Workshop Scheudle Week 2

Events for:
Monday, June 20th - Friday, June 24th

Monday, June 20th

9:00am **Introduction to workshop - SCGP 102**

9:30am **Peter Constantin - SCGP 102**

Title: TBA

Abstract: TBA

10:30am **Coffee Break - SCGP Cafe**

11:00am **Matt Novack - SCGP 102**

Title: An Intermittent Onsager Theorem

Abstract: For any regularity exponent s<1/2, we construct non-conservative weak solutions to the 3D incompressible Euler equations in the class C^0_t (H^s \cap L^{1/(1-2s)}). By interpolation, such solutions belong to C^0_t B^{r}_{3,infty} for r approaching 1/3 as s approaches 1/2. Hence this result provides a new proof of the flexible side of the Onsager conjecture, which is independent from that of Isett. Of equal importance is that the intermittent nature of our solutions matches that of turbulent flows, which are observed to possess an L^2-based regularity index exceeding 1/3. The proof employs an intermittent convex integration scheme for the 3D incompressible Euler equations. We employ a scheme with higher-order Reynolds stresses, which are corrected via a combinatorial placement of intermittent pipe flows of optimal relative intermittency.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Alex Blumenthal - SCGP 102**

Title: Exponential mixing for Pierrehumbert's alternating sine-shear flow with random phase shifts

Abstract: The Pierrehumbert alternating sine-shear flow is a classical example of an incompressible flow observed to mix passive scalars. Proving mixing for concrete is a substantial mathematical challenge in the dynamical systems and passive scalar mixing communities, and many open questions remain, even for simple toy models for which mixing is substantiated in numerical experiments. Recently, techniques from random dynamical systems theory have been applied to problems of this kind for typical realizations of incompressible flows subjected to noise (c.f. my previous joint work with Bedrossian and Punshon-Smith). In this talk I will review a recent result, joint with Michele Cot-Zelati and Rishabh Gvalani, proving exponentially fast mixing (a.k.a. H^{-1} decay) for all passive scalars, under the alternating shear flow with randomly chosen phase shifts. To our knowledge, this example provides the first-ever example of a universal exponentially-fast mixer on the periodic box which is uniformly smooth (C^∞) in space and time.

2:30pm **Susan Friedlander (remote) - Zoom**

Title: On Moffatt's Magnetic Relaxation Equations

Abstract: We investigate the stability properties for a family of equations introduced by Moffatt to model magnetic relaxation. These models preserve the topology of magnetic streamlines, contain a cubic nonlinearity, and yet have a favorable L^2 energy structure. We consider the local and global in time well-posedness of these models and establish a difference between the behavior as t goes to infinity with respect to weak and strong norms.
This is joint work with Rajendra Beekie and Vlad Vicol.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Daniel Ginsberg - SCGP 102**

Title: The stability of model shocks and the Landau law of decay

Abstract: It is well-known that in three space dimensions, smooth solutions to the equations describing a compressible gas can break down in finite time. One type of singularity which can arise is known as a "shock", which is a hypersurface of discontinuity across which the integral forms of conservation of mass and momentum hold and through which there is nonzero mass flux. One can find approximate solutions to the equations of motion which describe expanding spherical shocks. We use these model solutions to construct global-in-time solutions to the irrotational compressible Euler equations with shocks. This is joint work with Igor Rodnianski.A

Tuesday, June 21st

9:00am **Half Talk: Vikram Giri - SCGP 102**

Title: Non-unique trajectories of Hamiltonian vector fields

Abstract: We'll discuss a construction of an autonomous Hamiltonian Sobolev vector field in 4 dimensions such that for a positive measure of initial points, there are at least two trajectories emanating from them. We'll also see that the Hamiltonian need not be preserved along such trajectories. We use convex integration techniques and the Ambrosio superposition principle in the proof. This is joint work with Massimo Sorella.

9:30am **Jonathan Mattingly - SCGP 102**

Title: A model for random Euler and Navier Stokes equations based on a randomized splitting of the dynamics.

