birkhoff attractor, whose study goes back to Birkhoff, Charpentier, and more recently, the work of Le Calvez. We show that for a generic convex billiard, the Birkhoff attractor will be « simple » (typically, a normally contracted manifold) when the dissipation is strong; on the contrary, we show that if the dissipation is mild, then the Birkhoff attractor is topologically « complicated » (an indecomposable continuum) and has rich dynamics (rotation set with non-empty interior, presence of horseshoes…).

Tuesday, November 14th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Sergey Bolotin - SCGP 102**

Speaker: Sergey Bolotin

Title: Two degenerate billiard-like problems

Abstract: The first problem comes from studying near collision orbits in celestial mechanics. In the limit when some masses tend to zero we obtain a system with elastic reflections. Sometimes this makes it possible to prove the existence of near collision chaotic solutions. The second problem appears when a Riemannian metric tends to infinity at the boundary of a domain. After regularization the geodesics resemble billiard trajectories. This problem is motivated by the quasiclassical approximation for a degenerate wave equation (work in progress with Dmitry Treschev).

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee

11:00am **Workshop: Susanna Terracini - SCGP 102**

Speaker: Susanna Terracini

Title: On the variational approach to expanding and oscillating solutions to the N-body problem

Abstract: We report on the functional-analytic approach to the search of unbounded trajectories in the N-body problem (hyperbolic, parabolic, parabolic-hypebolic, oscillating etc.). We explore the use of renormalised energies in various contexts together with other global variational and topological methods. The same approach is pursued in the search for symbolic dynamics in various relevant models of celestial mechanics.

12:00pm **Workshop: Group Photo - SCGP Lobby**

   Title: Group Photo

12:00pm **Workshop: Lunch - SCGP Cafe**

   Title: Lunch

2:30pm   **Workshop: Mark Levi - SCGP 102**

   Speaker: Mark Levi

   Title: Adiabatic invariants in billiards.

   Abstract: I will discuss some examples of adiabatic invariants in billiards with moving walls and in billiards with cusps with gravity.

3:30pm   **Workshop: Tea Time - SCGP Cafe**

   Title: Tea Time

4:00pm   **Workshop: Yaofeng Su - SCGP 102**

Speaker: Yaofeng Su

Title: Some new results for open dynamical systems.

Abstract: Open dynamical systems study the statistics of the first hitting time when trajectories escape through a subset, referred to as a "hole," in the phase space. I will discuss my recent work on open hyperbolic systems, addressing three main aspects: 1. When the "hole" in the phase space takes the shape of a ball in a Riemann manifold, we established Poisson limit laws for certain dissipative systems. Our conditions are loosely dependent on the Hausdorff dimension of the SRB measure. 2. For hyperbolic billiard systems, the natural "hole" resides on the boundary of billiard tables, corresponding to a strip-shaped subset in the phase space. We obtain the Poisson limit law for open billiard systems with arbitrarily slow mixing rates. 3. To obtain the convergence rates associated with Poisson limit laws, we prove a maximal-type large deviation result for arbitrarily slowly mixing expanding systems. I will describe how this result can be applied to the inducing method employed in ergodic theory, particularly focusing on its application to open billiard systems during this presentation. In contrast to the chaotic systems discussed before, non-chaotic systems do not exhibit Poisson limit laws for first hitting times. In this context, I will describe the statistics of first hitting times for such open systems and note that Riemann hypothesis can be rephrased in relation to a convergence rate of this statistical law. This is joint work with Prof. Leonid Bunimovich.

Wednesday, November 15th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Dmitry Dolgopyat - SCGP 102**

Speaker: Dmitry Dolgopyat

Title: Infinite volume mixing for billiards

Abstract: In many problems of statistical mechanics one is interested in mixing properties of extended observables for infinite measure preserving systems. In this talk I will discuss several notions of mixing introduced by Marco Lenci and show how the tools from finite volume dynamics could be used to analyze these notions. The talk is based on a joint work with Peter Nandori.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee

11:00am **Workshop: Mark Demers - SCGP 102**

Speaker: Mark Demers

Title: Projective Cones for Dispersing Billiards

Abstract: We describe the recent construction of Birkhoff cones which are contracted by the action of transfer operators corresponding to dispersing billiard maps. The explicit contraction provided by this construction permits the study of statistical properties of a variety of sequential and open billiards. We will discuss some applications of this technique to chaotic scattering and the random Lorentz gas. This is joint work with C. Liverani.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

2:30pm **Workshop: Lael Costa**

Speaker: Lael Costa

Title: Experiments in outer length billiards

Abstract: The familiar inner and outer (or dual) billiards in the plane may be defined in variational terms using the length and area, respectively, of orbits. This suggests two additional billiards variants: "inner with area" and "outer with length," both described by P. Albers and S. Tabachnikov. In this talk, I will mostly present the results of computer experiments with the outer length variant, as well as several preliminary results on the behavior of this curious system, made in collaboration with those named above.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Maxim Arnold - SCGP 102**

Speaker: Maxim Arnold

Title: Optimal property of the Symmedian point

Abstract: An inscribed Euclidean polygon can be considered as a hyperbolic ideal polygon in a natural way. We develop a new dynamical coordinatization of the symmedian point of a Euclidean triangle, and use it to prove that it simultaneously satisfies several minimality properties, within both Euclidean and hyperbolic geometry. We also introduce a generalization of this point to other ideal hyperbolic polygons and provide few constructions of these points, leading to seemingly new incidence results. This is a joint work with Carlos Arreche.

