Mass, the Einstein Constraint Equations, and the Penrose Inequality Conjecture: September 18-22, 2023

Events for:
Monday, September 18th - Friday, September 22nd

Monday, September 18th

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:45am **Workshop: Christina Sormani - SCGP 102**

Speaker: Christina Sormani

Title: Progress towards the intrinsic flat stability of the Positive Mass Theorem and Penrose Inequality

Abstract: We consider asymptotically flat three dimensional manifolds, M, with nonnegative scalar curvature. Schoen-Yau and Witten proved that such an M has nonnegative ADM mass and if the ADM mass is zero then M is isometric to Euclidean space. Huisken-Ilmanen and Bray proved that if M has an compact boundary which is a outermost minimal surface then the ADM mass is strictly positive and satisfies the Penrose Inequality. Suppose we have a sequence of such M_j with ADM mass converging to 0, can one prove M_j converges to Euclidean space in some sense? In joint work with Lee we presented counter examples for smooth and Gromov Hausdorff convergence and proposed pointed intrinsic flat convergence at appropriately chosen base points. After reviewing the definition of intrinsic flat convergence, we will discuss two key approaches to such a project and then survey known results in this direction including work by Dan Lee, Lan-Hsuan Huang, Iva Stavrov, Brian Allen, Raquel Perales, Giacomo del Nin and others. Link to citations.

11:00am **Workshop: Spyros Alexakis - SCGP 102**

Speaker: Spyros Alexakis

Title: Squeezing a fixed amount of gravitational energy to arbitrarily small scales, in U(1) symmetry

Abstract: We show that for the 3+1 vacuum Einstein equations it is possible to squeeze a fixed amount m of gravitational energy to arbitrarily small scales in two dimensions (leaving the third dimension un-squeezed, by imposing U(1) symmetry), and produce smooth solutions of the equations in a "box" whose size is fixed, and independent of the amount of squeezing. We discuss the relationship of the result with the (loosely formulated) hoop conjecture, which predicts that squeezing a fixed amount of gravitational energy to below its Schwarzschild radius in "all directions" must necessarily result in trapped surface formation. Our method of proof relies on utilizing the reduction of the Einstein equations in U(1) symmetry to construct solutions to the equations with very large amplitude for the Bondi news function, making it inversely proportional to the area of "squeezing''. Joint work with N. Carruth.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Po-Ning Chen - SCGP 102**

Speaker: Po-Ning Chen

Title: A quasilocal charged Penrose inequality

Abstract: In this talk, we will discuss a quasi-local Penrose inequality with charges for time-symmetric initial data of the Einstein-Maxwell equation. Namely, we derive a lower bound for Brown-York type quasi-local mass in terms of the horizon area and the electric charge. The inequality we obtained is sharp in the sense that equality holds for surfaces in the Reissner-Nordström manifold. This talk is based on joint work with Stephen McCormick.

2:30pm **Workshop: Annachiara Piubello - SCGP 102**

Speaker: Annachiara Piubello

Title: Estimates on the Bartnik mass and their geometric implications.

Abstract: In this talk, we will discuss some estimates on the Bartnik quasi-local mass for data with nonnegative Gauss curvature and positive mean curvature. This estimate is in terms of the area, the total mean curvature, and a quantity measuring the roundness of the metric. If the ratio between the maximum and the minimum of the Gauss curvature approaches 1, we show that our estimate converges to the sharp bound derived by Miao (2009) for round spheres with positive mean curvature. Furthermore, if the total mean curvature approaches 0, our estimate tends towards the sharp bound found by Mantoulidis and Schoen (2015) for the black hole horizon case. We will then discuss some geometric implications. This is joint work with Pengzi Miao.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Sven Hirsch - SGCP 102**

