Small Scale Dynamics in Fluid Motion

Events for:
Monday, June 13th - Friday, June 17th

Monday, June 13th

9:00am **Introduction to workshop - SCGP 102**

9:30am **Alexei Mailybaev - SCGP 102**

Speaker: Alexei Mailybaev

Title: Spontaneous stochasticity and renormalization group in discrete multi-scale dynamics

Abstract: We study an initial value problem for a class of ideal scale-invariant systems with discrete time, by introducing a regularization and noise at small scales. Such systems are motivated by scale-invariant physical models, like equations for an ideal fluid regularized by viscosity and microscopic fluctuations. We propose a qualitative theory describing the vanishing regularization (inviscid) limit in this problem as an attractor of the renormalization group (RG) operator. The RG operator reflects symmetries and interactions of the ideal scale-invariant system. It acts on evolution maps of regularized systems, therefore, different regularizations provide different initial conditions for the RG dynamics. A fixed-point RG attractor defines a unique dynamics of the ideal system, akin to shock solutions in viscous conservation laws. Otherwise, a chaotic attractor defines a Markov kernel of spontaneously stochastic solutions. These are intrinsically probabilistic solutions solving deterministic equations of the ideal system with deterministic initial conditions. The results are illustrated with solvable models. This is a joint work with Artem Raibekas.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Sam Punshon-Smith - SCGP 102**

Speaker: Sam Punshon-Smith

Title: Using regularity to estimate Lyapunov exponents

Abstract: I will discuss a new method for estimating Lyapunov exponents for hypoelliptic diffusions from below using local regularity of stationary measures on the projective bundle. For damped driven truncations of nonlinear conservative SPDE in fluctuation dissipation scaling, I will outline a general strategy for proving positivity of the top Lyapunov exponent that can be applied to Galerkin truncations of the stochastic Navier Stokes equations on T^2. I will also discuss ongoing work extending this method to the Grassmannian bundle to give information on the sum of the first k Lyapunov exponents, giving a possible strategy for estimating growth of attractor dimension from below.

12:00pm **Lunch - SCGP Cafe**

1:30pm **In-Jee Jeong - SCGP 102**

Speaker: In-Jee Jeong

Title: Critical Sobolev ill-posedness for incompressible Euler

Abstract: We present a quantitative and robust proof that the incompressible Euler equations are strongly ill-posed in critical Sobolev spaces, in the sense that norm inflation and nonexistence occur for critical initial data. The argument is based on combining the Key Lemma of Kiselev-Sverak with certain stability estimates for dyadic bubbles. This strategy extends to related incompressible and inviscid fluid models and has consequences for the dissipative counterparts. Based on joint works with Tarek Elgindi, Tsuyoshi Yoneda, and Junha Kim.

2:30pm **Klas Modin - SCGP 102**

Speaker: Klas Modin

Title: Zeitlin’s model: old and new

Abstract: In 1991 Vladimir Zeitlin used quantization theory to discretize two-dimensional ideal hydrodynamics. Contrary to other discretizations, Zeitlin's model preserves the underlying Lie-Poisson geometry. It also brings forth a dictionary between hydrodynamical notions and concepts in linear algebra and Lie theory. With an emphasis on these connections, I shall present Zeitlin's model and discuss its long-time qualitative behavior.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Discussion/Open problem - SCGP 102**

Tuesday, June 14th

9:00am **Half Talk: Julia Domingues Lemos - SCGP 102**

Speaker: Julia Domingues Lemos

Title: based approach for time-correlated closures of turbulence models

Abstract: Shell models of turbulence are a class of dynamical deterministic systems used to model energy cascade and other key aspects of the Navier-Stokes equation. Some, such as the Sabra model, also present intermittency. We study a way to rescale the variables of the Sabra model, where we observe hidden symmetries and universal distributions. We then use such fine distributions to write closures, i.e., missing expressions for some of the Sabra variables. Our closures rely on approximating probability density functions using a Gaussian Mixture Model, which makes them probabilistic by nature and allows us to write time-correlated closures. We also provide a framework where other Machine Learning tools can be employed with reduced black-box aspects. This is a joint work with Alexei A. Mailybaev.