Abstract: We introduce a model for the generic behavior of the Euler equations by introducing a random splitting of the dynamics. Conceptually this allows for the separation of the energy pumping terms and those random perturbations ensuring they dynamics is generic. We will primarily consider the finite dimensional approximation by restricting the dynamics to a finite (but large) collection of Fourier modes (that is to say, we consider the finite dimensional Galerkin Approximations). We begin by splitting the dynamics into a large number of small dimensional random dynamical systems obtained by running a three dimensional ODE for a short but random amount of time. These small systems dynamical systems each conserve the energy and the entropy of the systems. Hence the composition of entire collection of these maps conserve energy and entropy and can be show to be a good approximation of the dynamics on an arbitrary finite time interval as the mean of the random step size is taken to zero. For fixed step size mean, the random dynamics have a unique invariant measure both in the Euler setting and when fix time independent forcing is added along with some (even partial) dissipation. We also show that the randomly split Euler system is chaotic in the sense that it almost surely has positive Lyapunov exponents. This is mainly joint work with Omar Melikechi (Duke) and Andrea Agazzi (Pisa).

10:30am **Coffee Break - SCGP Cafe**

11:00am **Dallas Albritton - SCGP 102**

Title: Non-uniqueness of Leray solutions of the forced Navier-Stokes equations

Abstract: In a seminal work, Leray demonstrated the existence of global weak solutions to the Navier-Stokes equations in three dimensions. Are Leray's solutions unique? This is a fundamental question in mathematical hydrodynamics, which we answer in the negative, within the `forced' category, by exhibiting two distinct Leray solutions with zero initial velocity and identical body force. This is joint work with Elia Brué and Maria Colombo.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Mihaela Ignatova - SCGP 102**

Title: Long Time Behavior of Electroconvection Models

Abstract: We present two electroconvection models describing the interaction between a surface charge density and a fluid in a two-dimensional situation. We compare these models with the surface quasi-geostrophic equation in bounded domains and recall some recent results. For the first model, the global existence results can be obtained for bounded domains and for the torus. In the latter case, in joint work with graduate student E. Abdo, we proved that the long time asymptotic state of the system is finite dimensional, if body forces are applied to the fluid, and a singleton solution in the absence of fluid body forces. In the whole space, in the absence of forcing, we obtain optimal decay rates. For the more challenging second model, corresponding to electroconvection through porous media, we proved global existence for subcritical and for small data cases.

2:30pm **Alexander A. Kiselev - SCGP 102**

Title: Regularity of vortex and SQG patches

Abstract: I will review some recent progress on regularity properties of vortex and SQG patches. In particular, I will present an example of a vortex patch with continuous initial curvature that immediately becomes infinite but returns to C^2 class at all integer times only without being time periodic. The proof involves derivation of a new system describing the patch evolution in terms of arc-length and curvature. The talk is based on a work joint with Xiaoyutao Luo.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Federico Pasqualotto - SCGP 102**

Title: Singularity formation in the Boussinesq equations

Abstract: In this talk, I will first review existing results on singularity formation in incompressible and inviscid fluids. I will then describe a new mechanism for singularity formation in the Boussinesq equations. The initial data we choose is smooth except at one point, where it has Hölder continuous first derivatives. This is joint work with Tarek Elgindi (Duke University).

