Workshop Scheudle

Events for:
Monday, June 27th - Friday, July 1st

Monday, June 27th

9:00am **Lorenzo Mazzieri - SCGP 102**

Title: Geometric inequalities and potential theory

Abstract: We describe through some selected examples an approach based on potential theory toward the proof of relevant geometric inequalities, holding in classical and curved frameworks. Time permitting, we also discuss some applications of interest in general relativity, including the positive mass theorem and the Riemannian Penrose inequality.

10:00am **Christina Sormani - SCGP 102**

Title: Compactness and Stability Conjectures

Abstract: Gromov has conjectured that a sequence of compact three dimensional Riemannian manifolds with nonnegative scalar curvature converges in the intrinsic flat sense to a limit space with generalized nonnegative scalar curvature. He has also conjectured the stability of the scalar torus rigidity theorem: that a sequence of three tori with scalar >-1/j has a subsequence which converges in the intrinsic flat sense to a flat torus. We will survey examples and results in this direction including joint work with Allen and Perales introducing a notion we call VADB convergence that has been applied by Cabrera Pacheco, Perales, and Ketterer to prove the stability of the scalar torus rigidity theorem. We will suggest a number of open problems applying VADB convergence to test both these conjectures and others.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Daniel Stern - SCGP 102**

Title: Harmonic maps and extremal Schrodinger operators on higher-dimensional manifolds

Abstract: I'll discuss recent progress on the existence theory for harmonic maps from arbitrary Riemannian manifolds of dimension >2 to a large class of target manifolds, including 3-manifolds of positive Ricci curvature and manifolds of dimension > 3 with positive isotropic curvature. As a special case, every Riemannian manifold (M^n,g) of dimension n>2 admits a family of nontrivial stationary harmonic maps u_k to the standard spheres S^k for k>2, smooth away from a singular set of dimension at most n-7 for k sufficiently large. I'll explain how these maps give rise to solutions of an optimization problem for Schrodinger operators related to work of Grigor'yan-Netrusov-Yau and Grigor'yan-Nadirashvili-Sire, generalizing to higher dimensions the maximization of Laplace eigenvalues on surfaces of fixed conformal type. (Based on joint work with Mikhail Karpukhin.)

12:30pm **Lunch - SCGP Cafe**

2:00pm **Jinmin Wang - SCGP 102**

Title: Dihedral rigidity conjecture and Stoker's problem

Abstract: The Stoker Conjecture states that the dihedral angles of a convex Euclidean polyhedron completely determine the angles of each face. In this talk, I will present my recent work joint with Zhizhang Xie and Guoliang Yu that answers positively to the Stoker Conjecture in all dimensions. Our work proves a more general dihedral rigidity theorem, which concerns the comparison of scalar curvature, mean curvature, and dihedral angles for convex polyhedrons, or more general, manifolds with polytope boundary. We use index theory on manifolds with polytope boundary and the Dirac operator methods.

3:00pm **Zhizhang Xie - SCGP 102**

Title: On Gromov’s dihedral extremality and rigidity conjectures

Abstract: Gromov’s dihedral extremality and rigidity conjectures concern comparisons of scalar curvature, mean curvature and dihedral angle for manifolds with corners (and more generally manifolds with polyhedral boundary). They have very interesting consequences in geometry and mathematical physics. The conjectures themselves can in some sense be viewed as “localizations” of the positive mass theorem. In this talk, I will explain some recent work on positive solutions to these conjectures. The talk is based on my joint papers with Jinmin Wang and Guoliang Yu.

