Riemannian Convergence Theory workshop
Talk Schedule

Events for:
Monday, November 9th - Friday, November 13th

Monday, November 9th

10:00am Gilles Carron - SCGP 102

Title: Integral pinched curvature and the topology of some 3-manifolds

Abstract: I will report on a joint work with V. Bour. The famous work of R. Hamilton on the Ricci flow gives a classification of closed Riemannian 3-manifolds with non negative Ricci curvature. We will give an analogous classification for closed 3-manifolds whose negative part of the Ricci curvature is small in an integral sense. Our results are based on a Bochner type argument, the classification of closed 3 manifolds carrying a metric with positive scalar curvature and a topological argument.

11:00am Coffee Break - SCGP Cafe

12:30pm Lunch - SCGP Cafe

2:30pm Jeff Viaclovsky - SCGP 102

Title: On the moduli space of critical metrics on connected sums of Einstein four-manifolds

Abstract: In joint work with Gursky, critical metrics for quadratic Riemannian functionals on certain connected sums was previously obtained using a gluing procedure. After discussing some background of this gluing problem, I will present some recent results regarding the computation of the quadratic term in the expansion of the Kuranishi map for this construction in the case of the blow-up of CP^2 at a point. The computation is quite non-trivial, since (i) this term is fundamentally a global geometric invariant and is not determined alone by local geometric invariants, and (ii) this term depends crucially on the explicit nonlinear structure of the critical equation. There is an interesting connection between these critical metrics and a family of Kahler constant scalar curvature metrics with edge-cone singularities along two divisors.

3:30pm Tea - SCGP Lobby

Tuesday, November 10th

10:00am Ruobing Zhang - SCGP 102

Title: Part 1: Quantitative Nilpotent Structure and Regularity of Collapsed Einstein Manifolds

Abstract: In this talk we discuss the regularity of Einstein manifolds and more generally manifolds with just bounded Ricci curvature, in the collapsed setting. A key tool in the regularity theory of noncollapsed Einstein manifolds is the following: If a bigger geodesic ball on an Einstein manifold M^n is sufficiently Gromov-Hausdorff-close to a ball in $\dR^n$, then in fact the curvature on a smaller ball is uniformly bounded. No such results are known in the collapsed setting, and in fact it is easy to see without more such results are false. It turns out that the failure of such an estimate is related to topology. Our main theorem is that for the above setting in the collapsed context, there is a correspondence between a priori curvature estimates and the maximality of local nilpotent rank. There are generalizations of this result to bounded Ricci curvature and even just lower Ricci curvature. This is joint work with Aaron Naber.

11:00am Ruobing Zhang - SCGP 102

Title: Part 2: Quantitative Nilpotent Structure and Regularity of Collapsed Einstein Manifolds

Abstract: In this talk we discuss the regularity of Einstein manifolds and more generally manifolds with just bounded Ricci curvature, in the collapsed setting. A key tool in the regularity theory of noncollapsed Einstein manifolds is the following: If a bigger geodesic ball on an Einstein manifold M^n is sufficiently Gromov-Hausdorff-close to a ball in $\dR^n$, then in fact the curvature on a smaller ball is uniformly bounded. No such results are known in the collapsed setting, and in fact it is easy to see without more such results are false. It turns out that the failure of such an estimate is related to topology. Our main theorem is that for the above setting in the collapsed context, there is a correspondence between a priori curvature estimates and the maximality of local nilpotent rank. There are generalizations of this result to bounded Ricci curvature and even just lower Ricci curvature. This is joint work with Aaron Naber.

12:00pm Lunch - SCGP Cafe

1:00pm   SCGP Weekly Talk - SCGP Rm 102

Speaker: Hans-Joachim Hein

Title: Riemannian convergence theory

Abstract: I will give an introduction to the topic of this week's SCGP workshop. The basic notion of a sequence of Riemannian manifolds converging to a limit manifold (or more generally metric space) was developed by Anderson, Cheeger, Fukaya, Gromov, and others in the 1980s. There has been a steady stream of further developments, with much recent activity such as the proof of the "codimension 4 conjecture" by Cheeger and Naber last year. Work so far has focused on problems involving sectional or Ricci curvature, with particularly strong applications to Kahler geometry and 3-manifolds. Challenges for the future include Ricci flow in higher dimensions and problems involving scalar curvature. Rather than surveying as many results as possible, I will try to explain a few basic ideas, examples, and applications in a way accessible to interested nonspecialists (such as the speaker).

