Number Theory and Physics

Events for:
Monday, October 24th - Friday, October 28th

Monday, October 24th

9:00am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: Karen Yeats

Title: Numbers from quantum field theory

Abstract: Many interesting numbers including multiple zeta values and values of elliptic polylogarithms and even more exotic things appear in quantum field theory computations. This makes quantum field theory a fascinating playground for number theorists interested in these kinds of numbers and their structures, and in the other direction better understanding the number theoretic and geometric structures appearing here will let us better understand quantum field theory and gives new tools to compute values of practical interest to physicists. I will start at the beginning with concretely how to compute Feynman periods and Feynman integrals in straightforward cases. I will then survey some of what is known and what has been computed, focusing on the quantum field theory side, but also touching on related numbers in string theory. In my final lecture I will speak on graph invariants with the same symmetries as Feynman periods, what we know about them and what they tell us about these interesting numbers, including work of myself and collaborators.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102/ZOOM**

Speaker: Grant Remmen

Title: Amplitudes and the Riemann Zeta Function

Abstract: In this talk, I will connect physical properties of scattering amplitudes to the Riemann zeta function. Specifically, I will construct a closed-form amplitude, describing the tree-level exchange of a tower with masses m^2_n = \mu^2_n, where \zeta(\frac{1}{2}\pm i \mu_n) = 0. Requiring real masses corresponds to the Riemann hypothesis, locality of the amplitude to meromorphicity of the zeta function, and universal coupling between massive and massless states to simplicity of the zeros of \zeta. Unitarity bounds from dispersion relations for the forward amplitude translate to positivity of the odd moments of the sequence of 1/\mu^2_n. Based on https://arxiv.org/abs/2108.07820.

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Matilde Lalin

Title: Sums of certain arithmetic functions over Fq[T] and symplectic and orthogonal distributions

Abstract: In 2018 Keating, Rodgers, Roditty-Gershon and Rudnick established relationships of the mean-square of sums of the divisor function dk(f) over short intervals and over arithmetic progressions for the function field Fq[T] to certain integrals over the ensemble of unitary matrices when q?? . We study similar problems leading to integrals over the ensembles of symplectic and orthogonal matrices when q?? . This is joint work with Vivian Kuperberg.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102/ZOOM**

Speaker: Steve Gonek

Title: Euler Products and the Riemann Hypothesis

Abstract: We investigate the approximation of the Riemann zeta function by short truncations of its Euler product in the critical strip. We then construct a parameterized family of non-analytic functions that approximate the zeta function to the right of the critical line. With the possible exception of finitely many zeros off the critical line, each function in the family satisfies a Riemann Hypothesis. Moreover, when the parameter is not too large, the functions in the family have about the same number of zeros as the zeta-function, their zeros are all simple, and the zeros ?repel?. The structure of these functions makes the reason for the simplicity and repulsion of their zeros apparent. Computer calculations suggest that the zeros of functions in the family are remarkably close to those of the zeta function, even for small values of the parameter. We also show that if the Riemann Hypothesis holds for the Riemann zeta function, then the zeros of these functions indeed converge to those of the zeta-function as the parameter increases and that, between consecutive zeros of the zeta-function, the functions tend to twice the zeta function.

3:30pm **Tea - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Sieg Baluyot

Title: The recipe for moments of L-functions and characteristic polynomials of random matrices

Abstract: In 2005, Conrey, Farmer, Keating, Rubinstein, and Snaith formulated a 'recipe' that leads to precise conjectures for the asymptotic behavior of integral moments of various families of L-functions. They also proved exact formulas for moments of characteristic polynomials of random matrices, and observed that these formulas have an almost identical form to their conjectures for the corresponding families of L-functions. In this talk, we will discuss recent progress towards their conjectures and survey a few applications of their formulas. These will include joint work with Brian Conrey and Caroline Turnage-Butterbaugh.

Tuesday, October 25th

9:00am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102**

Speaker: Karen Yeats

Title: Numbers from quantum field theory II

Abstract: Many interesting numbers including multiple zeta values and values of elliptic polylogarithms and even more exotic things appear in quantum field theory computations. This makes quantum field theory a fascinating playground for number theorists interested in these kinds of numbers and their structures, and in the other direction better understanding the number theoretic and geometric structures appearing here will let us better understand quantum field theory and gives new tools to compute values of practical interest to physicists. I will start at the beginning with concretely how to compute Feynman periods and Feynman integrals in straightforward cases. I will then survey some of what is known and what has been computed, focusing on the quantum field theory side, but also touching on related numbers in string theory. In my final lecture I will speak on graph invariants with the same symmetries as Feynman periods, what we know about them and what they tell us about these interesting numbers, including work of myself and collaborators.