4:00pm **Coffee Break - SCGP Cafe**

Tuesday, June 28th

9:00am **Richard Bamler - SCGP 102**

Title: Towards a theory of Ricci flow in dimension 4 (and higher)

Abstract: The Ricci flow (with surgery) has proven to be a powerful tool in the study of 3-dimensional topology — its most prominent application being the verification of the Poincaré and Geometrization Conjectures by Perelman about 20 years ago. Since then further research has led to a satisfactory understanding of the flow and surgery process in dimension 3. In dimensions 4 and higher, on the other hand, Ricci flows have been understood relatively poorly and a surgery construction seemed distant. Recently, however, there has been some progress in the form of a new compactness and partial regularity theory for higher dimensional Ricci flows. This theory relies on a new geometric perspective on Ricci flows and provides a better understanding of the singularity formation and long-time behavior of the flow. In dimension 4, in particular, it may eventually open up the possibility of a surgery construction or a construction of a "flow through singularities". The goal of this talk will be to describe this new compactness and partial regularity theory and the new geometric intuition that lies behind it. Next, I will focus on 4-dimensional flows. I will present applications towards the study of singularities of such flows and discuss several conjectures that provide a possible picture of a surgery construction in dimension 4. Lastly, I will discuss potential topological applications.

10:00am **Christos Mantoulidis (no recording) - SCGP 102**

Title: Decomposing PSC 4-manifolds

Abstract: We will discuss a decomposition result involving 0- and 1-surgeries for closed, oriented, topologically PSC 4-manifolds.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Claude LeBrun - SCGP 102**

Title: Kodaira Dimension and the Yamabe Problem, Revisited

Abstract: Dimension four provides a surprisingly idiosyncratic setting for the interplay between scalar curvature and differential topology. This peculiarity becomes especially pronounced when discussing the Yamabe invariant (or “sigma constant”) of a smooth compact manifold; and Seiberg-Witten theory makes this especially apparent for those 4-manifolds that arise as compact complex surfaces. For compact complex surfaces of Kaehler type, I showed in the late 1990s that the sign of the Yamabe invariant is always determined by the Kodaira dimension, and moreover calculated the Yamabe invariant exactly in all cases where it is non-positive. In this talk, I will describe recent joint work with Michael Albanese that generalizes these results to all complex surfaces of non-Kaehler type. However, the complex surfaces of class VII actually violate many of the expected patterns, and navigating around this hazard represents a key aspect of our story.

12:30pm **Lunch - SCGP 102**

2:00pm **David Maxwell - SCGP 102**

Title: Sobolov-Class Asymptotically Hyperbolic Manifolds and the Yamabe Problem

Abstract: We consider asymptotically hyperbolic manifolds whose metrics have Sobolev-class regularity. Building on prior work by Allen, Isenberg, Lee, and Allen-Stavrov in the Hölder category, we introduce two new function spacesfor metrics potentially having a large amount of interior differentiability measured in Sobolev scales, but whose regularity implies only a Hölder continuous conformal structure. We establish Fredholm theorems for elliptic operators arising from metrics in these families.To demonstrate utility of our methods, we solve the Yamabe problem in this category. As a special limiting case, we show that the asymptotically hyperbolic Yamabe problem is solvable so long as the metric admits a $W^{1,p}$ conformal compactification, with p greater than the dimension of the manifold.(Joint with Paul T. Allen, Lewis & Clark and John M. Lee, University of Washington)

3:00pm **Jim Isenberg (remote) - Zoom**

Title: Strong Cosmic Censorship and the Stability of AVTD Behavior for Polarized T^2-Symmetric Space-Times

Abstract: Penrose's Strong Cosmic Censorship Conjecture asserts that for generic initial data satisfying the Einstein constraint equations, the maximal spacetime development of that initial data cannot be smoothly extended past a Cauchy horizon into a region where determinism breaks down. This conjecture remains one of the outstanding unresolved issues in mathematical relativity. One of the most effective tools for proving that Strong Cosmic Censorship holds for a family of cosmological solutions of Einstein's equations is to verify that these solutions all exhibit Asymptotically Velocity Term Dominated (AVTD) behavior in a neighborhood of their Big Bang singularities. After presenting some background history of known results and conjectures concerning the behavior of the gravitational field near the Big Bang in cosmological solutions of Einstein's equations, we discuss recent work with Ellery Ames, Florian Beyer, and Todd Oliynyk in which we prove that polarized T^2- Symmetric vacuum solutions in a neighborhood of Kasner solutions all exhibit AVTD behavior close to the initial singularity. We also discuss our very recent extension of these results to the case of non-vanishing cosmological constant.