Wednesday, June 22nd

9:00am **Half Talk: Anuj Kumar - SCGP 102**

Title: Optimizing scalar transport using branching pipe flows

Abstract: We consider the problem of "wall-to-wall optimal transport" in which we attempt to maximize the transport of a passive temperature field between hot and cold plates. Specifically, we optimize the choice of the divergence-free velocity field in the advection-diffusion equation amongst all velocities satisfying an enstrophy constraint (which can be understood as a constraint on the power required to generate the flow). Previous work established an a priori upper bound on the transport, scaling as the 1/3-power of the flow's enstrophy. Recently, Tobasco & Doering (Phys. Rev. Lett. vol.118, 2017, p.264502) and Doering & Tobasco (Comm. Pure Appl. Math. vol.72, 2019, p.2385--2448) constructed self-similar two-dimensional steady branching flows saturating this bound up to a logarithmic correction to scaling. This logarithmic correction appears to arise due to a topological obstruction inherent to two-dimensional steady branching flows. We present a construction of three-dimensional "branching pipe flows" that eliminates the possibility of this logarithmic correction and therefore identifies the optimal scaling as a clean 1/3-power law. These branching pipe flows are not merely mathematical constructs but in principle, can be engineered. Our flows resemble previous numerical studies of the three-dimensional wall-to-wall problem by Motoki, Kawahara & Shimizu (J. Fluid Mech. vol.851, 2018, p.R4). However, using an unsteady branching flow construction, it appears that the 1/3 scaling is, in fact, optimal in two dimensions as well. We discuss the underlying physical mechanism that makes the branching flows "efficient" in transporting heat. Finally, we discuss the implications of our result to the heat transfer problem in the Rayleigh--Benard convection and the problem of anomalous dissipation in a passive scalar, and we propose a few conjectures in these directions.

9:30am **Anna Frishman - SCGP 102**

Title: Mean flow and fluctuations in two opposing limits of quasi-geostrophic flow

Abstract: Persistent large scale flows in geophysical and astrophysical settings are often sustained by small scale turbulence, rather than the other way around. I will discuss two idealized settings where analytical progress on such mean-flow-turbulence interactions and statistics is possible. Namely, I will compare and contrast the steady state statistics in two limits of the shallow water quasi-geostrophic equation (absent differential rotation): the limit of infinite deformation radius, corresponding to 2D Navier-Stokes, and that of zero deformation radius, called the asymptotic model. For both, I will present analytical results rooted in the quasi-linear approximation, along with supporting evidence from direct numerical simulations. I will also show some surprising features of the asymptotic model, including the influence of the inverse energy transfer on the direct cascade.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Michele Coti Zelati - SCGP 102**

Title: Nonlinear inviscid damping and shear-buoyancy instability in the two-dimensional Boussinesq equations

Abstract: We investigate the long-time properties of the two-dimensional inviscid Boussinesq equations near a stably stratified Couette flow. We prove that the system experiences a shear-buoyancy instability: the density variation and velocity undergo inviscid damping while the vorticity and density gradient grow. The result holds at least until a natural, nonlinear timescale. Notice that the density behaves very differently from a passive scalar, as can be seen from the inviscid damping and slower gradient growth. The proof relies on several ingredients: (A) a suitable symmetrization that makes the linear terms amenable to energy methods and takes into account the classical Miles-Howard spectral stability condition; (B) a variation of the Fourier time-dependent energy method introduced for the inviscid, homogeneous Couette flow problem developed on a toy model adapted to the Boussinesq equations, i.e. tracking the potential nonlinear echo chains in the symmetrized variables despite the vorticity growth.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Gregory Falkovich - SCGP 102**

Title: Multi-mode correlations as signature of turbulence.

Abstract: The general problem is what is the most sensitive and sensible statistical characteristics distinguishing turbulence from thermal equilibrium. We consider turbulence of capillary water waves and the so-called Fibonacci chain of interacting modes which models both incompressible turbulence and sets of resonantly interacting waves, We show that in turbulent cascades there exist multi-mode correlations of arbitrary order.

2:30pm **Anna Mazzucato - SCGP 102**

Title: Irregular transport and loss of regularity for transport equations.

