Integrability vs. Non-integrability in Statistical Mechanics Workshop Talk Schedule

Events for:
Monday, March 2nd - Friday, March 6th

Monday, March 2nd

9:15am **Welcome Message**

9:30am **Jacques H.H. Perk - 102**

Title: Integrability vs Non-integrability: Exact Results, Scaling, Corrections to Scaling, Natural Boundaries

Abstract: First I shall discuss a number of old and recent results relevant to the comparison of integrable and non-integrable models of statistical mechanics. Some recent theorems prove that scaling persists under certain small non-integrable perturbations, but more research remains to be done. Next I shall review our results on the susceptibility of the 2D Ising model and how they are derived. Unlike the free energy and correlation functions, which are given as D-finite expressions, the susceptibility exhibits structure incompatible with D-finiteness, albeit that it can be calculated with algorithms of polynomial complexity. For example, there is clear evidence that the zero-field susceptibility on the lattice as a function of complex temperature has a natural boundary, but that this natural boundary disappears in the scaling limit.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Craig Tracy - 102**

Title: On the Magnetic Susceptibility of the 2D Ising Model

Abstract: We discuss the status of the natural boundary conjecture for the Ising susceptibility.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Nicholas Witte - 102**

Title: "Correlations of the planar Ising model"

Abstract: The role of integrable systems, particularly the Painlev\'e transcendents, is pivotal in the description of the planar Ising model correlation functions. This began with the identification of Painlev\'e III in the scaled, critical correlations of $\square$ lattice Ising model by Wu, McCoy, Tracy, Barouch in 1976 and of Painlev\'e VI in the finite $ N $ lattice model for the diagonal correlations of $\square$ lattice Ising model by Jimbo and Miwa in 1980. A summary of the current understanding of extensions to their results in three distinct directions will be given: to other planar lattices, beyond the diagonal correlations, and the $ \lambda $ extensions. This will involve Picard's solution to the sixth Painlev\'e equation, multi-variate extensions of the sixth Painlev\'e equation, i.e. the Garnier systems, and their associated isomonodromic systems. While such identifications are elegant achievements, there is greater practical significance in that they allow one to bring to bear the machinery of integrable systems theory and derive efficient techniques to compute and study the correlations.

2:30pm **Jean-Marie Maillard - 102**

Title: Algebraic Statistical mechanics Subtitle: To be or not to be integrable ...

Abstract: "We first recall that Yang-Baxter integrability is canonically associated with algebraic varieties preserved by (generically infinite order) birational symmetries. This explains the emergence of algebraic varieties of Kodaira dimension zero (abelian varieties, Calabi–Yau manifolds, ...) on these integrable models, and underlines two different integrability concepts: Yang-Baxter integrability and integrability of the birational symmetries. We then recall some of our results on the n-fold integrals occurring in the decomposition of the full susceptibility of the square Ising model. These n-fold integrals are holonomic, solutions of highly selected linear differential operators, their factors being associated with elliptic curves, modular forms, Calabi-Yau ODEs, and more generally special differential Galois groups. These n-fold integrals, corresponding to series with integer coefficients, are also seen to be diagonal of rational functions, being, thus, the simplest transcendental generalization of algebraic functions. A canonical decomposition of such selected linear differential operators with special differential Galois groups is given. All these algebraic, arithmetic and differential highly selected structures correspond, or generalize, the concept of ""modularity"" encountered in Calabi-Yau manifolds. We also recall how the isogenies of these algebraic varieties correspond to exact generators of the renormalization group. We finally try to characterize the transcendental nature of the full susceptibility of the square Ising model, which is a non-holonomic series with integer coefficients which can actually be obtained with an algorithm of polynomial complexity. Recalling the Chazy III equation which has (circle) natural boundaries for its solutions and which is known to occur in the theory of modular forms, we explore a scenario where the full susceptibility could actually be solution of a nonlinear ODE with a Painleve property.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Tony Guttmann**