4:00pm **Coffee Break - SCGP Cafe**

Wednesday, June 29th

10:00am **Joachim Lohkamp - SCGP 102**

Title: Removal of Singularities

Abstract: Minimal hypersurfaces are important to study general scalar curvatureconstraints. The occurence of singularitiesis an infamous problem one may solve from an inductive dimensionalreduction scheme with built-in regularization. We will discuss thisparticular process and indicated some ways how it can be applied.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Dan Lee - SCGP 102**

Title: The positive mass theorem with boundaries, complete ends, and scalar curvature shields

Abstract: In joint work with M. Lesourd and R. Unger, we give a non-spinor proof of the spacetime positive mass theorem with weakly outer trapped boundary. It turns out that this implies a version of the Riemannian positive mass theorem that does not require nonnegative scalar curvature everywhere, so long as the negative scalar curvature is “shielded” from the asymptotically flat end by a region with sufficiently positive scalar curvature. By an argument of Lesourd, Unger, and Yau, this latter fact also implies the Riemannian positive mass theorem for arbitrary complete ends.

12:30pm **Lunch - SCGP Cafe**

2:00pm **Pengzi Miao - SCGP 102**

Title: Mass and capacitary potential of asymptotically flat 3-manifolds

Abstract: I will discuss some relation between the mass and positive harmonic functions decaying to zero on asymptotically flat 3-manifolds.

3:00pm **David Wiygul - SCGP 102**

Title: Asymptotic estimates for the Bartnik mass of small metric balls

Abstract: For all the various definitions of quasilocal mass in general relativity it is natural to investigate their behavior on regions shrinking to a point. In particular one can seek a Taylor expansion of the mass of geodesic balls of small radius and fixed center. I will present some recent estimates for the Bartnik mass in this regime, including one that is obtained by estimating the ADM mass of asymptotically flat static vacuum extensions with "small" Bartnik boundary data to second order in the data.

4:00pm **Coffee Break - SCGP Cafe**

Thursday, June 30th

9:00am **Bernhard Hanke - SCGP 102**

Title: Lipschitz rigidity for scalar curvature

Abstract: Let M be a closed connected smooth spin manifold of even dimension n, let g be a Riemannian metric of regularity W^{1,p}, p > n, on M whose distributional scalar curvature in the sense of Lee-LeFloch is bounded below by n(n-1), and let f : (M,g) \to S^n be a 1-Lipschitz continuous map of non-zero degree to the standard round n-sphere.
Then f is a metric isometry. This generalizes a result of Llarull (1998) and answers a question of Gromov (2019) in his ``four lectures''. Our proof, which is a joint work with Simone Cecchini and Thomas Schick, combines spectral properties of Dirac operators for metrics with low regularity and twisted with Lipschitz bundles with the theory of quasiregular maps due to Reshetnyak.

10:00am **Guoliang Yu - SCGP 102**

Title: Higher index theory and scalar curvature

Abstract: I will give an introduction to higher index theory of Dirac operators and its applications to scalar curvature.In particular, I will discuss my recent joint work with Shmuel Weinberger and Zhizhang Xie on higher index theory at infinityand its applications to Gromov's compactness conjecture on scalar curvature.

