

# Birational Complexity of Algebraic Varieties: December 12-16, 2022

Events for:  
Monday, December 12th - Friday, December 16th

Monday, December 12th
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9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Alexander Duncan - SCGP 102**

**Speaker:** Alexander Duncan

**Title:** Finite subgroups of Cremona groups and representation dimension

**Abstract:** Cremona groups are infamously large. In particular, even the plane Cremona group cannot be embedded into a linear algebraic group. Their finite subgroups are much more manageable, but still not completely understood even in the rank 2 case over the complex numbers. However, lacking a complete classification, one may attempt to find bounds for their complexity. Over a number field, one can consider the orders of the finite subgroups. Over general fields, there is the Jordan constant. I consider the minimal dimension of a faithful representation. This is joint work with C. Urech.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Julia Schneider - SCGP 102**

**Speaker:** Julia Schneider

**Title:** Birational maps of Severi-Brauer surfaces, with applications to Cremona groups of higher rank

**Abstract:** We describe the group of birational transformations of a non-trivial Severi-Brauer surface over a perfect field, proving that if it contains a point of degree 6, then it is not generated by elements of finite order. We then use this result to study Mori fibre spaces over the field of complex numbers and deduce that any group of cardinality at most  $C$  (the field of complex numbers) is a quotient of the Cremona group of rank at least 4. Moreover, we prove that the 3-torsion in the abelianization of the Cremona group of rank at least 4 is uncountable. This is based on a joint work with J. Blanc and E. Yasinsky.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:00pm **Workshop Speaker: Frank Gounelas - SCGP 102**

**Speaker:** Frank Gounelas

**Title:** Curves on K3 surfaces

**Abstract:** I will discuss recent results (joint with Chen and Chen-Liedtke) on existence and deformation theory of curves of geometric genus  $g \geq 0$  on complex projective K3 surfaces.

3:00pm **Tea Time - SCGP Cafe**

**Title:** Tea Time

3:30pm **Workshop Speaker: Lena Ji - SCGP 102**

**Speaker:** Lena Ji

**Title:** Rationality of real conic bundles with quartic discriminant curve

**Abstract:** Clemens–Griffiths introduced the classical intermediate Jacobian obstruction to rationality for complex threefolds in their proof of the irrationality of the cubic threefold. Recently, over non-closed fields, Hassett–Tschinkel (over  $\mathbb{R}$ ) and Benoist–Wittenberg (over  $k$ ) refined this obstruction using torsors over the intermediate Jacobian. In this talk, we describe this obstruction in the setting of conic bundle threefolds. In particular, we study rationality over the real numbers for a specific class of these conic bundles. This talk is based on joint work with S. Frei–S. Sankar–B. Viray–I. Vogt and on joint work with M. Ji.

<b>Tuesday, December 13th</b>
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9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Nathan Chen - SCGP 102**

**Speaker:** Nathan Chen

**Title:** Curves on very general hypersurfaces

**Abstract:** In this talk, we will explore the degrees of curves on very general hypersurfaces. Time permitting, we will explain some applications to measures of irrationality. Some of this will be joint with David Yang.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Evgeny Shinder - SCGP 102**

**Speaker:** Evgeny Shinder

**Title:** Derived categories of varieties with rational singularities

**Abstract:** Given a resolution of rational singularities  $Y \rightarrow X$ , there are old Bondal-Orlov conjectures relating derived categories of coherent sheaves on  $Y$  and on  $X$ . I will explain what is known about these conjectures and prove that the image of the pushforward map generates the derived category of  $X$ . The proof boils down to verifying that under some conditions, fibers of the resolution are  $\mathcal{O}$ -acyclic, which is based on a Hodge theory argument. This is joint work with Mirko Mauri.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:00pm **Workshop Speaker: Alena Pirutka - SCGP 102**

**Speaker:** Alena Pirutka

**Title:** Some uniform bounds for Chow groups

**Abstract:** In this talk we will discuss uniform bounds for the torsion subgroup in the Chow group of cycles of codimension two for families of varieties over number fields. This is joint work with F. Charles.