Speaker: Sven Hirsch

Title: Hawking mass monotonicity for initial data sets

Abstract: An interesting feature of General Relativity is the presence of singularities which can happen in even the simplest examples such as the Schwarzschild spacetime. However, in this case the singularity is cloaked behind the event horizon of the black hole which has been conjectured to be generically the case. To analyze this so-called Cosmic Censorship Conjecture Penrose proposed in 1973 a test which involves Hawking's area theorem, the final state conjecture and a geometric inequality on initial data sets (M,g,k). For k=0 this so-called Penrose inequality has been proven by Huisken-Ilmanen via inverse mean curvature flow and by Bray using the conformal flow, but in general the question is wide open. We will present several appraoches to generalize the Hawking mass monotonicity formula to arbitrary initial data sets including a new one based on double null foliations. For this purpose, we start with recalling spacetime harmonic functions and their applications which have been introduced together with Demetre Kazaras and Marcus Khuri in the context of the spacetime positive mass theorem.

Tuesday, September 19th

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:45am **Workshop: Mu-Tao Wang - SCGP 102**

Speaker: Mu-Tao Wang

Title: Criteria for the definition of angular momentum in general relativity

Abstract: I shall discuss various validity criteria for definitions of angular momentum, with emphasis on supertranslation invariance and cross-section continuity. This is based on joint work with Po-Ning Chen, Daniel Paraizo, Robert Wald, Ye-Kai Wang, and Shing-Tung Yau.

11:00am **Workshop: Demetre Kazaras - SCGP 102**

Speaker: Demetre Kazaras

Title: On the stability of Llarull's Theorem in dimension three

Abstract: As a consequence of Llarull's influential 1998 manuscript, the unit n-sphere has the following characterization: it is the only Riemannian metric on S^n whose scalar curvature and distance function are both at least as large as the unit sphere's. Following a developing program of Sormani and Gromov, this talk considers the associated "stability" problem which probes the flexibility of Llarull's rigidity statement. In joint work with Allen and Bryden, we provide a solution in dimension 3. The main result states that a sequence of Riemannian 3-spheres becoming closer and closer to satisfying the hypothesis of Llarull's Theorem must converge to the unit 3-sphere in the Sormani-Wenger intrinsic flat sense, so long as the sequence satisfies a uniform Cheeger constant bound. The argument relies on a new proof of Llarull's Theorem in dimension 3 which was inspired by the Positive Mass Theorem.

12:00pm **Workshop: Group Photo - SCGP Lobby**

Title: Group Photo

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop & SCGP Weekly Talk Speaker: Gerhard Huisken - SCGP 102**

Speaker: Gerhard Huisken

Title: Inverse mean curvature flow and 3-manifolds

Abstract: Inverse mean curvature flow expands the mean-convex boundary of a domain in normal direction with speed equal to the inverse of the mean curvature. The talk will explain how smooth and weak solutions of the flow sweeping out regions of Riemannian 3-manifolds relate lower bounds on Ricci- and scalar curvature with global properties of the manifold such as its volume growth near infinity. Examples are isoperimetric inequalities, an alternative approach to a recent non-existence result for Ricci-pinched 3-manifolds and applications to the concepts of mass and quasi-local mass.

2:30pm **Workshop: Raquel Perales - SCGP 102**

Speaker: Raquel Perales

Title: Rigidity of mass-preserving 1-Lipschitz maps from integral current spaces into Euclidean space

Abstract: We will prove that given an n-dimensional integral current space and a 1-Lipschitz map, from this space onto the n-dimensional Euclidean ball, that preserves the mass of the current and is injective on the boundary, then the map has to be an isometry. We deduce as a consequence the stability of the positive mass theorem for graphical manifolds as originally formulated by Huang--Lee--Sormani. (Joint work with G. Del Nin).

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: TeaTime

4:00pm **Workshop: Pengzi Miao - SCGP 102**

Speaker: Pengzi Miao

Title: On manifolds with controlled mass-to-capacity ratio

Abstract: On an asymptotically flat 3-manifold with boundary, it has been shown recently that the mass-to-capacity ratio is bounded below by one minus the square root of the normalized Willmore functional of the boundary, provided the manifold has nonnegative scalar curvature and simple topology. This inequality indicates manifolds with the mass-to-capacity ratio bounded above by one is worthy of study. In this talk, we discuss properties of such manifolds.