11:00am **Coffee Break - SCGP Cafe**

11:30am **Demetre Kazaras - SCGP 102**

Title: Comparison geometry and spacetime harmonic functions

Abstract: Comparison theorems are the basis for our geometric understanding of Riemannian manifolds satisfying a given curvature condition. A remarkable example is the Gromov-Lawson toric band inequality, which bounds the distance between the two sides of a Riemannian torus-cross-interval with positive scalar curvature by a sharp constant inversely proportional to the scalar curvature's minimum. We will give a new qualitative version of this and similar band-type inequalities in dimension 3 using the notion of spacetime harmonic functions, which recently played the lead role in our recent proof of the positive mass theorem. Other applications include new versions of Bonnet-Meyer's diameter estimate for positive Ricci curvature manifolds and Llarull's theorem which do not require a completeness assumption. This is joint work with Sven Hirsch, Marcus Khuri, and Yiyue Zhang.

12:30pm **Lunch - SCGP Cafe**

2:00pm **Rudolf Zeidler - SCGP 102**

Title: Distance estimates under lower scalar curvature bounds in the spin setting and beyond

Abstract: We will discuss several situations in which lower scalar curvature bounds can be used to infer certain distance estimates using the Dirac operator on spin manifolds. This includes Gromov's question on the distance between the boundary components of „bands", rigidity properties of warped products, and related questions in the context of Witten's spinor proof of the positive mass theorem. Finally, we will contrast this with a recently obtained result in the non-spin setting concerning the non-existence of complete psc metrics on certain manifolds which admit a compact incompressible hypersurface without psc. Based on joint works with S. Cecchini and D. Räde.

3:00pm **Daniel Räde - SCGP 102**

Title: Scalar and mean curvature comparison for Riemannian bands

Abstract: We use μ-bubbles to compare Riemannian bands in scalarcurvature, mean curvature and width to warped products over scalar flatmanifolds with $\log$-concave warping functions. Furthermore we explainhow these comparison results can be used to prove two conjectures due toGromov resp. Rosenberg and Stolz in dimensions ≤ 7. Thistalk is, in part, based on joint work with S. Cecchini and R. Zeidler.

4:00pm **Coffee Break - SCGP Cafe**

6:00pm **Banquet - SCGP Cafe**

Friday, July 1st

9:00am **Carla Cederbaum - Zoom**

Title: Coordinates are messy

Abstract: Asymptotically Euclidean initial data sets (M,g,K) are characterized by the existence of asymptotic coordinates in which the Riemannian metric g and second fundamental form K decay to the Euclidean metric δ and to 0 suitably fast, respectively. Provided their matter densities satisfy suitable integrability conditions, they have well-defined (ADM-)energy, (ADM-)linear momentum, and (ADM-)mass as was shown by Bartnik in 1986. To study their (ADM-)angular momentum and (BORT-)center of mass, one usually assumes the existence of so-called Regge—Teitelboim coordinates. We will give examples of asymptotically Euclidean initial data sets which do not possess any Regge—Teitelboim coordinates and explain other “non-features” of the Regge—Teitelboim coordinate conditions. This is joint work with Melanie Graf and Jan Metzger. We will also explain the consequences of these findings for the definition of the center of mass, relying on joint work with Nerz and with Sakovich.

10:00am **Coffee Break - SCGP Cafe**

10:30am **Zhongshan An - SCGP 102**

Title: Local existence and uniqueness of static vacuum extensions of Bartnik boundary data

Abstract: The study of static vacuum Riemannian metrics arises naturally in differential geometry and general relativity. It plays an important role in scalar curvature deformation, as well as constructing Einstein spacetimes. Existence of static vacuum Riemannian metrics with prescribed Bartnik data — the induced metric and mean curvature of the boundary — is one of the most fundamental problems in Riemannian geometry related to general relativity. It is also a very interesting problem on the global solvability of a natural geometric boundary value problem. In this talk I will first discuss some basic properties of the nonlinear and linearized static vacuum equations and the geometric boundary conditions. Then I will present some recent progress towards the existence problem of static vacuum metrics based on joint works with Lan-Hsuan Huang.

11:30am **Lunch - SCGP Cafe**