3:00pm **Tea Time - SCGP Cafe**

**Title:** Tea Time

3:30pm **Workshop Speaker: Fumiaki Suzuki - SCGP 102**

**Speaker:** Fumiaki Suzuki

**Title:** Non-algebraic geometrically trivial cohomology classes over finite fields

**Abstract:** The Tate conjecture is a long-standing problem in arithmetic geometry, describing algebraic cycles on an algebraic variety in terms of Galois representation on étale cohomology with coefficients in  $\mathbb{Q}_\ell$ . In contrast, an integral analogue of the Tate conjecture is known to fail in general, and a natural question is whether the failure is always caused by geometry. Over a finite field, we construct the first counterexamples to this question: in codimension 2 on our examples, a geometric cycle map is surjective but an arithmetic cycle map is not. We also show positive results toward a conjecture of Colliot-Thélène and Kahn on the third unramified cohomology group for threefolds over a finite field. This is a joint work with Federico Scavia.

4:45pm - **SCGP 102**

**Speaker:** Various

**Title:** Speed Talks

<b>Wednesday, December 14th</b>
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8:30am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:00am **Workshop Speaker: Elham Izadi (ZOOM) - SCGP 102/ZOOM**

**Speaker:** Elham Izadi

**Title:** Hodge classes on the moduli space of  $W(E_6)$  covers and the geometry of  $A_6$

**Abstract:** The Kodaira dimensions of moduli spaces of principally polarized abelian varieties are known except in the case of dimension 6. One approach to understanding the geometry of moduli spaces of abelian varieties is to parametrize them with suitable moduli spaces of curves. We proved earlier that principally polarized abelian sixfolds are Prym-Tjurin varieties of covers of  $\mathbb{P}^1$  with monodromy  $W(E_6)$  and identified three divisors on the moduli space  $M(W(E_6))$  of  $W(E_6)$  covers that are not contracted by the Prym-Tjurin map. Here we introduce naturally occurring divisors on  $M(W(E_6))$  obtained from the irreducible representations of  $W(E_6)$ . We compute these divisors in terms of the three main divisors and give a concrete description of the ramification divisor of the Prym-Tjurin map. We also give a second, more elementary, proof of the dominance of the Prym-Tjurin map. This is joint work with Alexeev, Donagi, Farkas, Ortega.

10:00am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

10:30am **Workshop Speaker: Roya Beheshti - SCGP 102**

**Speaker:** Roya Beheshti

**Title:** Restrictions on rational surfaces in general Fano hypersurfaces

**Abstract:** I will talk about Moduli spaces of rational curves on Fano hypersurfaces and rational curves contained in them. We investigate when these moduli spaces are uniruled and when they do not contain many rational curves. One motivation for the study of these types of questions comes from the question of when Fano hypersurfaces are swept out by certain families of rational surfaces. This is based on joint work with Eric Riedl.

11:30am **Coffee Break - SCGP 102**

**Title:** Coffee Break

12:00pm **Workshop Speaker: Christian Urech - SCGP 102**

**Speaker:** Christian Urech

**Title:** Actions of Cremona groups on  $\text{CAT}(0)$  cube complexes

**Abstract:** Recently, in geometric group theory, isometric actions of groups on  $\text{CAT}(0)$  cube complexes have turned out to be powerful tools to study a broad range of groups. In this talk, I will explain the construction of  $\text{CAT}(0)$  cube complexes on which groups of birational transformations act by isometries and explain how to use these actions to deduce new and old group theoretical and dynamical results about Cremona groups. This is joint work with Anne Lonjou.

1:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

3:00pm **Tea Time - SCGP Cafe**

**Title:** Tea Time

## Thursday, December 15th

9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Stefan Schreieder - SCGP 102**

**Speaker:** Stefan Schreieder

**Title:** The diagonal of quartic fivefolds

**Abstract:** We show that a very general quartic fivefold over a field of characteristic different from 2 does not admit a decomposition of the diagonal, hence is not retract rational. This generalizes a result of Nicaise-Ottem, who showed stable irrationality over fields of characteristic zero. Joint work with Nebojsa Pavic.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Masahiro Nakahara - SCGP 102**