Wednesday, September 20th

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:45am **Workshop: Netta Engelhardt - SCGP 102**

Speaker: Netta Engelhardt

11:00am **Workshop: Hari Kunduri - SCGP 102**

Speaker: Hari Kunduri

Title: Existence and uniqueness of toric gravitational instantons

Abstract: Gravitational instantons are four-dimensional, non-compact, complete Ricci-flat Riemannian manifolds. Natural asymptotic conditions for these spaces include asymptotically flat (S^1 X R^3 with the flat metric), asymptotically locally Euclidean (ALE) and asymptotically Taub-NUT. I will discuss existence and uniqueness results for gravitational instatons admitting a torus symmetry. Solutions are characterised by data that encodes the fixed point sets of the torus action and for every admissible data there exists an instanton that is smooth up to possible conical singularities at the axes of symmetry. This leads to some qualitative differences with the analogous problem in the Lorentzian setting (i.e. stationary and axisymmetric vacuum black holes). I will also discuss generalisations to higher dimensions.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Benedito Leandro - SCGP 102**

Speaker: Benedito Leandro

Title: On the geometry of electrovacuum spaces in higher dimensions

Abstract: A classical question in general relativity is classifying regular static black hole solutions of the static Einstein-Maxwell equations (or electrovacuum/electrostatic system). We will provide some classification results for an electrovacuum system such that the electric field is parallel to the gradient of the lapse function. We will show that an n-dimensional locally conformally flat extremal electrovacuum space must be in the Majumdar-Papapetrou class. Moreover, we prove that an n-dimensional (super)subextremal electrovacuum space with fourth-order divergence-free Weyl tensor and zero radial Weyl curvature is locally a warped product manifold with (n-1)-dimensional Einstein fibers. Finally, a three-dimensional electrovacuum space with a third-order divergence-free Cotton tensor is also classified.

2:30pm **Workshop: Paul Allen - SCGP 102**

Speaker: Paul Allen

Title: The asymptotically hyperbolic Yamabe problem in the Sobolev setting

Abstract: Given a Riemannian metric, the well-studied Yamabe problem seeks to find a conformally related metric of constant scalar curvature. For conformally compact, asymptotically hyperbolic metrics, the problem has been previously studied in smooth, polyhomogeneous, and H\"older regularity settings. In this talk, we consider the Sobolev setting, where the metric might only admit a W^{1,p} compactification (with p greater than the dimension of the manifold). We present new families of function spaces suitable for metrics potentially having a large amount of interior differentiability, but whose Sobolev regularity implies only a H\"older continuous conformal structure at infinity. We then analyze the Yamabe equation using Fredholm theory for elliptic operators arising from metrics in these families. This work is joint with John M Lee and David Maxwell.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Melanie Graf - SCGP 102**

Speaker: Melanie Graf

Title: Initial data sets that do not satisfy the Regge–Teitelboim conditions

Abstract: As we all know, in General Relativity an “isolated system at a given instant of time” can be modeled as an asymptotically Euclidean initial data set (M,g,K). Such asymptotically Euclidean initial data sets (M,g,K) are characterized by the existence of asymptotic coordinates in which the Riemannian metric g decays to the Euclidean metric and the second fundamental form K and the mass and momentum densities μ and J decay to 0 suitably fast. As shown by Bartnik using harmonic coordinates this decay ensures the convergence of the (ADM-)energy. However, to obtain convergence of the (BORT-)center of mass one needs to additionally impose the Regge-Teitelboim conditions stipulating stronger decay of the odd parts of g, μ and J and the even part of K. We will see that, under certain circumstances, harmonic coordinates can be used as a tool in checking whether a given asymptotically Euclidean initial data set admits asymptotically Euclidean coordinates satisfying these Regge-Teitelboim conditions. This allows us to easily give examples of asymptotically Euclidean initial data sets which do not possess any Regge-Teitelboim coordinates. This is joint work with Carla Cederbaum and Jan Metzger.

