Special Holonomy Talk Schedule

Events for:
Tuesday, September 6th - Thursday, September 15th

Tuesday, September 6th

8:30am **Registration - SCGP Lobby**

9:15am **Introduction**

9:30am **Simon Donaldson - SCGP 102**

Title: Introduction to global questions around special holonomy.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Robert Bryant - SCGP 102**

Title: On families of special holonomy metrics defined by algebraic curvature conditions

Abstract: There are various methods known now for constructing more-or-less explicit metrics with special holonomy; most of these rely on assumptions of symmetry and/or reduction. Another promising method for constructing special solutions is provided by the strategy of looking for metrics that satisfy algebraic curvature conditions. This method often leads to a study of structure equations that satisfy an overdetermined system of partial differential equations, sometimes involutive sometimes not, and the theory of exterior differential systems is particularly well-suited for analyzing these problems. In this talk, I will describe the ideas and the underlying techniques needed from the theory of exterior differential systems, illustrate the application in the most basic cases, and describe the landscape for the research needed to carry out this program. A similar program is envisioned for finding special calibrated submanifolds of the associated geometries and, if time permits, I will describe some of this work and the initial results.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Bobby Acharya - SCGP 102**

Title: G2-manifolds and Particle Physics

Abstract: G2-holonomy spaces serve as well motivated models of the seven extra dimensions of space predicted by M-theory. I will review how this leads to a completely geometric picture of all the known elementary forces including gravity. Special kinds of singularity of G2-spaces play a crucial role in this picture, which one might regard as a generalisation of the McKay correspondence. I will try to describe some mathematical problems which have arisen from this work concerning the existence of G2-holonomy metrics with singularities and questions about the moduli space of G2-manifolds.

2:30pm **Johannes Nordström**

Title: Complete and conically singular G_2-manifolds of cohomogeneity one

Abstract: Bryant and Salmon's cohomogeneity 1 examples of complete, asymptotically conical G_2-manifolds provide a model for desingularising compact G_2-manifolds with conical singularities; however no examples of the latter are yet known, and there are also no further known examples of asymptotically conical G_2-manifolds. Theoretical physicists such as Cvetic-Gibbons-Lu-Pope and Brandhuber-Gomis-Gubser-Gukov have considered complete cohomogeneity 1 G_2-manifolds that are "asymptotically locally conical"--the model at infinity is a circle bundle over a cone--and which in a 1-parameter family converge to an asymptotically cylindrical manifold. However, only some of these families have been studied rigorously (Bazaikin-Bogoyavlenskaya). I will discuss joint work in progress with Foscolo and Haskins on these families, and some of their limits, which include a new asymptotically conical G_2-manifold and a conically singular G_2-manifold with locally conical asymptotics. The latter may provide an avenue to construction of compact G_2-manifolds with conical singularities.

3:30pm **Tea**

4:00pm **Sebastian Goette**

Title: Connected components of the moduli space of G2 manifolds

Abstract: The Crowley-Nordstrom nu-invariant distinguishes topological G2 structures on 7-manifolds. It takes values in Z/48. There is a Z-valued extension of nu for manifolds of holonomy G2. We will introduce both invariants and show how they can be computed for extra twisted connected sums using eta-invariants of Dirac operators. This allows us to exhibit examples of 7-manifolds M where the space of G2-holonomy metrics is disconnected. We will then talk about some related open questions and problems and sketch possible next steps in our research program.

5:00pm **Business Meeting: Collaboration P.I.s**

Wednesday, September 7th

9:30am **Simon Donaldson - SCGP 102**

Title: Introduction to formal aspects of gauge and submanifold theory

10:30am **Coffee Break**

11:00am **Eleonora Di Nezza - SCGP 102**

Title: The space of Kähler metrics on singular varieties

Abstract: The geometry and topology of the space of Kähler metrics on a compact Kähler manifold is a classical subject, first systematically studied by Calabi in relation with the existence of extremal Kähler metrics. Mabuchi then proposed a Riemannian structure on the space of Kähler metrics under which it (formally) becomes a non-positively curved infinite dimensional space. Chen later proved that this is a metric space of non-positive curvature in the sense of Alexandrov; its metric completion was characterized only recently by Darvas. In this talk we will talk about the extension of such a theory to the setting where the compact Kähler manifold is replaced by a compact singular normal Kähler space. As one application we give an analytical criterion for the existence of Kähler-Einstein metrics on certain mildly singular Fano varieties, an analogous to a criterion in the smooth case due to Darvas and Rubinstein. This is based on joint work with Vincent Guedj.

