Symplectic and Algebraic Geometry in the Statistical Physics of Polymers workshop Talk Schedule

Events for:
Monday, October 12th - Friday, October 16th

Monday, October 12th

9:00am **Jason Cantarella - SCGP 102**

Title: Grassmannians and Random Walks Absrtact: In this introductory talk, we give the basic identification between the Grassmann manifold of 2-planes in complex n-space and the space of (relatively) framed closed n-gons in 3-space. Using Haar measure on the Grassmannian defines a natural probability measure on polygon spaces, and gives a measure and a geometric structure to the space of random walks. This additional structure provides interesting tools for proving theorems about random walks, and we give several examples, followed by open questions.

10:30am **Coffee - SCGP Cafe**

11:00am **Clayton Shonkwiler - SCGP 102**

Title: The Symplectic Geometry of Polygon Space and How to Use It

Abstract: Building on the first talk, we describe the geometry of the space of closed polygons in 3-space with fixed edgelengths, giving a sense of how this fits into a larger symplectic and algebraic geometric story. The space of fixed edgelength polygons turns out to be a toric symplectic manifold, yielding a (nearly) global coordinate system. These coordinates are powerful tools both for proving theorems and for developing numerical techniques. We give examples of both, as well as a number of open questions.

12:15pm **Lunch - SCGP Cafe**

2:00pm **Alexander Grosberg - SCGP 102**

Title: Topological problems in polymer physics

Abstract: In the talk, I will make an attempt to formulate some unsolved problems in polymer physics in the form more accessible to mathematicians. The subjects will include: -- Equilibrium and non-equilibrium properties. What is the difference between equilibrium and steady state; -- Chain flexibility, excluded volume, and volume interactions; -- Phases and phase transitions in a single polymer chain; -- Lattice and off-lattice models; -- Peculiarities of main biopolymers, Proteins, DNA, and RNA.

3:15pm **Tea - SCGP Cafe**

3:45pm **Alexander Grosberg**

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Tuesday, October 13th

9:00am **Rebecca Goldin - SCGP 102**

Title: The Symplectic Geometry of Polygon Spaces

Abstract: In this talk, I'll introduce the polygon space from a symplectic point of view. In this context, polygon space arises as a "symplectic reduction", and in particular it carries a symplectic structure. I will explore how some integrals (in particular, the symplectic volume) can be calculated on these polygon spaces, at least on an Abelian version of the polygon space.

10:30am **Coffee - SCGP Cafe**

11:00am **Koya Shimokawa - SCGP 102**

Title: Knots and links in the simple cubic lattice

Abstract: We consider knots and links in the simple cubic lattice. We discuss step numbers needed to construct a given knot in unconfined and confined regions and exponential growth rate of knots in a tube region.

12:15pm **Lunch - SCGP Cafe**

1:00pm **SCGP Weekly Talk - Zohar Komargodski (Weizmann) - SCGP Rm 102**

Speaker: Zohar Komargodski (Weizmann)

Title: "The Space of Quantum Field Theories"

Abstract: We discuss several aspects of the space of Quantum Field Theories. Some physical observables are sensitive to the local geometry of this space and to its global properties.We review these constructions and their consequences. We discuss outstanding open questions.

2:00pm **Dr. Ben Howard - SCGP 102**

Title: Invariant Theory of Spatial Polygons

Abstract: Given a G-variety X where G is a algebraic group, in good situations the ring of G-invariant polynomials is finitely generated. In even better situations one can practically describe these invariants and maybe even determine the the polynomial relations among them. For example if G is the group of area preserving linear mappings of the plane, and X is space of all homogeneous quadratic functions $a\,x^2 + b\,xy + c\,y^2$, then the G-invariants are all polynomials in the discriminant $b^2 - 4\,ac$. Or, if G is the orthogonal group O(V) acting on n copies of O(V) where $n > \dim(V)$, then the invariants are the entries of the Gram matrix (the dot products) and the relations say that the rank of the Gram matrix is at most dim(V). Klyachko and Kapovich-Millson found that the space of closed n-gons in Euclidean space with prescribed side lengths, modulo translations and rotations, is identical to the quotient of n points on the projective line modulo Möbius transformations. We will explain how that works and give an explicit description of the invariant ring. (Joint work, in multiple papers, with J. Millson, A. Snowden, and R. Vakil.)

3:15pm **Tea - SCGP Cafe**

3:45pm **Ken Millett - SCGP 102**

Title: Sampling the Space Thick Polygons

Abstract: The theory of thick polygons will be described following which we will briefly review the classical methods of sampling the space of polygons and the limited information about thick polygons that results. Having thus demonstrated the need, we will look carefully at the new methods for sampling the space of thick polygons, describe how it is that one knows they are ergodic, and the review new information and questions that result from their application. In the context of this workshop, one wonders how these new methods can be understood from the Grassmannian or GIT perspective. Considering the elementary steps as operations on the appropriate compactification, how would one characterize their action?

Wednesday, October 14th

9:00am **Chris Manon - SCGP 102**

Title: Algebraic Geometry and Combinatorics of polygon spaces

Abstract: From the perspective of algebraic geometry, Moduli spaces of euclidean polygons are known as classical spaces from invariant theory, the moduli spaces of weighted point arrangements on the projective line. I will give a description of the construction of these spaces by Geometric Invariant Theory (GIT) in to ways: as a quotient of a product of projective lines, and as a quotient of the Grassmannian variety of 2-planes. Then I will describe some of the combinatorial gadgets inherited by polygon spaces from these quotient constructions, and how they can be used to draw conclusions about the geometry of polygons. In particular I will describe how the diagonal length polytopes can be derived from this perspective. Along the way I will describe how these algebraic and combinatorial techniques can be applied to other moduli spaces from algebraic geometry.