10:30am **Coffee Break - SCGP Cafe**

11:00am

Speaker: Emma Bailey

Title: Large deviation estimates for Selberg's central limit theorem and applications

Abstract: Selberg's central limit theorem gives that the logarithm of the Riemann zeta function taken at a uniformly drawn height in $[T, 2T]$ behaves as a complex centered Gaussian random variable with variance $\log\log T$. A natural question is to investigate how far the Gaussian decay persists. We present results on the right tail for the real part of the logarithm, where the absolute value of zeta is `unusually large', on the scale of the exponential of the variance. The result is in agreement with the corresponding (known) random matrix result, under the usual dictionary. This work is joint with Louis-Pierre Arguin.

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: German Sierra

Title: Quantum Computation of Prime Number Functions

Abstract: We propose a quantum circuit that creates a pure state corresponding to the quantum superposition of all prime numbers less than 2^n , where n is the number of qubits of the register. This Prime state can be built using Grover's algorithm, whose oracle is a quantum implementation of the classical Miller-Rabin primality test. The Prime state is highly entangled, and encodes number theoretical functions such as the distribution of twin primes or the Chebyshev bias. This algorithm can be further combined with the quantum Fourier transform to yield an estimate of the prime counting function, that allows for the verification of the Riemann hypothesis. Arithmetic properties of prime numbers are then, in principle, amenable to experimental verifications on quantum systems.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Giuseppe Mussardo

Title: Generalised Riemann Hypothesis and Brownian Motion

Abstract: TBA

3:30pm **Tea - SCGP Cafe**

4:00pm **- SCGP 102/ZOOM**

Speaker: Trevor Wooley

Title: Digging below the square-root barrier: subconvexity and exponential sums

Abstract: In most circumstances, proving estimates better than those tantamount to square-root cancellation for mean values of exponential sums remains a distant prospect. It is classical that this is possible for small moments of quadratic Weyl sums. In this talk, we describe progress for higher degree exponential sums associated with Vinogradov's mean value theorem. It transpires that a natural extension of the Main Conjecture in Vinogradov's mean value theorem delivers subconvex estimates for twisted moments at the critical exponent, and that such conclusions may be proved unconditionally in the cubic case.

Wednesday, October 26th

9:00am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102/ZOOM**

Speaker: Don Zagier

Title: Differential equations, periods, and integrality

Abstract: Many situations in both pure mathematics and mathematical physics lead to sequences of numbers {An | n ? 0} that satisfy a recursion of finite length with polynomial coefficients, or equivalently, for which the generating function ?(t) = P? n=0 Ant n satisfies a linear differential equation with polynomial coefficients. Sometimes a “miracle” occurs and the numbers satisfying this recursion with appropriate initial conditions turn out always to be integers, even though nothing in the recursion suggests why this should happen. An example is given by the Ap´ery numbers satisfying the 3-term recursion (n + 1)3An+1 ? (34n 3 + 51n 2 + 27n + 5)un + n 3 un?1 = 0 with A0 = 1 and A1 = 5, which played the central role in Ap´ery’s famous proof of the irrationality of ?(3); here it looks as though the denominator of An could be as large as n! 3 , but in fact all of the An are integers. If one changed the coefficients of the ODE even slightly, then this would not happen. A well-known conjecture says that this integrality can only occur if the differential equation of which ?(t) is a solution is of Picard-Fuchs type, or equivalently, if ?(t) is a period function (= integral of some differential form on the total space of a family of varieties over the projective t-line). We will discuss various aspects of this question, including its relationship to modular forms, and then in a little more detail a particularly interesting case arising from Witten’s 1-point 5-spin intersection numbers on the moduli space of curves of genus g. This is joint work with Di Yang.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102/ZOOM**

Speaker: Fernando Rodriguez-Villegas

Title: Hypergeometric functions and L-series

Abstract: The classical one-variable hypergeometric functions $_nF_{n-1}$ with rational parameter has a geometric origin. This means that they arise from a one-parameter family of motives. In particular, for each rational value of the parameter we obtain an L-function of rank n. For example, $_2F_1(1/2,1/2;1,t)$ corresponds in this way to the Legendre family of elliptic curves $E_t: y^2=x(x-1)(x-t)$. For each rational number $t\neq 0,1$ the rank 2 L-function is that of E_t. Hypergeometric motives represent a class of motives that is accessible for detail study and still large enough to cover a wide range of features. The talk will focus on the explicit calculation of their L-functions

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Andre Leclair

Title: The argument of the completed zeta function: From the computation of very high zeros to the Quantum Hall effect.