12:00pm **Lunch - SCGP Cafe**

1:15pm **Thomas Walpuski - SCGP 102**

Title: From G2 gauge theory to the ADHM Seiberg–Witten equation

Abstract: The compactness question for G2–instantons naturally leads one to consider the local problem around an associative submanifold. I will explain how, in a neighborhood of an associative submanifold, the G2–instanton equation can be understood as a Seiberg–Witten type equation. A result of Haydys then motivates introducing a second Seiberg–Witten type equation associated with the ADHM construction. I will end the talk by discussing a compactness theorem for the simplest of the ADHM Seiberg–Witten equations. This is joint work with Andriy Haydys.

2:30pm **Yng-Ing Lee - SCGP 102**

Title: On the existence of Lagrangians with special properties

Abstract: I would like to take this opportunity to discuss my interests and different approaches to studying the existence of minimal Lagrangians, or special Lagrangians in Calabi-Yau manifolds, as well as other Lagrangians with special properties such as Hamiltonian stationary Lagrangians and soliton solutions of Lagrangian mean curvature flow. This will be a survey talk and intended to serve as a first step on exchanging thoughts for future collaboration.

3:30pm **Tea**

4:00pm **Dominic Joyce - SCGP 102**

Title: Derived differential geometry and moduli spaces in differential geometry.

Abstract: Derived Differential Geometry (DDG) is the study of "derived manifolds" and "derived orbifolds", where "derived" is in the sense of the Derived Algebraic Geometry of Jacob Lurie and Toen-Vezzosi. They include ordinary manifolds and orbifolds, but also many spaces which are singular at the classical level. There are several approaches to DDG, due to Spivak, Borisov-Noel and myself, all more-or-less equivalent. The "Kuranishi spaces" studied in symplectic geometry by Fukaya-Oh-Ohta-Ono are a prototype notion of derived orbifold. I claim that moduli spaces M of solutions of a nonlinear elliptic p.d.e. on a compact manifold should naturally have the structure of a derived manifold (if solutions have no symmetries) or a derived orbifold (if solutions have finite symmetry groups, e.g. Deligne-Mumford stable J-holomorphic curves). This includes many very interesting problems -- instantons on 4-manifolds and other gauge-theoretic moduli problems, J-holomorphic curves in symplectic geometry and so on. In particular, it includes moduli spaces used to define enumerative invariants (Donaldson invariants, Gromov-Witten invariants, etc.). This is because compact, oriented derived manifolds and derived orbifolds have virtual cycles in homology, and these virtual cycles may be used to define the invariants. I also claim that many natural (partial) compactifications of such moduli spaces M, e.g. by including J-holomorphic curves with nodes, should naturally have the structure of derived manifolds or derived orbifolds with corners, where the boundary \partial M represents the extra singular solutions. I will outline a method to prove the existence of natural derived manifold and derived orbifold structures on differential geometric moduli spaces by a method of "universal families", based on Grothendieck's representable functors in algebraic geometry. That is, given a moduli problem, we define a notion of family F of solutions over a base derived manifold or orbifold S. A "universal family" is a family with a universal property w.r.t. all other families. If a universal family exists (I claim it should, under reasonable conditions) it is unique up to equivalence, and the base M of the family is the moduli space, with a derived manifold/orbifold structure.

6:00pm **Banquet - SCGP Cafe**

Thursday, September 8th

9:30am **Dave Morrison - SCGP 102**

Title: Moduli, calibrations, and singularities

Abstract: A very effective way to study singularities of Calabi-Yau threefolds is to express the singular space as a limit of smooth spaces. If the complex structure is held fixed while the Kähler class is varied, there are algebraic cycles (calibrated by a power of the Kähler form) which approach zero volume in the limit. If the Kähler class is held fixed while the complex structure is varied, there are ``vanishing cycles'' whose volumes approach zero in the limit. These vanishing cycles are conjectured to have special Lagrangian representatives under appropriate conditions, i.e., to be calibrated by the real part of a holomorphic 3-form. Both kinds of limit can be studied with techniques from algebraic geometry. In this talk, we will discuss the prospects for a similar picture holding in the case of G2 manifolds. In a limit of smooth metrics in which the volume of some associative or co-associative cycle approaches zero, what happens to the space?