6:00pm **Workshop: Dinner Banquet - SCGP Cafe**

Title: Dinner Banquet

Thursday, September 21st

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:45am **Workshop: Anna Sakovich - SCGP 102/ZOOM**

Speaker: Anna Sakovich

Title: A definition of the mass aspect function for weakly regular asymptotically hyperbolic manifolds

Abstract: In mathematical general relativity, the notion of mass has been defined for certain classes of manifolds that are asymptotic to a fixed model background. Typically, the mass is an invariant computed in a chart at infinity, which is related to the scalar curvature and has certain positivity properties. When the model is hyperbolic space, under certain assumptions on the geometry at infinity one can compute the mass using the so-called mass aspect function, a function on the unit sphere extracted from the term describing the leading order deviation of the metric from the hyperbolic background. This definition of mass, due to Xiaodong Wang, is a particular case of the definition by Chru?ciel and Herzlich which proceeds by taking the limit of certain surface integrals and applies to asymptotically hyperbolic manifolds with less stringent asymptotics. In this talk I will present our joint work with Romain Gicquaud aimed at defining the mass aspect function and the mass for asymptotically hyperbolic manifolds of low regularity. We show that in this setting one can use cut-off functions to define suitable replacements to the potentially ill-defined surface integrals of Chru?ciel and Herzlich. Moreover, we are able to define the mass aspect function as a distribution on the unit sphere for metrics having slower fall-off towards hyperbolic metric than those originally considered by Xiaodong Wang. The related notion of mass is well-behaved under changes of coordinates and we expect that the positivity can be proven.

11:00am **Workshop: Dan Lee - SCGP 102**

Speaker: Dan Lee

Title: The equality case of the positive mass theorem

Abstract: We will review the equality case of the positive mass theorem for initial data sets and discuss a recent proof with Lan-Hsuan Huang that essentially resolves the problem in the most general setting.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Francesca Oronzio - SCGP 102/ZOOM**

Speaker: Francesca Oronzio

Title: ADM mass and potential theory

Abstract: TBA

2:30pm **Workshop: Rodrigo Avalos - SCGP 102**

Speaker: Rodrigo Avalos

Title: A Q-curvature positive energy theorem with applications to rigidity phenomena

Abstract: In this talk we will present recent results related to a notion of energy, which is associated to fourth-order gravitational theories, where it plays an analogous role to that of the classical ADM energy in the context of general relativity. We shall show that this quantity obeys a positive energy theorem with natural rigidity in the critical case of zero energy. Furthermore, we will comment on how the resulting notion of energy is deeply connected to Q-curvature analysis, underlying positive energy theorems for the Paneitz operator as well as several rigidity phenomena in geometry associated to Q-curvature analysis.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Marcelo Disconzi - SCGP 102**

Speaker: Marcelo Disconzi

Title: From free-boundary fluids to the constraint equations

Abstract: We consider the free-boundary relativistic Euler equations with a physical vacuum boundary, which provide a basic model of isolated stars. We present a proof of local well-posedness of the Cauchy problem in the case where the fluid evolves on a fixed, pre-determined spacetime. Next, we will discuss how attempting to adapt our proof to the situation where coupling to Einstein's equations is considered leads to some interesting open questions about the constraint equations. This is joint work with Mihaela Ifrim and Daniel Tataru.

Friday, September 22nd

9:00am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:45am **Workshop: Aghil Alaee - SCGP 102**

Speaker: Aghil Alaee

Title: A new quasi-local mass

Abstract: We define a new gauge independent quasi-local mass and energy with respect to the Minkowski spacetime, and show its relation to the Brown-York Hamilton-Jacobi analysis. A quasi-local proof of the positivity, based on spacetime harmonic functions, is given for admissible closed spacelike 2-surfaces which enclose an initial data set satisfying the dominant energy condition. Rigidity is also established in that vanishing energy implies that the 2-surface arises from an embedding into Minkowski space, and conversely, the mass vanishes for any such surface. This is joint work with M. Khuri and S.T. Yau.

