Workshop Schedule

Events for:
Monday, March 8th - Friday, March 12th

Monday, March 8th

9:00am **Mitsuhiro Shishikura**

Title: Reconstructing the germs of irrationally indifferent germs from renormalized sequence

Abstract: We discuss how to analyze the germs of irrationally indifferent germs of high type using the renormalized sequences. Key difference from unimodal maps or critical circle maps is that the domain of definitions of renormalizations overlap each other. We overcome this difficulty using the idea of dynamical charts and Decjoy odometer symbolic dynamics which governs the the combinatorics of pasting

10:00am **Pierre Berger**

Title: gy in the Hénon family
Abstract**:** We will present tools in bifurcation theory and a new theory on Hénon-like renormalization nearby homoclinic tangencies or chain of heteroclinic tangencies to deduce several new phenomena:
- long renormalization strips in the space of Hénon-like families,
- parametric embedding of composed quadratic families into the Hénon family (Milnor swallow or Shrimp) or nearby any non-degenerate 2-parameter homoclinic unfolding,
- new examples of uniformly hyperbolic Hénon maps, with non trivial number of attracting cycles,
- existence of wandering Fatou component among polynomial automorphisms of $\mathbb R^2$, in a joint work with S. Biebler,
- $Aut_d(\mathbb R^2)$-universal family in the space of steady Euler flows (Beltrami vector fields of $\mathbb R^3$), in a work in progress with A. Florio and D. Peralta-Salas.

11:00am **Natasha Goncharuk**

Title: Bubbles and renormalization

Abstract: Take a cylinder of height h and glue its boundaries via an analytic circle diffeomorphism f. The modulus of the resulting complex torus is called the complex rotation number of $f+ih$. Its limit as $h \to 0$ is related (and sometimes equal) to the rotation number of f. Add one more parameter: consider the map that takes $a+ih$ to the complex rotation number of $f+a+ih$. The limit values of this map as $h \to 0$ form a fractal set ``bubbles''. Bubbles is a complex analogue to Arnold tongues. I will discuss the shapes and scaling of bubbles near rational and Diophantine rotation numbers, and the relation of these questions to renormalization operators in the space of conformal annuli maps. The talk is partly based on joint results with X.Buff and on joint results with M.Yampolsky.

2:00pm **Anatoly Dymarsky**

Title: Scale invariance in quantum field theory

Abstract: In the context of unitary relativistic quantum field theory fixed points of the RG flow must exhibit scale invariance, yet all known examples are in fact conformal invariant. This enhancement of symmetry indicates a fundamental property of unitary relativistic QFTs, yet the one which is difficult to properly justify. This problem is only solved in two dimensions and partial progress has been achieved in four. In the talk I will review this problem, together with some recent progress, and outline possible directions for the future.

3:00pm **Svetlana Jitomirskaya**

Title: On the critical almost Mathieu cocycle.

Abstract: The critical almost Mathieu cocycle is special in various ways. It represents a prototypical family of cocycles at the boundary between non-uniform hyperbolicity and reducibility. It is also the prototypical critical cocycle in the sense of Avila's global theory. Finally, it is the model behind the famous Hofstadter's butterfly and has been heavily studied in physics literature. Besides one Fields medal, it is linked to several Nobel prizes. We will present a complete proof of singular continuous spectrum for this family, for all phases, finishing a program with a long history, and based on a simple Fourier analysis and a new duality-type transform. We also present a result (with I. Krasovsky) that proves one half of the Thouless' one half conjecture from the early 80s: that Hausdorff dimension of the spectrum of the ubderlying Schrodinger operator (the bifurcation set for the family of cocycles) is bounded by 1/2 for all irrational frequencies. If time permits, we will also discuss recent progress towards the Thouless Catalan conjecture (joint with I. Krasovsky and L. Konstantinov).

Tuesday, March 9th

9:00am **Davoud Cheraghi**

Title: Complex Feigenbaum phenomena with degenerating geometries

Abstract: Renormalisation is a main focus of the theory of one-dimensional complex dynamics. It is connected to the central conjectures on the density of hyperbolicity and the local connectivity of the Mandelbrot set. For quadratic polynomials, there are two different types of renormalisations — primitive and satellite types. The primitive renormalisation has been successfully studied over the past few decades; the corresponding maps exhibit tame dynamical behaviour. The satellite type has a very different nature and remained mostly mysterious until recently. In this talk, we discuss the wide range of possibilities for the dynamics in presence of infinitely many satellite renormalisation structures.

10:00am **Theodore D Drivas**

Title: Spontaneously stochastic solutions in dynamical systems with singularities

Abstract: We consider a class of dynamical systems described by ordinary differential equations with an isolated singularity, where the singularity is characterized by the lack of Lipschitz continuity. Singularities are common in applications both for ODEs (e.g., particle collisions) and PDEs (e.g., finite-time blowup in fluid models). The fundamental obstacle is that solutions cannot be continued past the singularity uniquely: typically, there are infinitely many solutions. The conventional way to proceed is to define a regularization limit, such as vanishing viscosity or noise. It turns out that there are structurally stable situations when such a limit is not sensitive to a particular form of regularization. This is explained by the analysis of attractors for the rescaled (non-singular) dynamical system and their ergodic properties. What is even more surprising is that solutions in this limit may become probabilistic (spontaneously stochastic) with the unique probability measure past the singularity. We will present rigorous results and discuss applications of this phenomenon. This is joint work with Alexei A. Mailybaev (IMPA, Rio de Janeiro) and Artem Raibekas (UFF, Rio de Janeiro).