Title: Integrability, Solvability and Enumeration

Abstract: There are a number of seminal two-dimensional lattice models that are integrable, but have only been partially solved, in the sense that only some properties are fully known (e.g. the two-dimensional Ising model, where the free-energy is known, but not the susceptibility). Alternatively, critical properties are known for some lattices but not others. For example, the critical point of the self-avoiding walk model is known rigorously for the honeycomb lattice, but not for other lattices. Similarly for the q-state Potts model and both bond and site percolation. The critical manifold of the former is known only for some lattices, likewise the percolation threshold is known only for some lattices. A range of numerical procedures exist, based on exact enumeration, or other numerical work, such as calculating the eigenvalues of transfer matrices, which, when combined with various structural invariants seem to give exact results in those cases that are known to be exact, but can be used to give increasingly precise estimates in those cases which are not exactly known. Reasons for this partial success are not well understood. In this talk I will describe four such procedures, and demonstrate their performance, and speculate on their partial success.

Tuesday, March 3rd

9:30am **Jean-Michel Maillet - 102**

Title: Microscopic approach to correlation functions in 1D quantum critical models

Abstract: Starting from the finite volume form factors of local operators, we show how and under which hypothesis the c=1 free boson conformal field theory in two-dimensions emerges as an effective theory governing the critical regime of multi-point correlation functions in a large class of one dimensional massless quantum Hamiltonians. In our approach, in the large-distance critical regime, the local operators of the initial model are represented by well suited vertex operators associated to the free boson model. This provides an effective field theoretic description of the large distance behavior of correlation functions in 1D quantum critical models. We develop this description starting from the first principles and directly at the microscopic level, namely in terms of the properties of the finite volume matrix elements of local operators; it is based on the integrable models examples like XXZ spin-1/2 chain in the massless phase and 1D Bose gas at arbitrary repulsive coupling where all these computations can be done exactly.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Giuliano Niccoli**

Title: An Introduction to the Quantum Separation of Variables SOV for the exact solution of integrable quantum models

Abstract: I will describe the method of quantum separation of variables (SOV), first introduced by Sklyanin, for the analysis of the spectrum of integrable quantum model. SOV method can be applied to a general class of integrable quantum models which are not analizable by other methods, like Algebraic Bethe Ansatz (ABA), and it has as fundamental built-in feature the completeness of spectrum description. The method will be introduced in general for integrable quantum models associated to both Yang-Baxter and Reflections algebras of 6-vetex type. Then I will consider the lattice formulation of the sine-Gordon model as a concrete and relevant physical example to develop the SOV method. In particular, I will show the complete characterization of the spectrum (eigenvalues and eigenstates) of the sine-Gordon model in terms of the polynomial solutions of an associated Baxter equation. The analysis of the continuum limit will be mentioned. Then, I will focus on the results on the characterization of the quantum dynamics in the SOV framework. The lattice sine-Gordon model will be use as example to illustrate the determinant formulae obtained for scalar products and matrix elements of local operators. The universal characters of these results will be then pointed out and will be evidenced their similarity to determinant formulae appearing in the contest of ABA, like Izergin's and Slavnov's determinants. Finally, I will complete the seminar with some current projects concerning applications of the SOV method which go from the spectral analysis of others integrable quantum models to the computation of their correlation functions.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Karol Kozlowski - 102**

Title: Recent developments in the large-N analysis of correlation functions in the quantum separation of variables method

Abstract: The scalar products and certain correlation functions of models solvable by the quantum separation of variables can be expressed in terms of N-fold multiple integrals which can be thought of as the partition function of a one dimensional gas of particles evolving on a curve $\msc{C}$, trapped in an external potential V and interacting through repulsive two-body interactions of the type $\ln \big[\sinh[\pi \om_1(\la-\mu)]\cdot \sinh[\pi \om_2 (\la-\mu)] \big]$. The choice of the curve $\msc{C}$ and of the confining potential V determines a given model. The analysis of the large-N asymptotic behaviour of these integrals is of interest to the description of the continuum limit of the integrable model. In this talk, I shall report on recent developments in the large-N analysis of such integrals and discuss, on some specific examples, the form taken by the asymptotics. Part of the results that I will present issue from a joint work with G. Borot (Max-Planck Institut, Bonn, Germany) and A. Guionnet (MIT, Boston, USA). "