10:30am **Coffee Break**

11:00am **Andriy Haydys - SCGP 102**

Title: On degenerations of the Seiberg--Witten monopoles and G2-instantons

Abstract: A sequence of the Seiberg-Witten monopoles with multiple spinors on a three-manifold can converge after a suitable rescaling to a Fueter section, say I, only on the complement of a subset Z. I will discuss the following question: Which pairs (I,Z) can (or can not) appear as the limit of a sequence of the Seiberg--Witten monopoles? I will also address an analogous question for the sequences of G2-instantons.

12:00pm **Lunch - SCGP Cafe**

1:15pm **Anda Degeratu - SCGP 102**

Title: Quasi-Asymptotically Conical Geometries

Abstract: In this talk we introduce the class of quasi-asymptotically conical (QAC) geometries, a less rigid Riemannian formulation of the QALE geometries introduced by Joyce in his study of crepant resolutions of Calabi-Yau orbifolds. Our set-up is in the category of real stratified spaces and Riemannian geometry. Given a QAC manifold, we identify the appropriate weighted Sobolev spaces, for which we prove the finite dimensionality of the null space for generalized Laplacians as well as their Fredholmness. We conclude with applications to metrics with special holonomy. The methods we use are based on techniques developed in geometric analysis by Grigor'yan and Saloff-Coste, as well as Colding and Minicozzi, and Peter Li. We show that our geometries satisfy the volume doubling property and the Poincare inequality, and we use these properties to analyze the heat kernel behaviour of a generalized Laplacian and to establish Li-Yau type estimates for it. This is based on joint work with Rafe Mazzeo and work in progress with Ronan Conlon and Frederic Rochon.

2:30pm **Sergei Gukov - SCGP 102**

Title: Mathematics of fivebranes on smooth 4-manifolds

Abstract: Compactifying fivebranes on coassociative 4-manifolds leads to a class of 2d theories (vertex operator algebras), whose spectrum captures the information about smooth structures. In particular, it contains information about Donaldson invariants and the hope is that it can be strictly stronger. In this talk I will try to focus on mathematical aspects of this problem, and in particular write a new set of flow equations somewhat similar to Ricci-flow equations. From the viewpoint of G2 geometry, one can think of the invariants in this talk as "local" invariants, associated with a coassociative 4-cycle (no pun intended).

3:30pm **Tea**

4:00pm **Lorenzo Foscolo - SCGP 102**

Title: ALF spaces and collapsing Ricci-flat metrics on the K3 surface

Friday, September 9th

9:30am **Mark Haskins - SCGP 102**

Title: Collapse and special holonomy metrics.

Abstract: The Gromov-Hausdorff topology provides a natural way to compactify the set of metrics with lower bounds on their Ricci curvature, in particular for Ricci-flat metrics (and therefore for special or exceptional holonomy metrics). Such Ricci-limit spaces need not be manifolds and a fundamental question that has attracted much attention is what one can say about the geometric structure of such limit spaces. In the non-collapsed case, i.e. when the limit space does not drop dimension, the theory is now very well developed, as will be described in Jeff Cheeger's talk. By comparison the structure of collapsed Ricci-limit spaces is much less well understood. In the Ricci-flat setting at least, relatively few families of collasping Ricci-flat metrics have been constructed. In this talk we concentrate on the latter aspect and describe recent progress, work still in progress and future work on the construction of families of special holonomy metrics that collapse to a limit space of one dimension less. We focus on two cases: (i) the collapsing families of metrics on the K3 surface recently constructed by Foscolo and (ii) collapsing families of metrics on G2 holonomy spaces (work in progress joint with Foscolo and Nordstrom). Understanding such 1-dimensional collapse in the context of G2 holonomy metrics is intimately related to a rigorous mathematical understanding of an important phenomenon in physics: the weak coupling limit of M-theory compactified on a G2 holonomy space being Type IIA String Theory on a 6-dimensional space. At various points inspiration for our work has already come from previous work of physicists, including members of our Collaboration. The K3 case serves as a good warmup for the much more involved G2 holonomy case and I will give an overview of Foscolo’s work in that setting. In the G2 case I will outline what we have already achieved, what remains to be done and some prospects for the future. My talk is also closely related to the talks of Johannes Nordstrom and Song Sun.