11:00am **Charles Tresser**

Title: RG: the hopes of K. Wilson about the ubiquity of RG: naivety or pure great vision?

1:00pm **Alexander Polyakov**

Title: In praise of quantum field theory

Abstract: Quantum field theory provides a universal language for very different areas of physics. I will try to demonstrate this by briefly discussing some funda-mental problems, both solved and unsolved.cThe problems will include (if time permits) critical phenomena and conformal bootstrap, quark confine-ment and gauge/string duality, cosmological con-stant and turbulence, and may be more.

2:15pm **Pavel Bleher**

Title: A Thouless-Like Effect in the Dyson Hierarchical Model with Continuous Symmetry Abstract. We study Dyson's vector valued hierarchical models with continuous symme- try. We prove a conjecture of Dyson which states a necessary and sufficient condition of the existence of a phase transition in the models under consideration. We also prove another Dyson's conjecture that in the presence of the phase transition the spontaneous magnetization vanishes at the critical point, i.e., there is no Thouless' effect. On the other hand, we prove that the distribution of the normalized mean spin at the critical temperature Tc with free boundary conditions tends to a uniform distribution on the unit sphere as the volume tends to infinity, a phenomenon which resembles the Thouless effect. We prove that the limit distribution of the normalized average spin is Gaussian for T > Tc, and it is non-Gaussian for T ? Tc. We also show that the density of the limiting radial distribution for T ? Tc is a nice analytic function which can be found as a unique solution of a nonlinear fixed point renormalization group equation. Finally, we determine various critical asymptotics and show that the divergence of the correlation length and magnetic susceptibility are super-polynomial as T ? Tc. This is a joint work with P ?eter Major, Budapest, Hungary.

Wednesday, March 10th

9:00am **Raphael Krikorian**

Title: On the accumulation of non-split separtrices by invariant circles

Abstract: A theorem by M.R. Herman,``Herman's last geometric theorem", asserts that if a smooth orientation and area preserving diffeomorphism f of the 2-plane $\R^2$ (or the 2-cylinder $\R/\Z\times\R$) admits a KAM curve C (a smooth invariant curve on which the dynamics of f is conjugated to a Diophantine translation) then C is accumulated by other KAM curves, the union of which covers a set of positive 2-dimensional Lebesgue measure in any neighborhood of C. In this talk we shall investigate whether such a phenomenon holds if, instead of being a KAM circle, the invariant set C is a (non-split) separatrix of a hyperbolic fixed point of f. This analysis might be useful for understanding symplectic diffeomorphisms with zero entropy or in the search of a smooth twist map admitting an isolated irrational invariant curve bounding two Birkhoff instability regions. The renormalization paradigm is an important element in our approach. This is a joint work with Anatole Katok.

10:00am **Michael Yampolsky**

Title: Renormalization convergence and rigidity of multicritical circle maps

Abstract: In a joint work with I. Gorbovickis we prove geometric convergence of renormalizations for C^3-smooth multicritical circle maps with rotation numbers of bounded type. From this, we obtain a C^{1+\alpha} rigidity theorem for such maps.

11:00am **Selim Ghazouani**

Title: Renormalisation of generalised interval exchange maps (and foliations on surfaces)

Abstract: Generalised interval exchange transformations (GIETs) are one-dimensional dynamical systems which naturally arise as first-return maps of flows/foliations on surfaces. Building upon work of Forni, Marmi-Moussa-Yoccoz and Khanin, I will attempt to draw a conjectural picture of a renormalisation theory that would give a satisfactory account of rigidity and geometric properties of GIETs. If time permits, I will try to highlight the important role that Lorentzian geometry should play in the development of such a theory.

Thursday, March 11th

9:00am **Sebastian Van Strien**

Title: Conjugacy classes of real analytic maps

Abstract: Avila-Lyubich-de Melo proved that the topological conjugacy classes of unimodal real-analytic maps are complex analytic manifolds, which laminate a neighbourhood of any such mapping without a neutral cycle. Their proof that the manifolds are complex analytic crucially depends on the fact that these are codimension-one in the space of unimodal mappings. In joint work with Trevor Clark, we show how to construct a “pruned polynomial-like mapping" associated to a real mapping. This gives a new complex extension of a real-analytic mapping. The additional structure provided by this extension, makes it possible to generalize this result of Avila-Lyubich-de Melo to interval mappings with several critical points. Thus we show that the conjugacy classes are complex analytic manifolds whose codimension is determined by the number of critical points. This result allows us to show density of real-hyperbolicity in families of real transcendental maps (such as trigonometric polynomials), even if the critical points of these maps are non-real. This generalises joint results with Lasse Rempe, and answers a question by Welington de Melo.