**Speaker:** Masahiro Nakahara

**Title:** Bijective Cremona transformations of the plane

**Abstract:** I will discuss birational self-maps of the projective plane over finite fields that induce permutations on the set of rational points. As a main result, we proved that no odd permutation arises over a non-prime finite field of characteristic two, which completes the investigation initiated by Cantat about which permutations can be realized this way. Main ingredients in our proof include the invariance of parity under groupoid conjugations by birational maps, and a list of generators for the group of such maps. This is joint work with Shamil Asgarli, Kuan-Wen Lai, and Susanna Zimmermann.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:00pm **Workshop Speaker: Kuan-Wen Lai - SCGP 102**

**Speaker:** Kuan-Wen Lai

**Title:** Estimates on the growth of irrationality for moduli spaces of K3 surfaces

**Abstract:** Similar to moduli spaces of curves, the moduli space of polarized K3 surfaces of genus  $g$  is of general type and thus is irrational for  $g$  sufficiently large. In this talk, I will introduce various estimates on how the irrationality grows with  $g$  in terms of the measure introduced by Moh and Heinzer. These estimates are built upon the modularity of the generating series of these moduli spaces in certain ambient spaces, as well as upon the existence of Hodge theoretically associated cubic fourfolds, Gushel–Mukai fourfolds, and hyperkähler fourfolds. This is joint work with Daniele Agostini and Ignacio Barros.

3:00pm **Tea Time - SCGP Cafe**

**Title:** Tea Time

3:30pm **Workshop Speaker: David Stapleton - SCGP 102**

**Speaker:** David Stapleton

**Title:** Minimal degree fibrations and the asymptotic degree of irrationality of divisors

**Abstract:** Given a smooth, polarized, projective variety  $(Y, H)$  it is natural to ask how the degree of irrationality of hypersurfaces  $X$  in  $|dH|$  relate to the geometry of  $Y$ . For example, if  $X$  is a hypersurface in projective space of sufficiently large degree then the fibers of any map computing the degree of irrationality of  $X$  are contained in lines in projective space. In this talk I discuss joint work in preparation with Levinson and Ullery. We show that if  $Y$  is arbitrary and  $d$  is sufficiently large then any map computing the degree of irrationality of  $X$  factors through a "minimal degree fibration of  $Y$  in curves." As a consequence we can compute the asymptotic degree of irrationality of  $X$  in terms of a natural invariant of  $(Y, H)$  and we show that the degree of irrationality of general complete intersections of sufficiently unbalanced degrees  $0 < d_1 < \dots < d_r$  is given asymptotically by the product of the degrees  $d_1 \dots d_r$ .

4:45pm **- SCGP 102**

**Speaker:** Various

**Title:** Speed Talks

6:00pm **Workshop Banquet - SCGP Cafe**



**Title:** Banquet

<b>Friday, December 16th</b>
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9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Susanna Zimmerman - SCGP 102**

**Speaker:** Susanna Zimmerman

**Title:** Open problems on Cremona groups

**Abstract:** In this talk I will present an overview on Cremona groups and present some open problems.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Claire Voisin - SCGP 102**

**Speaker:** Claire Voisin

**Title:** Geometry of the Abel-Jacobi map and stable birational invariants

**Abstract:** The Chow group of codimension 2 cycles homologous to 0 on a rationally connected manifold is known to be isomorphic to the corresponding intermediate Jacobian, as it is for divisors of any smooth projective variety. However, the geometry of the Abel-Jacobi map may differ from that of divisors, and contain some obstruction to stable rationality, for example, there may not exist a universal codimension 2 cycle. We prove that, as conjectured by Clemens, for a rationally connected threefold  $X$ , there exists a smooth projective surface  $S$  parameterizing 1-cycles on  $X$  so that the induced Abel-Jacobi map is an isomorphism between the Albanese variety of  $S$  and the intermediate Jacobian of  $X$ . We will also discuss the case of the cubic threefold, for which the existence of a universal codimension 2 cycle is unknown. We will explain how this problem is related to questions on integral Hodge classes on abelian varieties.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

3:00pm **Tea Time - SCGP Cafe**

**Title:** Tea Time