

# Schedule

Events for:  
**Monday, April 29th - Friday, May 3rd**

## Monday, April 29th

9:00am **Christina Sormani - SCGP 102**

**Speaker:** Christina Sormani

**Title:** Introduce topics

**Abstract:**

10:30am **Armando Cabrera - SCGP 102**

**Speaker:** Armando Cabrera

**Title:** "On the stability of the positive mass theorem for asymptotically hyperbolic graphs"

11:15am **Tristan Ozuch - SCGP 102**

**Speaker:** Tristan Ozuch

**Title:** Noncollapsed degeneration of Einstein 4-manifolds

12:00pm **Zhongshan An**

**Speaker:** Zhongshan An

**Title:** Ellipticity of Bartnik boundary data for vacuum spacetimes"

12:45pm **Lunch - SCGP Cafe**

2:15pm **Changliang Wang - SCGP 102**

**Speaker:** Changliang Wang

**Title:** Compactness Theorem for Rotationally Symmetric Riemannian Manifolds with Positive Scalar Curvature. Part 1

**Abstract:** Gromov and Sormani conjectured that sequences of compact Riemannian manifolds with nonnegative scalar curvature and area of minimal surfaces bounded below should have subsequences which converge in the intrinsic flat sense to limit spaces which have nonnegative generalized scalar curvature and Euclidean tangent cones almost everywhere. In a joint work with Jiewon Park and Wenchuan Tian, we confirm this conjecture in the rotationally symmetric case. We will report this work in two short talks, and this is the first of them.

3:00pm **Wenchuan Tian - SCGP 102**

**Speaker:** Wenchuan Tian

**Title:** Compactness Theorem for Rotationally Symmetric Riemannian Manifolds with Positive Scalar Curvature. Part 2

**Abstract:** Gromov and Sormani conjectured that sequences of compact Riemannian manifolds with nonnegative scalar curvature and area of minimal surfaces bounded below should have subsequences which converge in the intrinsic flat sense to limit spaces which have nonnegative generalized scalar curvature and Euclidean tangent cones almost everywhere. In a joint work with Jiewon Park and Changliang Wang, we confirm this conjecture in the rotationally symmetric case. We will report this work in two short talks, and this is the first of them.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **Anna Sakovich - SCGP 102**

**Speaker:** Anna Sakovich

**Title:** Low regularity aspects of the Jang equation

**Tuesday, April 30th**

9:00am **Abdelghani Zeghib - SCGP 102**

**Speaker:** Abdelghani Zeghib

**Title:** Conformal groups of pseudo-Finsler structures

**Abstract:**

10:00am **Tea - SCGP Cafe**

10:30am **Christian Ketterer - SCGP 102**

**Speaker:** Christian Ketterer

**Title:** Flat and intrinsic flat convergence of graphical tori with lower scalar curvature bounds. Part 1

**Abstract:** A celebrated theorem by Schoen-Yau and Gromov-Lawson states that Riemannian tori with nonnegative scalar curvature must be flat. Later Gromov conjectured that Riemannian tori with almost nonnegative scalar curvature should be close to flat tori w.r.t. to some weak Sobolov-type topology on the space of Riemannian metrics. In this talk we address Gromov's conjecture in 3D from the viewpoint of flat topology. In this first part, by adapting ideas of Huang and Lee, and Huang, Lee and Sormani, we show that the total volume of tori that are graphs w.r.t. to smooth functions on a fixed flat torus  $T^2$  converges to the volume of  $T^2$  if the lower scalar curvature bounds approaches  $0$ . We require 3 technical assumptions for the level sets that allow us to apply an ODE comparison principle for the surface volume of the level sets. From this, we deduce in a quantitative way that some level sets have small volume and the amount of level sets with big volume decreases as the scalar curvature goes to zero. Thus, the convergence of the volume follows. As consequence we see that our graph tori with almost nonnegative scalar curvature - considered as embedded submanifolds in  $T^2 \times \mathbb{R}^3$  - converge w.r.t. flat distance to  $T^2 \times \mathbb{R}^3$ .