10:00am **Dzmitry Dudko**

Title: Near-neutral renormalization in Complex Dynamics

Abstract: We will give an overview of near-neutral renormalization theories for quadratic polynomials and discuss how these theories relate to the renormalization of circle maps and to the problem of local connectivity of the Mandelbrot set. We will then discuss uniform a priori bounds for neutral polynomials that control how Siegel quasidisks degenerate when the rotation number becomes unbounded. Joint work with Misha Lyubich.

11:00am **Gregory Falkovich**

Title: Fibonacci turbulence

Abstract: Never is the difference between thermal equilibrium and turbulence so dramatic, as when a quadratic invariant makes the equilibrium statistics exactly Gaussian with independently fluctuating modes. That happens in two very different yet deeply connected classes of systems: incompressible hydrodynamics and resonantly interacting waves. I shall describe an information-theoretic analysis of turbulence in such strongly interacting systems. We introduce a beautifully simple yet rich family of discrete models with neighboring triplet interactions and show that it has families of quadratic conservation laws defined by the Fibonacci numbers. Depending on the single model parameter, three types of turbulence were found: single direct cascade, double cascade, and the first ever case of a single inverse cascade. We describe quantitatively how deviation from thermal equilibrium all the way to turbulent cascades makes statistics increasingly non-Gaussian and find the self-similar form of the one-mode probability distribution. We reveal where the information (entropy deficit) is encoded and disentangle the communication channels between modes, as quantified by the mutual information in pairs and the interaction information inside triplets.

4:00pm **Coffee/discussion Break**

4:30pm **Curtis McMullen, Math Colloquium**

Title: Billiards and the arithmetic of non-arithmetic groups

Friday, March 12th

9:00am **(CANCELLED) Marco Martens**

Title: TBA

10:00am **Daniel Smania**

Title: Infinitesimal deformations of one-dimensional maps

Abstract: Perhaps one of the main features of one-dimensional dynamics (either real or complex) is that the theory of deformations is rich. By this we mean that the topological classes of such maps often are infinite dimensional manifolds, but with finite codimension. Moreover for smooth families of maps inside a given topological class the associated family of conjugacies also moves on a smooth way. The derivative of this family of conjugacies with respect to the parameter is called an infinitesimal deformation. There are various applications in the study of renormalisation theory and linear response theory. The theory of holomorphic motions and its uses in complex dynamics is one the most famous incarnations of this phenomena. Lyubich used infinitesimal deformation in a crucial way to study the quadratic-like maps and renormalization theory on this setting, as well as Avila, Lyubich and de Melo in the study of generic behaviour of real-analytic unimodal maps. For real maps on the interval this phenomena also occurs, but our current understanding is far behind the complex setting. We will discuss the recents developments obtained in joint work with several collaborators: Viviane Baladi, Amanda de Lima and Clodoaldo Ragazzo.

11:00am **Igor Klebanov**

Title: Confining or Not?

Abstract: The problem of Color Confinement in Yang-Mills theory is one of the deepest problems in theoretical physics. There is convincing numerical evidence from Lattice Gauge Theory, yet the proof of Confinement in Asymptotically Free theories has not been found. I will briefly introduce the Confinement problem and review some results on large N theories using the gauge/gravity duality. I will then discuss two-dimensional SU(N) theory coupled to an adjoint Majorana fermion. I will show that, when the adjoint mass is sent to zero, the spectrum retains a mass gap but the confinement disappears. Using the Discretized Light-Cone Quantization, I will discuss the spectrum of color singlet states and exhibit certain threshold states. Similar threshold states are also present in a model with a massless adjoint and a massive fundamental fermion. They provide new evidence for the lack of confinement. When the adjoint mass is turned on, the theory becomes confining, and the spectrum of bound states becomes discrete.

1:30pm **Giovanni Forni**

Title: Ergodic properties of Billiards in Polygons, Renormalization and Hodge theory

Abstract: We survey old and new results on the ergodic theory of rational and non-rational billiards in polygons from the perspective of renormalization methods based on the so-called Teichmueller geodesic flow and related cocycles (Kontsevich--Zorich, twisted). We emphasize the role of Hodge theory in the analysis (just started in the case of the twisted cocycle) of their Lyapunov structure, which lies at the core of most of the results we will present.

2:30pm **Paul Wiegmann**

Title: Almost Mathieu equation and representation theory: does it help to understand singular continuous spectrum?

Abstract: Almost Mathieu equation is the case study of operators with singular continuous spectrum. Most of (extended) studies of the spectrum belongs to the domain of functional analysis. At the same time, incidentally or not, the problem of Almost Mathieu operator can be formulated in terms of the representation theory of the (quantum) group $SL_q(2)$: its spectrum can be seen as weights of $SL_q(2)$ cyclic representation and obeys certain algebraic equation often called the Bethe Ansatz. In the talk I review this (not-so-recent) developments and formulate the problem of quantitative description of the spectrum in terms of yet to be determine critical exponents.