9:30am **Gregory Eyink (remote) - Zoom**

Speaker: Gregory Eyink

Title: Onsager Theory for Momentum Cascade in Wall-Bounded Tubulence

Abstract: Experimental evidence shows that turbulent drag is either independent of Reynolds number Re (strong anomaly) or decaying morely slowly than the laminar rate ~1/Re (weak anomaly). This enhanced drag remains to be explained. Furthermore, existence of a strong anomaly for interior flows (pipes, channels, Taylor-Coulette or Rayleigh-Benard cells, etc.) seems empirically to require side-walls that are “hydraulically rough”, while a strong anomaly is observed in exterior flows (wakes of bluff bodies, jets through orifices, grid turbulence, etc.) even when the walls are smooth! To explain these various observations, we develop a dynamical theory of the infinite Reynolds number limit in the spirit of the renormalization group. To regularize UV divergences of velocity-gradients in the bulk, we can use the standard approach of spatial coarse-graining/low-pass filitering/mollifying which removes eddies of size <?. Regularizing divergences right at the walls requires an additional operation, which we take to be "windowing out" the eddies at distance <h from the wall. The effective coarse-grained balance equations for the fluid momentum for finite h,?>0 have inviscid limits which correspond to weak Euler solutions. These balance equations contain, however, an unclosed term which represents "spatial cascade" of momentum to the wall. We study this turbulent momentum cascade by adapting prior approaches of Onsager and Duchon-Robert for energy cascade. First, we show that the direct inviscid limits of the viscous skin friction and the wall pressure exist, in the sense
of distributions. We then show that these inviscid limits can be obtained from the flux of momentum to the wall, the skin friction from the tangential component and the pressure from the normal component. However, when the limiting Euler solution satisfies the no-flow-through condition at the wall in a suitable sense, then anomalous skin friction vanishes. In that case, all of the drag in the infinite Reynolds-number limit arises from “form drag” due to pressure forces and the latter can be computed from the weak Euler solution in the flow interior. We discuss how these results explain the experimental observations and we mention some further applications of the theory.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Paco Torres de Lizaur - SCGP 102**

Speaker: Paco Torres de Lizaur

Title: Invariant manifolds in the Euler equation

Abstract: Consider a finite dimensional family of smooth divergence-free vector fields, parametrized by some manifold N, and with the property that the solutions of the Euler equation with initial condition in the family remain there for all time, defining an ODE on N. I will review recent results on this type of invariant manifolds of fluid velocities and on the eulerian dynamics they describe. In particular, I will show that any first order ODE on any closed manifold N, possibly after arbitrarily small smooth perturbation, can be found as an invariant family of velocity fields on some other Riemannian manifold M.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Vladmir Sverak - SCGP 102**

Speaker: Vladmir Sverak

Title: On self-similar singular solutions of the De Gregorio model

Abstract: The De Gregorio model was suggested in the 1990s by Salvatore De Gregorio as a modification of the well-known Constantin-Lax-Majda model. The modification involves adding an advection term, making the structure of the equation closer to the Euler equations. Similarly to the Constantin-Lax-Majda model, the De Gregorio model can play an important role in testing new ideas. The existence of self-similar singular solutions of the De Gregorio model on the real line was established a few years ago via a computer-assisted proof by Chen, Hou, and Huang. I will discuss recent joint work with Hao Jia on the existence of additional self-similar solutions. It turns out that the Chen-Hou-Huang solution is the first member of a countable family. The solutions have connections to other classical topics.