11:15am **Raquel Perales Aguilar - scgp 102**

**Speaker:** Raquel Perales Aguilar

**Title:** Flat and intrinsic flat convergence of graphical tori with lower scalar curvature bounds. Part 2

**Abstract:** In this second part we address Gromov's conjecture in 3D from the viewpoint of Sormani-Wenger intrinsic flat topology. We adapt Huang, Lee and Sormani and, Sormani's techniques to show that graphical tori as in the previous talk (where we do not necessarily fix a flat torus  $T^2$ ) subconverge w.r.t. SWIF distance to a fat torus. We define graphical fundamental domains. Then given a sequence of graphical tori with almost nonnegative scalar curvature we show that a subsequence of them and of their graphical fundamental domains converge w.r.t SWIF distance. The former to a current space  $(X, d, S)$  and the latter to a flat fundamental domain  $D$ . We then construct a map from which we deduce that  $X$  is a flat torus and  $D$  its fundamental domain

11:45am **lunch - SCGP Cafe**

1:00pm **SCGP Weekly Talk: Philippe LeFloch - 102**

**Speaker:** Philippe LeFloch

**Title:** A geometric analysis view on spacetimes: stability and singularities

**Abstract:** I will overview, in a non-technical manner, some recent mathematical results on the Einstein equations of general relativity, especially on understanding nonlinear geometry-matter interactions. Are self-gravitating matter fields nonlinearly stable near Minkowski spacetime? In what quantitative sense does the curvature provide a control of the spacetime geometry? What happens when two plane gravitational waves collide? Blog: philippelefloch.org

2:15pm **Shabnam Beheshti - SCGP 102**

**Speaker:** Shabnam Beheshti

**Title:** Well-posed viscous cosmology? Open questions and first steps

**Abstract:** It is known that viscous effects lead to nontrivial dynamical behaviour in homogeneous cosmological models such as FLRW and Bianchi spacetimes. Recent progress in well-posedness of certain Einstein-Navier-Stokes systems motivates revisiting a model first proposed by Lichnerowicz in 1967. We investigate the role of dynamic velocity in a cosmological background, demonstrating that the additional degree of freedom afforded by the associated fluid index may play a geometric role in the evolution of the fluid. Open questions for the more general setting of conformal fluids are also suggested.

3:00pm **Demetre Kazaras - scgp 102**

**Speaker:** Demetre Kazaras

**Title:** 4-dimensional cobordisms of positive scalar curvature and metric singularities

**Abstract:** We show that the bordism group of closed 3-manifolds with positive scalar curvature (psc) metrics is trivial by explicit methods. Our constructions are derived from scalar-flat  $K\{a\}$ -like ALE surfaces discovered by Lock-Viaclovsky. Next, we study psc 4-manifolds with metric singularities along points and embedded circles. Our psc null-bordisms are essential tools in a desingularization process developed by Li-Montoulidis. This allows us to prove a non-existence result for singular psc metrics on enlargeable 4-manifolds with uniformly Euclidian geometry. As a consequence, we obtain a positive mass theorem for asymptotically flat 4-manifolds with non-negative scalar curvature and low regularity.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **Andrea Mondino - SCGP 102**

**Speaker:** Andrea Mondino

**Title:** An optimal transport formulation of the Einstein equations of general relativity

**Abstract:** In the talk I will present a recent joint work with Stefan Suhr (Bochum) where we give an optimal transport formulation of the full Einstein equations of general relativity, linking the (Ricci) curvature of a space-time with the cosmological constant and the energy-momentum tensor. Such an optimal transport formulation is in terms of convexity/concavity properties of the Shannon-Boltzmann entropy along curves of probability measures extremizing suitable optimal transport costs. The result gives a new connection between general relativity and optimal transport (compare with recent related work by Robert McCann); moreover it gives a mathematical reinforcement of the strong link between general relativity and thermodynamics/information theory that emerged in the physics literature of the last years.