6:00pm **Workshop: Dinner Banquet - SCGP Cafe**

Title: Dinner Banquet

Thursday, November 16th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Maria Saprykina - SCGP 102**

Speaker: Maria Saprykina

Title: Nonconmutative coboundary equations over integrable dynamics

Abstract: In this joint work with Rafael de la Llave we prove an analog of Livshits theorem for real-analytic families of cocycles over an integrable system with values in a Banach algebra $\mathcal G$ or a Lie group. Namely, we consider an integrable dynamical system $f:\mathcal M \equiv\mathbb T^d \times [-1,1]^d\to \mathcal M$, $f(\theta, I)=(\theta + I, I)$, and a real-analytic family of cocycles $\eta_\epsilon : \mathcal M \to \mathcal G$, indexed by a complex parameter ϵ in an open ball $\mathcal E_\rho \subset \mathbb C$. We show that if η_ϵ is close to identity and has trivial periodic data, i.e., $$ \eta_\epsilon(f^{n-1}(p))\dots \eta_{\epsilon}(f(p))\cdot \eta_{\epsilon} (p)=Id $$ for each periodic point $p=f^n p$ and each $\epsilon \in \mathcal E_{\rho}$, then there exists a real-analytic family of maps $\phi_\epsilon: \mathcal M \to \mathcal G$ satisfying the coboundary equation $$ \eta_\epsilon(\theta, I)=(\phi_\epsilon\circ f(\theta, I))^{-1} \cdot \phi_\epsilon(\theta, I) $$ for all $(\theta, I)\in \mathcal M$ and $\epsilon \in \mathcal E_{\rho/2}$. We also show that if the coboundary equation above with an analytic left-hand side η_ϵ has a solution in the sense of formal power series in ϵ, then it has an analytic solution.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee

11:00am **Workshop: Alberto Abbondandolo - SCGP 102**

Speaker: Alberto Abbondandolo

Title: On symplectic capacities and volume

Abstract: I will give an overview talk on the present state of a conjecture of Viterbo concerning symplectic capacities and volume of convex bodies.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

2:30pm   **Workshop: Anastasiia Sharipova - SCGP 102**

Speaker: Anastasiia Sharipova

Title: Convex bodies with all characteristics planar

Abstract: I will show that in symplectic space smooth strongly convex bodies with all characteristics planar or all outer billiard trajectories planar are affine symplectic images of a ball.

3:30pm   **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm   **Workshop: Pazit Haim-Kizlev - SCGP 102**

Speaker: Pazit Haim-Kizlev

Title: On the existence of symplectic barriers

Abstract: Results concerning rigidity of Lagrangian submanifolds lie at the heart of symplectic topology, and have been intensively studied since the 1990s. These theories include some of the fundamental cornerstones of symplectic geometry today, such as the Arnold-Givental conjecture and Lagrangian Floer theory, which have significantly improved our understanding of the field, solving problems which previously seemed unattainable. An important feature of these theories is that, many times, intersections between Lagrangian submanifolds cannot be avoided using symplectic maps. An example for this phenomenon is the concept of Lagrangian Barriers, a form of symplectic rigidity introduced by Biran in 2001, which involves obligatory intersections of symplectic embeddings with Lagrangian submanifolds not derived from mere topology. Conversely, when considering submanifolds that do not locally contain Lagrangian submanifolds, there are results in the opposite direction - results regarding flexibility. These results indicate that, in many cases, it is possible to symplectically displace these submanifolds. In this joint work with Richard Hind and Yaron Ostrover, we reveal rigidity phenomena of non-Lagrangian submanifolds by introducing what appears to be the first illustration of Symplectic Barriers, a form of symplectic rigidity stemming from necessary intersections of symplectic embeddings with symplectic submanifolds (and in particular not Lagrangian). The key point is that Lagrangian submanifolds are not the sole barriers, and there exist situations where a symplectic submanifold does not exhibit flexibility.

Friday, November 17th

8:30am   **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Otto Vaughn Osterman - SCGP 102**

Speaker: Otto Vaughn Osterman

Title: Length Spectrum Rigidity in Dispersing Billiard Systems

Abstract: The problem of spectral rigidity in dispersing billiard systems is that of determining whether the set of perimeters of periodic orbits uniquely determines the billiard table up to isometry. We consider this problem for the class of dispersing billiard systems consisting of three scatterers in the plane satisfying the non-eclipse condition. My result is that the perimeters of a particular set of periodic orbits for two such systems are identical if and only if their collision maps are analytically conjugated to each other in some neighborhood of a particular homoclinic orbit.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee

11:00am **Workshop: Giovanni Forni - SCGP 102**

Speaker: Giovanni Forni

Title: Weakly mixing billiards in polygons

Abstract: According to numerical simulations (Artuso,Casati, Guarneri, Prosen, Wang, ...) the billiard in the random polygon is ergodic and mixing. The mathematical theory of billiards in polygons is rather well-developed for billiards in *rational* polygons (with limitations that will be explained), but there are few results on the ergodic theory of *typical* polygons. Kerckhoff-Masur-Smillie proved in 1986 that ergodic polygons are dense (in the space of polygons), by fast approximation based on their ergodicity result for rational polygons. In this talk we will present a joint result with Jon Chaika that weakly mixing polygons are also dense.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time