

# Recent Developments in the mathematical study of gauge Theory workshop Talk Schedule

Events for:  
Monday, October 17th - Friday, October 21st

## Monday, October 17th

9:30am **Timothy Perutz - SCGP 102**

**Speaker:** Timothy Perutz

**Title:** Fixed-point Floer homology in spaces of stable pairs over a Riemann surface

**Abstract:** I will report on joint work with graduate student Andrew Lee, motivated by a proposal that there should exist a “U(2)-Heegaard Floer theory”: an analogue of the Heegaard Floer theory for 3-manifolds, based on Lagrangian Floer theory not in the  $g$ -fold symmetric product of the Heegaard surface  $\Sigma$ , but in the moduli space  $M$  of rank 2 stable pairs over  $\Sigma$  (as studied by Bradlow and by Thaddeus, among others) of degree  $2g+2$  and small stability parameter. This is a Fano manifold of dimension  $3g$ ; when  $g=1$ , it is the blow-up of projective 3-space along a copy of  $\Sigma$ . I expect such a theory will have an Atiyah-Floer type relationship with a gauge-theoretic Floer theory for the 3-manifold, based on a version of the U(2) Seiberg-Witten equations, but so far neither of these theories have been constructed. As an exploratory exercise, we develop a Floer theory for fibered 3-manifolds based on fixed-point Floer homology in  $M$ . We compute the resulting groups in genus 1; our argument requires a continuity principle for Floer homology, and a precise knowledge of quantum cohomology for  $M$ .

10:30am **Coffee break - SCGP 102**

11:00am **Yalong Cao - SCGP 102**

**Speaker:** Yaolong Cao

**Title:** Donaldson-Thomas gauge theory for Calabi-Yau 4-folds

**Abstract:** Donaldson-Thomas theory on Calabi-Yau 3-folds is a complexification of Chern-Simons-Floer theory on 3-manifolds. In this talk, we will discuss a complexification of Donaldson theory on complex oriented 4-folds (i.e. CY4). This is based on my PhD thesis under supervision of Naichung Conan Leung.

12:00pm **Lunch - SCGP 102**

2:15pm **Shinichiroh Matsuo - SCGP 102**

**Speaker:** Shinichiroh Matsuo

**Title:** ASD connections and mean dimension

**Abstract:**

3:30pm **Tea time - SCGP 102**

4:00pm **Alex Waldron - SCGP 102**

**Title:** Long-time existence for Yang-Mills flow

**Tuesday, October 18th**

9:00am **Rafe Mazzeo - SCGP 102**

**Title:** Some analytic aspects of the Kapustin-Witten equations with Nahm pole boundary conditions

10:00am **Coffee break - SCGP 102**

10:30am **Aliakbar Daemi - SCGP 102**

**Speaker:** Aliakbar Daemi

**Title:** Sutures and Higher Rank Bundles

11:30am **Lunch**

1:00pm **SCGP Weekly Talk Tom Mrowka - SCGP 102**

**Title:** An approach to the Four Color Theorem via Gauge Theory and Three Manifold Topology.

**Abstract:** A few years ago Peter Kronheimer and I realized that some of the invariants of knots in three manifolds coming from gauge theory had a generalization to invariants of knotted trivalent graphs. One of these invariants plausibly can be used to attack the four color theorem. In this talk I will sketch in rather broad terms some of these ideas. These invariants connect the four color problem to an important line of work combining gauge theory and 3 dimensional topology.

2:15pm **Benjamin Sibley - SCGP 102**

**Speaker:** Benjamin Sibley

**Title:** Limits and bubbling sets for the Yang-Mills flow on Kähler manifolds

**Abstract:** Fixed-point Floer homology in spaces of stable pairs over a Riemann surface

3:30pm **Tea - SCGP Lobby**

**Wednesday, October 19th**

9:30am **Thomas Walpuski - SCGP 102**

**Title:**  $G_2$ -instantons over twisted connected sums

**Abstract:** Donaldson and Thomas's visionary article "Gauge Theory in Higher Dimensions" initiated a program to study gauge theory in the context of special holonomy and, in particular, on  $G_2$ -manifolds. I will explain how the quest for a higher dimensional version of Chern-Simons theory naturally leads to  $G_2$ -geometry. In joint work with H. Sá Earp, we introduced a method to construct  $G_2$ -instantons over compact  $G_2$ -manifolds arising as the twisted connected sum (TCS) of a matching pair of building blocks. After reviewing the TCS, I will discuss our main result and explain how to interpret it in terms of certain Lagrangian subspaces of a moduli space of stable bundles on a K3 surface. Finally, I will talk about how to use our construction to produce a rather concrete example of a  $G_2$ -instanton over twisted connected sum discovered by Crowley and Nordström. Time permitting, I will also discuss ideas for implementing Mukai duality for TCS  $G_2$ -manifolds (currently being investigated in joint work with A. Kovalev).