**Wednesday, May 1st**

9:00am **Xianzhe Dai - SCGP 102**

**Speaker:** Xianzhe Dai

**Title:** Entropy Rigidities for RCD Space

**Abstract:** Volume entropy is a fundamental geometric invariant defined as the exponential growth rate of volumes of balls in the universal cover. It is a very subtle invariant which has been extensively studied in geometry, topology and dynamical systems. RCD spaces are the most general metric spaces which one can still talk about Ricci curvature lower bounds (and still in the Riemannian category). They contain the Ricci limit spaces and have attracted intensive attention recently. We will report some of our recent joint work with Chris Connell, Jesus Nunez-Zimbron, Requel Perales, Pablo Suarez-Serrato and Guofang Wei about the generalization to RCD spaces of the volume entropy rigidity results, including the famous rigidity result of Besson-Courtois-Gallot.

10:00am **Tea - SCGP Cafe**

10:30am **Stephanie Alexander - SCGP 102**

**Speaker:** Stephanie Alexander

**Title:** Alexandrov geometry for Lorentzian-pseudometrics

**Abstract:** "We extend the Alexandrov theory of metric spaces with curvature bounds to the not-just-causal Lorentzian setting. Our motivation is the outstanding contribution of Alexandrov geometry to the global theory of Riemannian spaces. We seek a comparison theory that governs both causal and spacelike geodesics because these are the scaffolding of the global geometry and topology of the space. We introduce Lorentzian-pseudometrics, which take possibly negative distances, as the correct analogue of metric for this program. Curvature bounds are defined by comparison of triangles whose sides have arbitrary causal character. Among our examples: Sectional curvature bounds (in the sense of Andersson-Howard) are extended to  $C^{1,1}$ -Lorentzian metrics. We show that the purely timelike synthetic curvature bounds defined by Kunzinger-S'ammann are consistent with our timelike/spacelike bounds. As a test case, we prove the following analogue for Lorentzian-pseudometrics of a fundamental theorem of metric geometry: A Minkowski cone over a geodesic metric space  $M$  is a Lorentzian-pseudometric space with curvature bounded below by  $0$  if and only if  $M$  is an Alexandrov space of curvature bounded below by  $\kappa_1$  (similarly for bounds above)."

11:45am **Stacey Harris - SCGP 102**

**Speaker:** Stacey Harris

**Title:** Simple Characterization of the Causal Boundary and Continuous Extension of the Spacetime Metric

**Abstract:** This is an ongoing examination of the Causal Boundary of a spacetime  $M$  in terms of a foliation  $F$  by observer worldlines (timelike curves), without any assumed symmetries for  $M$ . The idea is to reflect spacetime geometry in time-dependent fields (Riemannian metric and drift-form) on the leaf-space  $Q (= M/F)$  of the foliation. In this stage of the set-up (inspired by interior Schwarzschild), the worldlines are assumed to be future-incomplete, and conditions are obtained which will force the Future Causal Boundary of  $M$  to be spacelike, diffeomorphic to  $Q$ , and attached to  $M$ ---represented as  $(a, b) \times Q$ ---as  $\{b\} \times Q$  in  $(a, b] \times Q$ . With that in hand, then the continuous extension of the spacetime metric to the Boundary depends simply on an integral condition of sectional curvature along the observer worldlines. Future stages of this set-up could be aimed at future-complete foliations, emulating either Robertson-Walker with a spacelike Future Causal Boundary or Minkowski space with a null Future Causal Boundary.

12:45pm **lunch - SCGP Cafe**

2:15pm **Clemens Saemann - SCGP 102**

**Speaker:** Clemens Saemann

**Title:** Lorentzian length spaces

**Abstract:** We introduce an analogue of the theory of length spaces into the setting of Lorentzian geometry and causality theory. The role of the metric is taken over by the time separation function, in terms of which all basic notions are formulated. In this way we recover many fundamental results in greater generality, while at the same time clarifying the minimal requirements for and the interdependence of the basic building blocks of the theory. A main focus of this work is the introduction of synthetic curvature bounds, akin to the theory of Alexandrov and CAT(k)-spaces, based on triangle comparison. Applications include Lorentzian manifolds with metrics of low regularity, closed cone structures, and warped products of a line with a (Riemannian) length space. Moreover, an application to the low regularity (in)-extendibility of spacetimes is given and if time permits we give an outlook on a notion of convergence of such spaces.