10:30am **Coffee - SCGP Cafe**

11:00am **Tetsuo Deguchi - SCGP 102**

Title: Statistical physics of topological polymers studied through quaternions

Abstract: Polymers with various topological structures are recently synthesized in chemistry such as double-ring polymers, polymers of theta curves and those of complete bipartite graphs. We call them topological polymers. By generating random walks with an algorithm making use of quaternions we study statistical and dynamical properties of topological polymers. For instance, we show that the ratio of the hydrodynamic radius to the gyration radius of a given polymer increases as the topology becomes more complex. We suggest that the ratio can be measured in experiments. Here we remark that the method of quaternions is closely related to equilateral polygons in symplectic geometry.

12:15pm **Lunch - SCGP Cafe**

2:00pm **Tom Needham - SCGP 102**

Title: The Geometry of Framed Path and Loop Spaces

Abstract: An important problem in biology is to determine the optimal alignment to compare the shapes of biomolecules, given the space positions of their atoms. This problem has an efficient, well-known solution when the proteins have the same number of atoms. However, the solution to the problem is not so clear if one wishes to compare proteins with a different number of atoms. In this talk we propose a new geometric solution. We will consider various moduli spaces of framed paths and loops in Euclidean space. A framed path is a space curve with a unit normal vector field and points in these moduli spaces are Euclidean similarity classes of framed paths. We will show that each of these moduli spaces has the structure of an infinite-dimensional Riemannian manifold and that they are closely related to infinite-dimensional versions of classical manifolds---namely, spheres and Grassmannians. This gives smooth versions of the descriptions of polygon spaces given by Hausmann and Knutson and generalizes the work of Michor, Mumford, Shah and Younes on spaces of plane curves. As an application of this structure, we will give an algorithm for comparing shapes of framed paths in a Euclidean similarity-invariant way and give an example using protein backbones from the Protein Database. As another application, we will discuss the classification of critical points of some natural functionals on the space of closed framed loops. One of the functionals is closely related to the potential energy function of an elastic rod, introduced by Kirchhoff, and thus to vortex filament evolution and the nonlinear Schrödinger equation.

3:15pm **Tea - SCGP Cafe**

3:45pm **Joanna Sulkowska - SCGP 102**

Title: Protein and RNA geometry.

Abstract: Overview of complex topology in biomolecules: knots, slipknots, lassos, and genus analysis. What do we want to know about the role of topology in biomolecules?

Thursday, October 15th

9:00am **Neal Madras - SCGP 102**

Title: Self-Avoiding Walks: Problems and Progress

Abstract: will introduce self-avoiding walks on regular lattices, such as the simple cubic lattice. I will review what is known, highlight some of the main open problems, and describe some recent progress in the field.

10:30am **Coffee - SCGP Cafe**

11:00am **Chris Soteros - SCGP 102**

Title: Pattern Theorems, Entanglement Complexity and Confinement Effects for Lattice Polymer Models

Abstract: TBA

12:15pm **Lunch - SCGP Cafe**

2:00pm **Steve Plotkin - SCGP 102**

Title: Generalized Euclidean distance to understand polymer uncrossing and polymer knotting: Problems, solutions, and more problems.

Abstract: TBA

3:15pm **Tea - SCGP Cafe**

3:45pm **Chaim Even-Zohar - SCGP 102**

Title: Random Knots from Petal Diagrams, and their Invariants

Abstract: Random curves in space and how they are knotted give an insight into the behavior of "typical" knots and links, and are expected to introduce the probabilistic method into the mathematical study of knots. One leading approach to this subject is based on closed random walks. However, several other random knots models have been proposed, and indeed the choice of the random model may affect the nature of the resulting knots. In the talk, I will focus on a new, more combinatorial model for generating curves at random, based on petal projections, developed by Adams et al. (2012). In work with Hass, Linial and Nowik, we found explicit formulas for the distribution of the linking number of a random two-component link. We also found formulas for the moments of two finite type invariants of knots, the Casson invariant and another coefficient of the Jones polynomial. These are the first precise formulas of this sort in any model for random knots or links. All necessary background, and the above terms will be explained. Joint work with Joel Hass, Nati Linial, and Tahl Nowik.

Friday, October 16th

9:00am **Bertrand Duplantier - SCGP 102**

Title: Statistical Mechanics of Anchored Polymer Chains

Abstract: We review some aspects of the statistical mechanics of several models of polymer chains attached to a wall, and the related entropic forces. These include the Gaussian chain, the freely-jointed chain, the worm-like chain and the framed DNA chain. Applications to protein translocation and to DNA pulling and twisting experiments will be briefly described. We shall end with the interesting case of a freely-jointed chain in a half-space, which happens to have non-trivial combinatorics and symplectic structure.

10:30am **Coffee - SCGP Cafe**

11:00am **Open Problems Session - Moderator Jason Cantarella - SCGP 102**

12:15pm **Lunch - SCGP Cafe**

4:00pm **Reading Seminar in Geometric Analysis**