Abstract: I will present recent results concerning examples of loss of regularity for solutions to linear transport equations with advecting field in Sobolev spaces below the Lipscitz class. I will discuss how this loss is generic and can be made instantaneous and total (that is, there exists smooth initial data for which the solution leaves instantaneously any Sobolev space of positive order). This is joint work with Giovanni Alberti, Gianluca Crippa, Gautam Iyer, and Tarek Elgindi.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Jiahong Wu - SCGP 102**

Title: Stabilizing phenomena for incompressible fluids

Abstract: talk presents several examples of a remarkable stabilizing phenomenon. The 3D incompressible Navier-Stokes equation with dissipation in only one direction is not known to always have global solutions even when the initial data are small. However, when this Navier-Stokes is coupled with the magnetic field in the magneto-hydrodynamic system, solutions near a background magnetic field are shown to be always global in time. The magnetic field stabilizes the fluid. The results of T. Elgindi and T. Hou's group (see a recent review paper of Drivas and Elgindi, arXiv: 2203.17221v1) show that the 3D Euler can blow up in a finite time. Even small data would not help. But when the 3D Euler is coupled with the non-Newtonian stress tensor in the Oldroyd-B model, small smooth data always lead to global and stable solutions. Solutions of the 2D Navier-Stokes in R^2 with dissipation in only one direction are not known to be stable, but the Boussinesq system involving this Navier-Stokes is always stable near the hydrostatic equilibrium. The buoyancy forcing helps stabilize the fluid. In all these examples the systems governing the perturbations can be converted to damped wave equations, which reveal the smoothing and stabilizing effect.

6:00pm **Banquet**

Thursday, June 23rd

9:00am **Half Talk: Trinh Nguyen - SCGP 102**

Title: The inviscid limit of Navier-Stokes for domains with curved boundaries

Abstract: TBA

9:30am **Steve Shkoller - Zoom**

Title: The geometry of shock formation for Euler

Abstract: I will describe a new geometric approach for the shock formation problem for the multi-dimensional Euler equations that provides uniform estimates for the solution along the entire hypersurface on which the shock forms. This, in turn, allows for a complete description of the solution along this hypersurface of first singularities.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Vlad Vicol - SCGP 102**

Title: Formation and development of singularities for the compressible Euler equations

Abstract: We give a complete description of the formation and development of singularities for the compressible Euler equations in two space dimensions, under azimuthal symmetry. Our proof applies mutatis mutandis in the drastically simpler situations of one-dimensional flows, or multi-dimensional flows with radial symmetry. We prove that for smooth and generic initial data with azimuthal symmetry, the 2D compressible Euler equations yield a local in time smooth solution, which in finite time forms a first gradient singularity, the so-called pre-shock. We then show that a discontinuous entropy producing shock wave instantaneously develops from the pre-shock. Simultaneous to the development of the shock, two other characteristic surfaces of higher-order cusp-type singularities emerge from the pre-shock. These surfaces have been termed weak discontinuities by Landau and Lifshitz [17, Chapter IX, §96], who conjectured their existence. We prove that along the characteristic surface moving with the fluid, a weak contact discontinuity is formed, while along the slowest surface in the problem, a weak rarefaction wave emerges. The constructed solution is the unique solution of the Euler equations in the class of entropy-producing weak solutions with azimuthal symmetry and with regularity determined by the fact that it arises from a generic pre-shock. The talk is based on joint work with T. Buckmaster, T. Drivas, and S. Shkoller.

12:00pm **Lunch - SCGP Cafe**

1:30pm **Roman Shvydkoy - SCGP 102**

Title: Mechanisms for energy conservation in Onsager (super)critical fluids

Abstract: When the regularity of a solution to the Euler and Navier-Stokes equation is higher than 1/3 (in a certain Besov sense), then the solution is known to conserve energy. This threshold is also dimension-independent. At the same time for 3D Euler the convex integration method provided examples of solutions in regularity 1/3 - epsilon that do not conserve energy (P. Isett, 2016). Finding examples that belong to the critical regularity class 1/3 constitutes what is called the Onsager conjecture. In this talk we will highlight several special mechanisms that allow to prove energy conservation for incompressible fluids that fall under the Onsager threshold. In particular, this provides a list of scenarios one would like to avoid when attempting to construct critical energy-dissipative examples.