2:30pm **Bian Wu - SCGP 102**

Speaker: Bian Wu

Title: On stable solutions to the Euler equations in convex planar domains

Abstract: On convex planar domains, given an initial vorticity with one sign, I will talk about the regularity and geometric properties of the dynamically stable solutions to the Euler equations in the coadjoint orbit of the initial vorticity. These flows have elliptic stagnation points. Under some nondegeneracy conditions on the data, we show they are Holder continuous and have convex level curves. We also give a detailed description for the set of stagnation points. If the initial vorticity has nice level set topology, these stable solutions are in the L^\infty-strong closure of the coadjoint orbit. We also demonstrate the sharpness of most assumptions we made.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Discussion/Open problems - SCGP 102**

Wednesday, June 15th

9:00am **Half Talk: Karim Khalil - SCGP 102**

Speaker: Karim Khalil

Title: TBA

Abstract: TBA

9:30am **Tom Hou (remote) - Zoom**

Speaker: Tom Hou

Title: Potentially singular behavior of 3D incompressible Navier-Stokes equations

Abstract: Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in nonlinear PDEs. In this talk, I will present some new numerical evidence that the 3D Navier-Stokes equations develop nearly self-similar singular scaling properties with maximum vorticity increased by a factor of 10^7. This potentially singular behavior is induced by a potential finite time singularity of the 3D Euler equations. Unlike the Hou-Luo blowup scenario, the potential singularity of the 3D Euler and Navier-Stokes equations occurs at the origin. We have applied several blowup criteria to study the potentially singular behavior of the Navier-Stokes equations. The Beale-Kato-Majda blow-up criterion, the blowup criteria based on the growth of enstrophy and negative pressure, the Ladyzhenskaya-Prodi-Serrin regularity criteria all seem to imply that the Navier-Stokes equations develop a potential finite time singularity. Moreover, some preliminary computational results using the dynamic rescaling formulation show that the potential singular solution is nearly self-similar.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Tristan Buckmaster (remote) - Zoom**

Speaker: Tristian Buckmaster

Title: Smooth Imploding Solutions for 3D Compressible Fluids

Abstract: Building upon the pioneering work of Merle-Rodnianski-Szeftel, we construct exact, smooth self-similar imploding solutions to the 3D isentropic compressible Euler equations for ideal gases for all adiabatic exponents. For the particular exponent 7/5, corresponding to diatomic gasses, akin to the result of Merle-Raphael-Rodnianski-Szeftel, we show the existence of a sequence of smooth, self-similar imploding solutions. In addition, we provide simplified proofs of linear stability and non-linear stability which allows us to construct asymptotically self-similar imploding solutions to the compressible Navier-Stokes equations with density independent viscosity for the adiabatic exponent 7/5 and density bounded from below.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Boris A. Khesin (remote) - Zoom**

Speaker: Boris A. Khesin

Title: Hamiltonian geometry and the golden ratio in the Euler hydrodynamics

Abstract: The binormal (or vortex filament) equation provides the localized induction approximation of the 3D incompressible Euler equation. We present a Hamiltonian framework for the binormal equation in higher-dimensions and its explicit solutions that collapse in finite time. On the other hand, by going to lower dimensions, we observe a curious appearance of the golden ratio in the motion of point vortices in the plane. This is a joint work with C.Yang and H.Wang.

2:30pm **Daniel Peralta-Salas - SCGP 102**

Speaker: Daniel Peralta-Salas

Title: Lagrangian complexity in 3D steady Euler flows.

Abstract: I will present some recent (and not so recent) results on the dynamics of smooth stationary solutions to the 3D Euler equation. This includes Arnold's structure theorem (and its implications) and recent developments on the universality of Beltrami flows obtained in joint work with P. Berger and A. Florio.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Discussion/Open problems - SCGP 102**

Thursday, June 16th

9:00am **Half Talk: Patrick Heslin - SCGP 102**

Speaker: Patrick Heslin

Title: Two-Point Boundary Value Problems on Diffeomorphism Groups

Abstract: V. Arnold observed in his seminal paper that solutions of the Euler equations for ideal fluid motion can be viewed as geodesics of a certain right-invariant metric on the group of volume-preserving diffeomorphisms, $D_\mu(M)$. In their celebrated paper Ebin and Marsden provided the formulation of the above in the H^s Sobolev setting. Here they proved that the space of H^s volumorphisms can be given the structure of a smooth, infinite dimensional Hilbert manifold. They illustrated that, when equipped with a right-invariant L^2 metric, the geodesic equation on this manifold is a smooth ordinary differential equation. They then applied the classic iteration method of Picard to obtain existence, uniqueness and smooth dependence on initial conditions. In particular, the last property allows one to define a smooth exponential map on $D_\mu^s(M)$ in analogy with the classical construction in finite dimensional Riemannian geometry. Hence, the work of Arnold, Ebin and Marsden allows one to explore questions of ideal fluid motion armed with tools from Riemannian geometry. In this talk I will present some results about the behaviour of geodesics on these Hilbert manifolds, when they are framed as solutions to a two-point boundary value problem.