2:30pm **Veronique Terras - 102**

Title: SOS and 8-vertex models - Recent developments

Abstract: We review recent developments on the SOS and 8-vertex/XYZ models and towards the exact computation on their finite-size form factors/correlation functions. We in particular discuss the solution of the antiperiodic SOS model by separation of variables and its connection to quasi-periodic 8-vertex models.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Nikolai Kitanine - 102**

Title: XXZ spin chains with generic boundaries

Wednesday, March 4th

9:30am **Alexander Zamolochikov - 102**

Title: Primus inter pares: Integrable Field Theories Among Common Ones.

Abstract: Integrable Quantum Filed Theories (IQFT) fill a subspace in the space of generic QFT. The tangent to this subspace is of course the collection of local operators which, when taken as infinitesimal perturbations, preserve integrability (the IQFT analog to the space of exactly marginal operators in CFT). We show that in 2D any IQFT has infinitely many such "integrable" perturbations which are in one-to one correspondence with local and semi-local integrals of motion, and moreover admit explicit construction in terms of the associated currents. In special case (sine-Gordon model) interpretation in terms of form-factor bootstrap can be obtained. Based on joint work with Fedor Smirnov.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Sergei Lukyanov - 102**

Title: Exact overlaps in the Kondo problem

Abstract: It is well known that the ground states of a Fermi liquid with and without a single Kondo impurity have an overlap which decays as a power law of the system size, expressing the Anderson orthogonality catastrophe. Ground states with two different values of the Kondo couplings have, however, a finite overlap in the thermodynamic limit. This overlap, which plays an important role in quantum quenches for impurity systems, is a universal function of the ratio of the corresponding Kondo temperatures. In the talk we will discuss an exact formula for this overlap.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Tetsuji Miwa - 102**

Title:

2:30pm **Fedor Smirnov - 102**

Title: One-point functions in sinh-Gordon model and on-shell Yang-Yang action.

Abstract: "I shall explain the relation of the one-point functions for the the sinh-Gordon model with its generalized Gibbs ensemble. This relation leads to the definition of the on--shell Yang-Yang action which depends on the ""temperatures"" related to the higher integrals of motion. The one-point functions are given by the Hessian determinants of the Yang-Yang action."

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Andreas Kluemper - 102**

Title: Non-linear integral equation approach to sl(2|1) integrable network models

Thursday, March 5th

9:30am **Helen Au-Yang - 102**

Title: CSOS Model & Chiral Potts Model: Degeneracy of the Eigenspace and Quantum Loop Algebra

Abstract: The transfer matrices of two ?2 models commute with one another, as their monodromy matrices satisfy a Yang–Baxter equation with the six-vertex R- matrix as the intertwiner. This Yang–Baxter equation then determines the commutation relations of the elements of the monodromy matrices. We show the reason why in the superintegrable case the eigenspace is degenerate, but not in the general case. We then show that the eigenspaces of special CSOS models descending from the chiral Potts model are also degenerate. We use finite-size calculations to understand the completeness of the spectrum, and the likelihood of the existence of an L(sl2) quantum loop algebra (or subalgebra) in these models. We will also discuss several open problems.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Tetsuo Deguchi - 102**

Title: Singular solutions and the completeness of the XXX and XXZ spin chains through the loop algebra symmetry

Abstract: We give exotic solutions such as singular strings and non self-conjugate strings numerically in the XXX spin chain, and discuss with them the completeness of the spectrum under the string hypothesis. We show numerical evaluation of the form factors with strings, motivated with application to the quantum dynamics. For the completeness of the XXZ spin chain at roots of unity we review some exact results of the sl(2) loop algebra symmetry and recall the numerical exotic solutions associated with it. We also review the loop algebra symmetry in the superintegrable chiral Potts model.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Paul Fendley - 102**