11:00am **Workshop: Markus Wolff - SCGP 102**

Speaker: Markus Wolff

Title: Ricci-Flow on surfaces along the standard light cone in the $3+1$ Minkowski spacetime

Abstract: Identifying any conformally round metric on the 2-sphere with a unique cross section on the standard lightcone in the $3+1$-Minkowski spacetime, we gain a new perspective on the standard lightcone in the $3+1$-Minkowski spacetime, we gain a new perspective on $2d$-Ricci flow on topological spheres. It turns out that in this setting, Ricci flow on $2d$-Ricci flow on topological spheres. It turns out that in this setting, Ricci flow is equivalent to a null mean curvature flow first studied by Roesch–Scheuer along null hypersurfaces. Exploiting this equivalence, we can translate well-known results from $2d$-Ricci flow first proven by Hamilton into a full classification of the singularity models for null mean curvature flow in the Minkowski lightcone. Conversely, we obtain a new proof of Hamilton’s classical result using only the maximum principle.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Eric Ling - SCGP 102**

Speaker: Eric Ling

Title: On the cosmological constant appearing as an initial condition for nonhomogeneous inflationary models

Abstract: Milne-like spacetimes are a class of k = -1 inflationary FLRW spacetimes which admit continuous extensions through the big bang. Under suitable assumptions on the scale factor, the cosmological constant appears as an initial condition for Milne-like spacetimes. More precisely, the energy density and pressure approach the equation of state of a cosmological constant for points approaching the big bang. However, Milne-like spacetimes can only approximate our universe since our universe is not perfectly isotropic or homogeneous. In this talk, we show how the cosmological constant appears as an initial condition even for nonhomogeneous versions of Milne-like spacetimes. Applications to inflationary theory are discussed along with some questions on forming an initial value problem for such models. Moreover, these results have analogues in the k = 0 FLRW setting with eternal inflation; this is joint work with Ghazal Geshnizjani and Jerome Quintin.

2:30pm **Workshop: Albachiara Cogo - SCGP 102**

Speaker: Albachiara Cogo

Title: Maximal Surfaces over exterior domains and approximation of Null Hypersurfaces.

Abstract: Maximal Surfaces are spacelike hypersurfaces of a Lorentzian manifold which are critical points of the area functional. They are very important tools in General Relativity and can be studied by applying non-linear PDEs techniques since the Euler-Lagrange equation of the variational problem of maximization of the area is a quasi-linear elliptic PDE that geometrically describes the vanishing of the mean curvature. Given the behavior of some simple solutions in Minkowski Spacetime, it seems natural to investigate when sequences of Maximal Surfaces on exterior domains converge to Null Hypersurfaces. We will present some developments in this direction, starting with a discussion on the existence of solutions over exterior domains in some Asymptotically Flat Spacetimes.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Caterina Valcu - SCGP 102/ZOOM**

Speaker: Caterina Valcu

Title: Solving initial data for Kaluza-Klein spacetimes

Abstract: We study the constraint equations for Einstein equations on manifolds of the form $\mathbb{R}^{n+1}\times\mathbb{T}^m$, where $\mathbb{T}^m$ is a flat m-dimensional torus. Spacetimes with compact directions were introduced almost a century ago by Theodor Kaluza and Oskar Klein as an early attempt of unifying electromagnetism and general relativity in a simple, elegant way. The aim of this article is to construct initial data for the Einstein equations on manifolds of the form $\mathbb{R}^{n+1}\times \mathbb{T}^m$, which are asymptotically flat at infinity, without assuming any symmetry condition in the compact direction. We use the conformal method to reduce the constraint equations to a system of elliptic equation and work in the near CMC (constant mean curvature) regime. The main new feature of the proof is the introduction of new weighted Sobolev spaces, adapted to the inversion of the Laplacian on product manifolds. Classical linear elliptic results need to rigorously proved in this new setting. This is joint work with Cécile Huneau.