

# Toric Kahler Geometry workshop Talk Schedule

Events for:  
Monday, October 5th - Friday, October 9th

## Monday, October 5th

10:00am Vestislav Apostolov - Math 5127

**Title:** Toric aspects of the Einstein--Maxwell equations

**Abstract:** In this talk, I will explain how the geometry of conformally Kähler, Hermitian metrics  $(g, J)$  with  $J$ -invariant Ricci tensor and constant scalar curvature fits into the general framework of toric geometry of Abreu's equation, developed by M. Abreu, S. Donaldson, Zhou-Zhu, etc. This class of metrics, which includes CSC Kähler and Einstein metrics, provides in real dimension four a Riemannian counter-part of the Einstein—Maxwell equations in General Relativity. As an application of the theory, I will prove that on a compact toric 4-orbifold with second Betti number equal to  $2$ , any torus-invariant Hermitian metric with  $J$ -invariant Ricci tensor and constant scalar curvature must be obtained as a compactification of an explicitly defined family of metrics. This is a work in progress with G. Maschler.

11:00am Coffee - SCGP Cafe

11:30am Simon Donaldson - SCGP 102

**Title:** Toric manifolds and K-stability

**Abstract:** The talk will give an overview of the speaker's (old) papers on constant scalar curvature toric surfaces and outline some of the main arguments in the existence proofs. Tuesday lecture.

12:30pm Lunch - SCGP Cafe

2:15pm Brian Weber - Math P131

**Abstract:** It is known that Kahler reduction of scalar-flat toric surfaces leads to a pair of linear degenerate-elliptic equations  $x(f_{xx} + f_{yy}) - f_x = 0$  on the polytope. Vice-versa, solving these equations on the polytope give scalar-flat 4-manifolds. To the end of finding all admissible solutions, we study global aspects of the linear degenerate-elliptic equations of this form. Such PDE have been studied locally, but we obtain a new global classification of solutions, which amounts to a Liouville-type theorem. This provides a classification of scalar-flat metrics under certain natural conditions.

3:30pm **Tea - SCGP Lobby**

**Tuesday, October 6th**

11:00am **An-Min Li - SCGP 313**

**Title:** Extremal metrics on toric surfaces (I)

12:00pm **Lunch - SCGP Cafe**

1:00pm **SCGP Weekly Talk -Gabor Szekelyhidi - SCGP Rm 102**

**Speaker:** Gabor Szekelyhidi

**Title:** Extremal metrics on toric manifolds

**Abstract:** Extremal metrics were introduced by Calabi in the 1980s as a notion of canonical metric on Kahler manifolds, generalizing Kahler-Einstein metrics. Through the works of Donaldson and Chen-Li-Sheng we have a fairly complete existence theory on toric surfaces, but in higher dimensions and non-toric situations our knowledge is very incomplete. I will give an overview of some of the basic ideas in this field.

3:30pm **Tea - SCGP Lobby**

4:00pm **Simon Donaldson Joint talk with Mathematics Department - SCGP 102**

**Title:** Complete zero scalar curvature Kahler surfaces"

**Abstract:** The focus of the talk will be a family of explicit metrics, which are deformations of the Taub-NUT metric. In the first part we will explain some of the context and possible relevance of these metrics as blow-up limits. In the last part we will describe an elementary approach to the construction which extends to other equations, including the 2-dimensional affine maximal equation.

## Wednesday, October 7th

10:00am **Gabor Szekelyhidi - SCGP 313**

**Title:** Kahler-Einstein metrics on T-varieties

**Abstract:** " T-varieties are generalizations of toric varieties, where one only has an effective action of a lower dimensional torus. In this talk I will discuss the complexity-one situation, where the dimension of the torus is one less than that of the manifold. Work of Ilten-Suss gives a combinatorial description of equivariant test-configurations in this setting, which will necessarily have toric central fibers. In particular, using joint work with Datar, this leads to new examples of Kahler-Einstein 3-folds. In the talk I will focus on the description of complexity-one T-varieties which underlies this work"

11:00am **Coffee - SCGP Cafe**

11:30am **Li Sheng - SCGP 102**

**Title:** Extremal metrics on toric surfaces (II)

12:30pm **Lunch - SCGP Cafe**

3:30pm **Tea - SCGP Lobby**

6:30pm **Workshop Banquet**

## Thursday, October 8th

11:30am **Bohui Chen - SCGP 313**

**Title:** Extremal metrics on toric surfaces (III)

12:30pm **Lunch - SCGP Cafe**

2:15pm **Hongnian Huang - Math P131**

**Title:** Calabi flow on admissible structures

**Abstract:** In this talk, we will discuss the Calabi flow on admissible structures defined by Apostolov, Calderbank, Gauduchon and Tonnesen-Friedman. The admissible structures can be seen as generalizations of toric varieties and Calabi's ansatz. We will show that on some admissible Kaehler manifolds, one can obtain the uniform  $C^0$  norm of the Kaehler potentials of the Calabi flow by adapting the techniques developed in the study of the Calabi flow on toric manifolds with uniform Sobolev constant bounds. This ongoing research is motivated by the discussions with Vestislav Apostolov.

3:30pm **Tea - SCGP Lobby**

4:00pm **An-Min Li Mathematics Department Colloquium - SCGP 102**

**Title:** Extremal Metrics on Toric Manifolds and Affine Techniques

**Abstract:** In a sequence of papers, Donaldson initiated a program to study the extremal metrics on toric manifolds and solved the problem for cscK metrics on toric surfaces. For toric manifolds, the equation of extremal metrics can be reduced to a real 4th-order partial differential equation on the Delzant polytope, called the Abreu equation, which is similar to the maximal equation in affine geometry. The affine techniques play important role. In joint papers with Bohui Chen and Li Sheng we apply the affine techniques to extend the existence result in dimension 2 to extremal metrics. In this talk we explain our main idea and methods.

**Friday, October 9th**

10:00am **Eveline Legendre - Math 5127**

**Title:** The Einstein-Hilbert functional and the Sasaki-Futaki invariant

11:00am **Coffee - SCGP Cafe**

11:30am **Tamas Darvas - SCGP 102**

**Title:** Energy properness and geometry of the space of Kahler metrics

**Abstract:** In the 90's, Tian introduced a notion of properness in the space of Kähler metrics in terms of Aubin's J-energy for Mabuchi's K-energy and formulated several conjectures on the relation between properness and Kähler-Einstein metrics. In joint work with Y. Rubinstein we disproved one of these conjectures, and proved the remaining ones. Our results extend to a variety of canonical metrics, in particular Kähler-Einstein edge metrics and Kähler-Ricci solitons. Lastly, we formulated a corresponding conjecture for constant scalar curvature metrics and reduce it to a regularity question on minimizers of the K-energy

12:30pm **Lunch - SCGP Cafe**

2:30pm **Tristan Clifford Collins - math 4-130**

**Title:** The J-equation on toric varieties.

**Abstract:** I will discuss the solvability of the J-equation, which defines the critical point of Chen-Donaldson's J-functional. It is known that there do not exist solutions to the J-equation in general -- a notion of algebro-geometric stability has been proposed by Lejmi-Szekelyhidi which is conjectured to be equivalent to the convergence of the flow. I will discuss a proof of this conjecture on toric varieties. This work is joint with G. Szekelyhidi.

3:30pm **Tea - SCGP Lobby**