9:30am **Gerard Misiolek - SCGP 102**

Speaker: Gerard Misiolek

Title: Conjugate points in fluid motion

Abstract: e 1960's V. Arnold showed that a flow of an ideal fluid traces out a geodesic curve in the group of volume-preserving diffeomorphisms. He posed a number of questions concerning existence and properties of conjugate points along fluid flows. After describing the relevant geometric background I will give a brief overview of some more recent results concerning the structure and distribution of these points.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Steve Preston - Zoom**

Speaker: Steve Preston

Title: Conjugate point criteria in 2D hydrodynamics

Abstract: I will present a new criterion for finding conjugate points along the geodesics in the area-preserving diffeomorphism group that come from steady 2D Euler flows, which generalizes the criterion of Misiolek. For rotationally symmetric flows, this criterion captures all of the known conjugate points, and in a few situations it can be checked more easily. I will also show how to answer several questions posed by Drivas, Misiolek, Shi, and Yoneda in the recent paper "Conjugate and cut points in ideal fluid motion."

12:00pm **Lunch - SCGP Cafe**

1:30pm **Uriel Frisch (remote) - Zoom**

Speaker: Uriel Frisch

Title: TBA

Abstract: TBA

2:30pm **Alexander Schnirelman (remote) - Zoom**

Speaker: Alexander Schnirelman

Title: Asymptotic geometry and asymptotic dynamics of 2-d ideal incompressible fluid

Abstract: Consider the flow of the ideal incompressible fluid in a bounded 2-dimensional domain M. It is described by the velocity field u(x,t). How does this field behave as the time t goes to infinity? In this talk I concentrate on the geometrical side of this problem. The configuration space of the fluid is the group D of volume preserving diffeomorphisms with the energy metric. As a metric space, D has infinite diameter (Eliashberg & Ratiu); therefore it makes sense to consider its asymptotic space (or asymptotic cone) at infinity (Gromov). Its structure turns out to be quite rich, and provides valuable clues on the actual long-time behavior of the flows.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Discussion/Open Problems - SCGP 313**

Friday, June 17th

9:00am **Half Talk: Yupei Huang - SCGP 102**

Speaker: Yupei Huang

Title: On the steady states of 2D incompressible Euler Equations near Bahouri-Chemin Patch

Abstract: Bahouri-Chemin patch is an important singular steady-state for 2D incompressible Euler equations in the torus. In this talk, we consider steady states near the Bahouri-Chemin patch. More precisely, we use two different ways to approximate the semi-linear elliptic equations governing the Bahouri-Chemin patch. In one way, we get smooth steady states close to the Bahouri-Chemin patch in the sense of tthe Hölder norm of the velocity field, and in the other way, we obtain singular steady states near the Bahouri-Chemin in the sense of the continuous norm of the velocity field. This is the joint work with Tarek Elgindi.

9:30am **Jonguk Yang - SCGP 102**

Speaker: Jonguk Yang

Title: Renormalization of Dynamical Systems.

Abstract: It has been observed that two seemingly very different dynamical systems can bear striking resemblances when viewed at sufficiently small scales. Drawing motivations from physics, Feigenbaum (and independently, Collet and Tresser) introduced renormalization in the mid 1970's as a conjectural explanation of this phenomenon. Since then, this idea has been successfully applied to a wide variety of fundamentally important examples of dynamical systems, leading to deep and rigorous mathematical theories that describe their long-term behaviors. In this talk, I will outline the general structure of a fully developed renormalization theory. The aim will be to emphasize intuition over formal and technical details, and to avoid giving specifics that excessively narrow the scope of the discussion.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Discussion - SCGP 102**

11:30am **Lunch - SCGP Cafe**