3:30pm **Tea - SCGP Lobby**

Wednesday, November 11th

10:00am **Kota Hattori - SCGP 102**

Title: The nonuniqueness of the tangent cone at infinity of Ricci-flat manifolds

Abstract: For a complete Riemannian manifold (M,g), the pointed Gromov-Hausdorff limit of (M, r^2g, p) as r to 0 is called a tangent cone at infinity, if it exists. By the Gromov's Compactness Theorem, there is a tangent cone at infinity for every complete Riemannian manifolds with nonnegative Ricci curvatures. Moreover if it is Ricci-flat, with Euclidean volume growth and having at least one tangent cone at infinity with a smooth cross section, then the uniqueness holds by the result of Colding and Minicozzi. In this talk I will show an example constructed by Anderson-Kronheimer-LeBrun, which asserts the assumption of the volume growth is essential for their uniqueness theorem.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Brian Weber - SCGP 102**

Title: Regularity and Convergence on certain Toric 4-manifolds without Ricci Curvature Conditions

Abstract: Recently, under the condition of bounded scalar curvature, all blowup models for degenerations of toric K ?ahler manifolds in dimension 4 were classified by using the Donaldson-Joyce instanton construction along with a new Liouville-type theorem that applied to the resulting linear geometric PDEs. Under mild conditions on the symplectic structure, these blowup models are precisely the twisted Taub-NUT metrics discovered by Donaldson. In this talk, we explore the implications of this classification for regularity of compact toric K ?ahler 4-manifolds, Gromov-Hausdorff convergence of sequences, and the possible degeneration of these manifolds. Notably, neither Ricci curvature nor volume-growth controls are necessary to the blowup model classification, and therefore much can be said about convergence in a very general setting.

12:30pm **Lunch - SCGP Cafe**

3:30pm **Tea - SCGP Lobby**

4:00pm **Gao Chen - SCGP 102**

Title: Almost flat Calabi-Yau surface.

Abstract: SU(2) and U(1) are the only nontrivial holonomy groups acting freely. This property makes the theory of almost flat Calabi-Yau surfaces and Riemann surfaces very special. In this talk, I will discuss this special theory and its application to the classification of gravitational instantons,i.e. noncompact complete Calabi-Yau surface with faster than quadratic decay curvature.

Thursday, November 12th

10:00am **Jeff Cheeger - SCGP 102**

Title: Part 1: Regularity of Manifolds of Bounded Ricci Curvature and the Codimension 4 Conjecture

Abstract: TBA

11:00am **Coffee Break - SCGP Cafe**

11:30am **Jeff Cheeger - SCGP 102**

Title: Part 2: Regularity of Manifolds of Bounded Ricci Curvature and the Codimension 4 Conjecture

Abstract: TBA

12:30pm **Lunch - SCGP Cafe**

2:30pm **Bing Wang - SCGP 102**

Title: Regularity scale and convergence of the Calabi flow.

Abstract: This is a joint work with H.Z. Li and K. Zheng. We define regularity scales as alternative quantities of (max M |Rm|) ^ {?1} to study the behavior of the Calabi flow. Based on estimates of the regularity scales, we obtain convergence theorems of the Calabi flow on extremal K¨ahler surfaces, under the assumption of global existence of the Calabi flow solutions. Our results partially confirm Donaldson's conjectural picture for the Calabi flow in complex dimension 2. Similar results hold in high dimension with an extra assumption that the scalar curvature is uniformly bounded.

3:30pm **Tea - SCGP Lobby**

Friday, November 13th

10:00am **Simon Donaldson - SCGP 102**

Title: Co-associative fibrations of G2 manifolds

Abstract: I will describe a plausible model for the geometry of certain collapsed limits of 7-dimensional manifolds with exceptional holonomy G_2. The structure considered is a "Kovalev-Lefschetz fibration", with K3 fibres, and in the limit the fibres shrink to points. The limiting geometry is given locally by a stationary 3-dimensional submanifold in a space of indefinite signature. The first part of the talk will review background in G_2 geometry.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Thomas Walpuski - SCGP 102**

Title: Degenerations of G2-manifolds and vanishing associatives

12:30pm **Lunch - SCGP Cafe**

2:30pm **Xiuxiong Chen - SCGP 102**

Title: Limits of the Kahler-Ricci flow

Abstract: TBA

3:30pm **Tea - SCGP Lobby**

4:00pm **End Of Workshop**