3:00pm **Eric Ling - SCGP 102**

**Speaker:** Eric Ling

**Title:** Spacetime Extensions of the Big Bang

**Abstract:** In this talk we show that a large class of  $k = -1$  inflationary FLRW spacetimes dubbed 'Milne-like' admit continuous spacetime extensions through the big bang. For these spacetimes the big bang is a coordinate singularity analogous to how the  $r = 2m$  event horizon in Schwarzschild is a coordinate singularity. The geometry of the big bang coordinate singularity for Milne-like spacetimes is that of a lightcone in a spacetime conformal to Minkowski space. We discuss how the mathematics of these Milne-like spacetimes may provide connections to problems in cosmology.

3:30pm **Tea Time - SCGP Cafe**

4:00pm **David Maxwell - SCGP 102**

**Speaker:** David Maxwell

**Title:** The Conformal Method in Low Sobolev Regularity

**Abstract:** This talk we discuss the CMC conformal method and its application to constructing low Sobolev regularity solutions of the constraint equations. In Sobolev scales, the method can be adapted to metrics that have one weak derivative and have just enough regularity to be continuous. Technical improvements needed to accomplish this goal lead to a simplified theory of the conformal method, even for smooth solutions

**Thursday, May 2nd**

9:00am **Bruno Le Floch - SCGP 102**

**Speaker:** Bruno Le Floch

**Title:** Global evolution of self-gravitating matter in T2 symmetry

**Abstract:** Compressible matter obeying the Euler equation naturally forms shock wave singularities. These back-react on the spacetime geometry, leading to weakly regular metrics whose curvature is only defined in the sense of distributions. I will explain how to formulate the Einstein and Euler equations at this level of regularity for spacetimes that admit two commuting Killing vectors (technically, T2 or Gowdy symmetry). I will then present a global stability theory for such spacetimes with weak regularity. This is joint work with P.G. LeFloch (Sorbonne Univ).

10:00am **Tea - SCGP Cafe**

10:30am **Chao Li - SCGP 102**

**Speaker:**

**Title:** Constrained deformations of manifolds with positive scalar curvature metrics

**Abstract:** We present a series of results concerning the interplay between the scalar curvature of a manifold and the mean curvature of its boundary. In particular, we give a complete topological characterization of those compact 3-manifolds that support Riemannian metrics of positive scalar curvature and mean-convex boundary and, in any such case, we prove that the associated moduli space of metrics is path-connected. The methods we employ are flexible enough to allow the construction of continuous paths of positive scalar curvature metrics with minimal boundary, and to derive similar conclusions in that context as well. This talk is based on a joint work with Alessandro Carlotto.

11:45am **Shouhei Honda - SCGP 102**

**Speaker:** Shouhei Honda

**Title:** Embedding of metric measure spaces in  $L^2$  via heat kernel

**Abstract:** Berard-Besson-Gallot proved that any closed Riemannian manifold can be embedded in  $L^2$  via the heat kernel and that the original Riemannian metric can be approximated by the pull-back metrics. In this talk we generalize this theorem to singular spaces, so-called RCD(K, N) metric measure spaces. Combining the Gromov-Hausdorff compactness of the moduli space of noncollapsed spaces with Reifenberg flatness, we prove a quantitative sharp convergence result for the pull-back metrics, which is new even for closed Riemannian manifolds. This is a joint work with L. Ambrosio, J. W. Portegies and D. Tewodrose.