Abstract: This talk naturally separates into two distinct parts, though both parts are based on the argument of the completed zeta function $\pi^{-s/2} \Gamma(s/2) \zeta (s)$. The first part deals with an exact transcendental equation satisfied by individual zeros on the critical line, which depends only on the zero number, i.e. the integer that enumerates the zeros. Combined with the Euler product formula we can calculate very high zeros, in particular the google-th zero, to a certain degree of accuracy depending on home many primes are kept in the Euler product. In the second part we will use the same function to propose a phenomenological formula for the transverse resistivity in the Quantum Hall effect. We describe the consequences of the Riemann Hypothesis for this hypothetical formula. We also argue that the random properties of the zeros arise from the disordered landscape experienced by the electrons, again assuming our phenomenological formula.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Andrea Trombettoni

Title: Holographic Realization of the Prime Number Quantum Potential

Abstract: I present a discussion of the experimental realization with holographic techniques of the prime number quantum potential, defined as the potential entering the single-particle Schrödinger Hamiltonian with eigenvalues given by the first N prime numbers. A spatial light modulator is used to tailor the potential to the desired shape. We also implement the potential having as eigenvalues the first lucky numbers, a sequence of integers generated by a different sieve than the familiar Eratosthenes's sieve used for the primes. These results increase in perspective the possibilities to set up quantum systems for arithmetic manipulations or mathematical tests involving prime numbers.

3:30pm **Tea - SCGP Cafe**

4:00pm **- SCGP102**

Speaker: Vladimir Korepin

Title: Transcendental numbers in spin chains

Abstract: We shall start with XXX antiferromagnet on the infinite lattice at zero temperature. We shall consider time independent correlation functions. For spin 1/2 these can be represented as polynomials [with rational coefficients] of the values of Riemann zeta at odd arguments. We shall also mention different generalizations. This is joint work with Kun Hao.

Thursday, October 27th

9:00am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102/ZOOM**

Speaker: Miranda Cheng

Title: Mock Modular Forms and Physics

Abstract: In this talk I will introduce mock modular forms, and give examples of how these fascinating functions appear in various areas of mathematics and physics, in an intricate and at times mysterious way.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102/ZOOM**

Speaker: Charles Creffield

Title: Quantum simulation of the Riemann zeta function using a driven ion

Abstract: The non-trivial zeros of the Riemann zeta function are central objects in number theory, and have also attracted the attention of physicists working in random matrix theory and quantum chaos for many years. In this talk I will show how a quantum simulation technique termed "Floquet engineering" can be used to modify the spectrum of a periodically-driven system, so that its quasienergies correspond to the imaginary components of the Riemann zeros. I will go on to show how this has been implemented for a trapped ion qubit held in a Paul trap, allowing the position of the zeros to be determined by measuring the tunneling dynamics of the qubit. By scanning over the amplitude of the driving field, the locations of the Riemann zeros can be measured experimentally to a high degree of accuracy, providing a physical embodiment of these fascinating mathematical objects in the quantum realm. References: 1. "Riemann zeros from Floquet engineering a trapped-ion qubit", Ran He, et al., npj Quantum Inf 7, 109 (2021). 2. "Identifying the Riemann zeros by periodically driving a single qubit", Ran He, et al., Phys. Rev. A 101, 043402 (2020).