2:30pm **Michele Dolce - SCGP 102**

Title: On maximally mixed equilibria of two-dimensional perfect fluids

Abstract: The motion of a two-dimensional incompressible and inviscid fluid can be described as an area-preserving rearrangement of the initial vorticity that preserves the kinetic energy. In the infinite time limit, some irreversible mixing can occur and predicting what structures can persist is an issue of fundamental importance. Shnirelman introduced the concept of maximally mixed states (any further mixing would necessarily change their energy) and proved they are perfect fluid equilibria.We offer a new perspective on this theory by showing that any minimizer of any strictly convex Casimir, in a set containing Euler's end states, is maximally mixed. Thus, (weak) convergence to equilibrium cannot be excluded solely on the grounds of vorticity transport and conservation of kinetic energy. On the other hand, in the straight channel, we give examples of open sets of initial data which can be arbitrarily close to any shear flow in L1 of vorticity but do not weakly converge to them in the long time limit.
This is joint work with T.D. Drivas.

3:30pm **Coffee Break - SCGP Cafe**

4:00pm **Ayman Said - SCGP 102**

Title: The zero homogeneous Euler equation: long time behaviour and ageing in fluids.

Abstract: n this talk we are going to discuss recent results on the long time behaviour of the zero homogeneous Euler equation obtained jointly with T. M. Elgindi and R. Murray and their implication on the ageing phenomena in ideal fluids.

Friday, June 24th

9:00am **Half Talk: Marc Nualart - SCGP 102**

Title: On zonal steady solutions to the 2d Euler equations

Abstract: We comment on recent developments of the long-time dynamics and stationary structures of solutions to the incompressible Euler equations on planar domains. Afterwards, we focus on the incompressible Euler equations on the rotating unit sphere and we study stationary solutions that are near zonal flows. In particular, we construct a new family of non-zonal steady solutions arbitrarily close in analytic regularity to the second degree zonal Rossby-Haurwitz stream function, for any given rotation of the sphere.

9:30am **Siming He - SCGP 102**

Title: Suppression of Chemotactic Blow-up through Flows

Abstract: We study the advective-Patlak-Keller-Segel equations, which model chemotaxis phenomena of bacteria in a fluid stream. If the bacteria population is large and no fluid transportation is present, it is well-known that the solutions form Dirac-type singularities in finite time. We showed that strong hyperbolic or shear flows help suppress the blow-up of the equation on the plane or in the infinite channel. This result confirms the numerical observation made in Khan-Johnson-Cartee-Yao. This is joint work with Eitan Tadmor and Andrej Zlato\v{s}.

10:30am **Coffee Break - SCGP Cafe**

11:00am **Kyle Liss - SCGP 102**

Title: Quantitative convergence to equilibrium and some partial dissipation results for Euler-like SDE

Abstract: In this talk I will present some results on the existence of invariant measures and convergence rate to equilibrium for a class of nonlinear SDEs that include damped-driven truncations of the 2d/3d Euler equations and Sabra shell model. If the diffusion is hypoelliptic and the damping acts on every degree of freedom, then it is well known that the system admits a unique invariant measure that attracts all initial conditions exponentially fast. A natural problem is then to study how the convergence rate to equilibrium scales in the zero-dissipation limit, which in the fluid setting amounts to understanding the convergence rate to equilibrium in the limit of infinite Reynolds number. In the first part of the talk I will discuss results in which we obtain an optimal estimate on the exponential convergence to equilibrium in the limit of zero dissipation. The proof is based on a PDE approach that hinges on quantitative hypoelliptic estimates for the stationary density as well as the associated time-dependent Kolmogorov equation. In the second part of the talk I will discuss the situation where the damping only acts on a proper subset of the degrees of freedom in the system. In this setting the existence of an invariant measure in a difficult question in general, as one must quantify carefully how the nonlinearity transfers energy from the undamped modes to the damped modes. I will discuss an approach for proving existence of an invariant measure based on time-averaged coercivity estimates and its application to some examples. This is joint work with Jacob Bedrossian.

12:00pm **Lunch - SCGP Cafe**