10:30am **Coffee break**

11:00am **Sergey Cherkis - SCGP 102**

**Speaker:** Sergey Cherkis

**Title:** Analysis of Yang-Mills Instantons on multi-Taub-NUT and Bows.

12:00pm **Lunch**

2:15pm **Nikita Nekrasov - SCGP 102**

**Title:** Crossed and spiked instantons, and the Compactness theorem

**Abstract:** Gieseker-Nakajima moduli spaces parametrize noncommutative  $U(n)$  instantons on  $\mathbb{R}^4$  and framed rank  $n$  torsion free sheaves on  $\mathbb{C}P^2$ . They also serve as local models of the moduli spaces of instantons on general four-manifolds. We study the generalization of gauge theory in which the four dimensional spacetime is a stratified space  $X$  immersed into a Calabi-Yau fourfold  $Z$ . The local model of the corresponding instanton moduli space is the moduli space of (noncommutative) instantons on origami spacetimes with  $X$  modelled on a union of (up to six) coordinate complex planes  $\mathbb{C}^2$  intersecting in  $Z$  modelled on  $\mathbb{C}^4$ . The instantons are shared by the collection of four dimensional gauge theories sewn along two dimensional defect surfaces and defect points. We also define several quiver versions of these moduli spaces, motivated by the considerations of sewn gauge theories on orbifolds  $\mathbb{C}^4/\Gamma$ . The main result is the compactness of the set of torus-fixed points, for various tori. It underlies the non-perturbative Dyson-Schwinger identities recently found to be satisfied by the correlation functions of qq-characters viewed as local gauge invariant operators in the  $N=2$  quiver gauge theories.

3:30pm **Tea time**

**Thursday, October 20th**

9:30am **Andriy Haydys - SCGP 102**

**Speaker:** Andriy Haydys

**Title:** Topology of the blow up locus for the Seiberg-Witten equation

10:30am **Coffee break**

11:00am **Aleksander Doan**

**Title:** Seiberg-Witten monopoles with multiple spinors on a surface times a circle

**Abstract:** I will discuss a generalisation of the 3-dimensional Seiberg-Witten equations which was introduced by Haydys and Walpuski in relation to the problem of defining the Casson invariant of  $G_2$ -manifolds. The main difference from the classical setting is the lack of compactness, caused by so-called Fueter sections. In my talk I will explain how to tackle this problem and count the solutions in the special case when the underlying 3-manifold is the product of a Riemann surface and a circle. The main ingredient is a holomorphic description of the moduli space of solutions and its compactification. It allows us to relate our problem to classical results on theta divisors and stable bundles over complex curves.

12:00pm **Lunch**

2:15pm **Ryosuke Takahashi - SCGP 102**

**Speaker:** Ryosuke Takahashi

**Title:** APS-type index theorem and the zero loci of  $\mathbb{Z}/2$  harmonic spinors

3:30pm **Tea time**

4:00pm **Bobby Acharya - SCGP 102**

**Title:** G2-instantons and Associative submanifolds in String and M theory

5:00pm **Banquet**

**Friday, October 21st**

9:30am **Adam Jacob - SCGP 102**

**Title:** Hermitian-Yang-Mills connections over asymptotically cylindrical Kahler manifolds

**Abstract:** In this talk I will discuss an extension of the Theorem of Donaldson-Uhlenbeck-Yau to the non-compact setting. Given a complete Kahler manifold  $X$  with a cylindrical end, I will prove existence of a Hermitian-Yang-Mills connection on a reflexive sheaf over  $X$ . I will demonstrate how to achieve exponential decay of a solution along the cylindrical end, which has applications to the study of G2-instantons. This is joint work with T. Walpuski.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Dimirti Markushevich - SCGP 102**

**Title:** Algebro-geometric bubble-tree compactification of moduli of rank 2 bundles over algebraic surfaces

**Abstract:** A gauge-theoretic bubble-tree compactification of moduli of  $SU(2)$ -instantons on a 4-manifold was constructed in the works of Donaldson, Taubes and Feehan. In the talk, approaches to constructing an algebro-geometric analog of it will be discussed. In gauge theory, the objects on the boundary of the compactification are obtained by gluing ASD connections on the original 4-manifold  $S_0$  and on finitely many copies  $S_i$  of the 4-sphere forming a tree of "bubbles". In our analog, trees of bubbles become reducible algebraic surfaces, with 4-spheres replaced by complex projective planes, possibly blown up at finitely many points, and the ASD connections are substituted by holomorphic vector bundles. The completeness and separatedness of the compactification and relations to stability of bundles over bubble trees are discussed. This is a joint work with Alexander Tikhomirov and Günther Trautmann. The part on stability bases on the PhD thesis of Piero Coronica.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Yuanqi Wang - SCGP 102**

**Title:** A critical elliptic theory and its applications in higher-dimensional gauge theory

**Abstract:** The celebrated result of Lockhart-McCormick says that on a non-compact complete manifold, a first order elliptic operator (with proper asymptotic conditions) is Fredholm between weighted Sobolev spaces if and only if the weight is not an indicial root. We show that a proper weighted Sobolev-theory exists even when the weight is an indicial root, and give the index formula. We also discuss some applications to singular  $G_{\{2\}}$ -instantons which converges to their tangent cones in polynomial rates.