10:30am **Coffee Break**

11:00am **Simon Salamon - SCGP 102**

Title: Manifolds with holonomy Sp(n)Sp(1)

Abstract: This holonomy group from Berger's list characterizes the class of quaternion-kaehler (or "nearly hyperkaehler") manifolds of real dimension 4n. Each such manifold M carries a parallel 4-form, and is Einstein but is not in general Kaehler. In the case of positive scalar curvature, the twistor space (a 2-sphere bundle over M) is Kaehler, and it is an open question as to whether the only complete examples are homogeneous. This topic provides fascinating links between complex and Riemannian geometry that illustrate the power of spinor and twistor methods.

12:00pm **Lunch - SCGP Cafe**

1:15pm **Jeff Cheeger - SCGP 102**

Title: Ricci curvature: Some recent progress and open questions

Abstract: We will survey some recent (and less recent) progress on Ricci curvature and mention a few questions which remain open.

2:30pm **Song Sun - SCGP 102**

Title: Degeneration of Calabi-Yau metrics

Abstract: The complex structure moduli space of Calabi-Yau manifolds can be compactified using the Gromov-Haudorff topology, and a central question is to understand the structure of these Gromov-Hausdorff limits. We will focus on the ``non-collapsing" case, and explain the connection with algebraic geometry (joint work with S. Donaldson), and the recent example of a compact Calabi-Yau manifold with isolated conical singularities (joint work with H. Hein). A well-known application of the latter is the existence of special Lagrangian spheres on the smoothing of nodal Calabi-Yau varieties.

3:30pm **Tea**

4:00pm **Simon Donaldson - SCGP 102**

Title: Adiabatic limits of coassocative fibrations

Saturday, September 10th

Sunday, September 11th

Monday, September 12th

9:15am **Registration/Welcome - SCGP Lobby**

9:30am **Bobby Acharya - SCGP 102**

Title: Particle physics for mathmematicians

10:30am **Coffee - SCGP Cafe**

11:00am **Jacques Distler - SCGP 102**

Title: Geometric structures and supersymmetric theories

12:00pm **Lunch - SCGP Cafe**

1:15pm **Frederik Denef - SCGP 102**

Title: Supergravity theories and their compactification, I

2:30pm **Ronen Plesser - SCGP 102**

Title: String worldsheets

3:30pm **Tea**

4:00pm **Johannes Walcher - SCGP 102**

Title: D-branes, I

Tuesday, September 13th

9:30am **Jacques Distler - SCGP 102**

Title: Geometric structures and supergravity theories

10:30am **Coffee Break**

11:00am **Dave Morrison - SCGP 102**

Title: The role of special holonomy in string/M/F compactification

12:00pm **Lunch - SCGP Cafe**

1:15pm **Sakura Schafer-Nameki - SCGP 102**

Title: Gauge theories in M/string theory and Higgs bundles

2:30pm **Johannes Walcher - SCGP 102**

Title: D-branes, II

3:30pm **Tea**

4:00pm **Bobby Acharya - SCGP 102**

Title: The standard model of particle physics

5:00pm **Jacques Distler - SCGP 102**

Title: Geometric structures and supergravity theories

Wednesday, September 14th

9:30am **Ronen Plesser - SCGP 102**

Title: BPS and all that

10:30am **Coffee Break**

11:00am **Dave Morrison - SCGP 102**

Title: Calibrations and Branes

12:00pm **Lunch - SCGP Cafe**

1:15pm **Frederik Denef - SCGP 102**

Title: Supergravity theories and their compactification, II

2:30pm **Sakura Schafer-Nameki - SCGP 102**

Title: M-theory on CY, singularities and resolutions

3:30pm **Tea**

4:00pm **Jim Halverson - SCGP 102**

Title: More Gauge Enhancement in M on CY

Thursday, September 15th

9:30am **Bobby Acharya - SCGP 102**

Title: Beyond the standard model

10:30am **Coffee Break**

11:00am **Jim Halverson - SCGP 102**

Title: Gauge Enhancement in M on G2

12:00pm **Lunch - SCGP Cafe**

1:15pm **Bobby Acharya - SCGP 102**

Title: Codimension seven singularities in M-theory compactifications

2:30pm **Dave Morrison: Summary of the school - SCGP 102**

Title: Summary of the school

3:30pm **Tea**