Title: What integrability has to do with topology

Abstract: Integrable systems have applications ranging from experimental physics to profound mathematics. An example of the latter is the fundamental role of the Temperley-Lieb algebra of statistical mechanics in evaluating the Jones polynomial of knot and link invariants. A seemingly distinct example is discrete holomorphicity", which gives a powerful tool both for proving conformal invariance and finding integrable Boltzmann weights. In this talk I will explain the deep relations between these two examples. In particular, I will show how utilizing topological invariants enables discretely "holomorphic" quantities to be found easily. This allows both a deeper understanding of why they occur, and a great generalisation of where they occur. Applying these results to quantum spin chains yields exact zero modes, such as a Fibonacci zero mode in the hard-square/golden chain.

2:30pm **Giuseppe Mussardo - 102**

Title: Non-integrable Quantum Field Theories and Kink Confinement

Abstract: Mainly based on the class of universality of the Tricritical Ising Model I will describe several mechanisms that may or may not confine massive excitations of topological nature in a (1+1) quantum field theory.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Joint Math/SCGP Colloquium: Jesper Jacobson,"Integrability and non-integrability in hard lattices gases and the Ising model in a magnetic field" - SCGP 103**

Speaker: Jesper Jacobsen (Ecole Normale Suprieure Paris)

Title: Integrability and non-integrability in hard lattices gases and the Ising model in a magnetic field

6:00pm **Workshop Banquet**

Friday, March 6th

9:30am **Frank Goehmann - 102**

Title: Form factors and large distance asymptotics of correlation functions of the XXZ chain in the massive regime

10:30am **Coffee Break - SCGP Cafe**

11:00am **Jesper Jacobsen - 102**

Title: Critical manifolds in pure and quenched random systems from topological graph polynomials

Abstract: The critical temperature is only known analytically for the simplest two-dimensional models (Ising model), or for more complicated models (Potts and O(n) vector models) on the simplest possible lattices. The known critical temperatures are invariably given by simple algebraic curves. These results can be derived by duality arguments or integrability results, and some have recently been proved mathematically by the technique of discrete Holomorphicity. For Potts and (bond or site) percolation models on any desired two-dimensional lattice we define a graph polynomial that distinguishes between various topological sectors of the transfer matrix. Its roots turn out to give very accurate approximations to the critical temperatures, or even yield the exact result in the exactly solvable cases. This polynomial depends on a basis (unit cell) and its embedding into the infinite lattice. As the size of the basis is increased the approximation becomes increasingly accurate. This, on the one hand, gives strong evidence that the critical temperature for the lattices with no known analytical solution may not be algebraic numbers, and that conformal invariance will not have any counterpart in finite size (discrete holomorphicity). On the other hand, the method determines the critical temperature to unprecedented accuracy. For instance, we find p_c = 0.52440499916737 (4) for bond percolation on the kagome lattice, and p_c = 0.592746050803 (2) for site percolation on the square lattice. It also shows that the phase diagram of the Potts model in the antiferromagnetic regime has an intricate and highly lattice-dependent structure. Our method can be extended to O(n) models of loops, again providing exact results for solvable models and highly accurate approximations for models which are not. For self-avoding walks on the square lattice we thus find the critical monomer fugacity z_c = 0.3790522777533 (2), which is more accurate than the best estimates coming from series analysis. Finally, the method can be applied to quenched random systems. In models having a gauge symmetry, such as the +-J random-bond Ising model and the q-state Potts gauge glass, it provides very accurate estimates of the so-called Nishimori point. For instance, in the Ising case the critical probability for a frustrated bond is found to be p_N = 0.10929 (2).

12:00pm **Lunch - SCGP Cafe**

1:30pm **Carlo Meneghelli - 102**

Title: Light-cone lattice from the universal R-matrix

2:30pm **Barry McCoy - 102**

3:30pm **Coffee Break - SCGP Cafe**