12:45pm **Lunch - SCGP Cafe**

2:15pm **Michael Kiessling - SCGP 102**

**Speaker:** Michael Kiessling The Einstein-Infeld-Hoffmann legacy in mathematical relativity

**Abstract:** Einstein's General Relativity Theory is usually summarized, in a nutshell, as follows: "Matter tells spacetime how to curve, and spacetime curvature tells matter how to move." In particular, one learns that uncharged point particles of rest mass  $m > 0$  move along (time-like) geodesics. This law of geodesic motion is based on the notion of "test particle" — a theoretical fiction which assumes that the contribution of the very point particle to the curvature of spacetime can be ignored. In their Annals of Mathematics paper of 1938, Albert Einstein, Leo Infeld, and Banesh Hoffmann tried to go beyond this test-particle fiction by arguing that Einstein's gravitational field equations would also dictate the motion of point particles which "do tell spacetime how to curve." Point particles are associated with time-like singularities of spacetime and the electromagnetic fields these support. Needless to say that their paper was not mathematically rigorous by modern standards; worse, their claim cannot be true in the generality in which the claim was made. In this talk I first explain the physical background, then formulate the problem mathematically sharply, then state what we've managed to rigorously prove about it, and last not least I point out important open problems for mathematical relativists to work on. This project is a long-time collaboration with Shadi-Tahvildar Zadeh, and more recently also Annegret Burtscher.

3:00pm **Annegret Yvonne Burtscher - SCGP 102**

**Speaker:** Annegret Yvonne Burtscher

**Title:** Energy-momentum conservation for spacetimes with singularities

**Abstract:**

3:30pm **Tea Time - SCGP Cafe**

4:00pm **Guofang Wei - SCGP 102**

**Speaker:** Guofang Wei

**Title:** Semi-local simple connectedness of non-collapsing Ricci limit spaces

**Abstract:** Gromov-Hausdorff limits of manifolds with uniform Ricci curvature lower bounds are referred as Ricci limit spaces. Studying these spaces has been a very active field. Cheeger, Colding, and Naber have developed a rich theory on the regularity and geometric structure of these spaces, leading to many applications. On the other hand, the topology of these spaces could be very complicated. About twenty years ago, Sormani and I gave the first topological restriction, showing that the universal cover of any Ricci limit space does exist, but were not able to show that the universal cover is simply connected. The question is surprisingly complicated even in the non-collapsing case. In a very recent joint work with J. Pan, we solved this problem, showing that the non-collapsing Ricci limit spaces are in fact essential locally simply connected. This allows extension of results on fundamental groups for manifolds to limit spaces.

**Friday, May 3rd**

9:00am **Theodora Bourni - SCGP 102**

**Speaker:** Theodora Bourni

**Title:** "null mean curvature" flow and marginally outer trapped surfaces.

**Abstract:** In this talk we discuss a new second order parabolic evolution equation for hypersurfaces in space-time initial data sets, that generalizes mean curvature flow (MCF). In particular, the 'null mean curvature' - a space-time extrinsic curvature quantity - replaces the usual mean curvature in the evolution equation defining MCF. This flow is motivated by the study of black holes and mass/energy inequalities in general relativity. We present a theory of weak solutions using the level-set method and outline a natural application of the flow as a parabolic approach to finding outermost marginally outer trapped surfaces (MOTS), which play the role of quasi-local black hole boundaries in general relativity. This is joint work with Kristen Moore.

10:00am **Tea - SCGP Cafe**

10:30am **Marcus Khuri - SCGP 102**

**Speaker:** Marcus Khuri

**Title:** Stationary Vacuum Black Holes in Higher Dimensions

11:45am **Dan Lee - SCGP 102**

**Speaker:** Dan Lee

**Title:** Lower semicontinuity of ADM mass under Sormani-Wenger convergence

**Abstract:** In earlier work, we proved a lower semicontinuity result for ADM mass under  $C^0$  Cheeger-Gromov convergence. In this talk we discuss an extension of that result assuming a much weaker type of convergence. All of this is joint work with Jeffrey Jauregui.

12:45pm **Lunch - SCGP Cafe**

2:15pm **A. Shadi Tahvildar-Zadeh - SCGP 102**

**Speaker:** A. Shadi Tahvildar-Zadeh

**Title:** Quantum Laws of Motion for Singularities of Spacetime

3:30pm **Tea Time - SCGP Cafe**

4:00pm **Closing Discussion - SCGP 102**