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102/ZOOM**

Speaker: David Farmer

Title: The zeta function when it is particularly large

Abstract: What does the graph of the zeta function look like in a neighborhood of its largest values? That question cannot be answered by looking at graphs of the zeta function, because no particularly large values have ever been calculated. We will give an answer by combining results from analytic number theory and random matrix theory, illustrated by examples.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Jeff Lagarias

Title: Complex Equiangular Lines and the Stark Conjectures

Abstract: This talk describes aspects of an exciting connection made by physicists between an un- solved problem in quantum information theory and topics In algebraic number theory involv- ing class fields of real quadratic fields. The quantum information problem-existence of SIC- POVM's in given dimensions, is interpretable as a problem in combinatorial design theory- the existence of maximal sets of d 2 complex equiangular lines in C d . The connection with class fields and algebraic number theory was made by physicists. My former student Gene Kopp recently uncovered a surprising, deep (unproved!) connection of these sets with the Stark con- jectures. For infinitely many dimensions d he predicts the existence of maximal equiangular sets, constructible by a specific recipe starting from suitable Stark units, in the rank one case. Numerically computing special values at s = 0 of suitable L-functions then permits recovering the units numerically to high precision, then reconstructing them exactly, then testing they satisfy suitable extra algebraic identities to yield a construction of the set of equiangular lines. It has been carried out for d = 5, 11, 17 and 23.

3:30pm **Tea - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Zhao Zhang

Title: From coprime quantum spin ladder to partially integrable spin chain

Abstract: We present a quantum spin ladder made with N coupled chains of spin-1/2, with its degrees of freedom bearing a number theoretic interpretation of square-free numbers. The rung and leg Hamiltonian introduces competing phases of the ground state. One of them is given by the superposition of degrees of freedom encoding the first N prime numbers. This phase can be described by an emergent spin chain that is a minimal generalisation to the antiferromagnetic XXZ spin chain with enlarged local Hilbert space. Such a Hamiltonian is known be be partially integrable, due to the generic violation of Yang-Baxter's equation. Nevertheless, we identify new Bethe Ansatz integrable excited states in the non-integrable Krylov subspace, which leads to slow thermalisation that interpolates between integrability and the eigenstate thermalisation hypothesis.

Friday, October 28th

9:00am **Breakfast - SCGP Cafe**

9:30am **- SCGP 102/ZOOM**

Speaker: Masatoshi Suzuki

Title: On the screw function of the Riemann zeta function

Abstract: The notion of screw functions was introduced in functional analysis in the 1940s. In this talk, we first explain what the screw function is. Then we define the screw function associated with the Riemann zeta function and describe its properties. Typical results are several equivalent conditions for the Riemann hypothesis in terms of the screw function including an analog of the so-called Weil's positivity criterion or Yoshida's non-degenerate criterion.

10:30am **Coffee Break - SCGP Cafe**

11:00am **- SCGP 102/ZOOM**

Speaker: Alain Connes

Title: Zeta and the archimedean prime

Abstract: I will describe recent results involving the role of the prolate spheroidal wave functions and operator, obtained in collaboration with C. Consani concerning the low lying zeros and in collaboration with H. Moscovici concerning the ultraviolet behavior of zeros (of zeta in both cases).

12:00pm **Lunch - SCGP Cafe**

1:15pm **- SCGP 102**

Speaker: Alessandro Fazzari

Title: Weighted statistics of the Riemann zeta function

Abstract: We will discuss classical statistics for the Riemann zeta function when the averages are tilted by powers of zeta on the critical line. In particular, we will focus on the weighted value-distribution of the Riemann zeta function, and statistics for its non-trivial zeros. This weighted investigation blends together the theory of moments and that of n-th level density, and allows for a better understanding of the interplay between zeros and large values of zeta.

2:15pm **- SCGP Cafe**

Title: Break

2:30pm **- SCGP 102**

Speaker: Brad Rodgers

Title: The alternative hypothesis and some extremal problems related to mimicking zeta zero statistics

Abstract: The Alternative Hypothesis concerns a hypothetical (and unlikely) arrangement of the zeros of the Riemann zeta function in which zeros are spaced as though they lie in a lattice. Ruling out the alternative hypothesis would have a number of interesting consequences in number theory. In this talk I will discuss joint work with J. Lagarias and independent work of T. Tao showing that what is presently known about the spacing of zeros of the zeta function is not sufficient to rule out the Alternative Hypothesis. This is done via construction of explicit counterexamples. I hope to also advertise some open problems coming mainly from analysis and probability which are related to this perspective.

3:30pm **Tea - SCGP Cafe**

4:00pm **- SCGP 102**

Speaker: Ghaith Hiary

Title: TBA

Abstract: TBA