

# SuperGeometry and SuperModuli

Events for:

Monday, March 27th - Friday, March 31st

Monday, March 27th

9:00am **Workshop: Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop: Giovanni Felder (ZOOM) - SCGP 102/ZOOM**

**Speaker:** Giovanni Felder

**Title:** Towards a mathematical description of superstring perturbation theory

**Abstract:** Superstring amplitudes, describing the scattering of superstrings, have perturbative power series expansions in the coupling constants, whose coefficients are integrals over moduli spaces of supercurves. These integrals over non-compact superspaces are expected to (conditionally) converge, in contrast with the case of Feynmann diagrams of Quantum Field Theory, where integrals diverge and require renormalization. In this talk I will review the definition of these objects, present the challenges for a mathematical description (integration over middle dimensional cycles, GSO projection, dependence on the regularization), and explain how to address them in the simplest non-trivial case of genus 2 contribution to the vacuum amplitude of the type II superstring in 10 dimensional Minkowski space. The talk is based on joint work with David Kazhdan and Alexander Polishchuk.

10:30am **Workshop: Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop: Anton Zeitlin - SCGP 102**

**Speaker:** Anton Zeitlin

**Title:** Super-Teichmueller spaces: coordinates and applications

**Abstract:** The Teichmüller space parametrizes Riemann surfaces of fixed topological type and is fundamental in various mathematics and physics contexts. It can be defined as a component of the moduli space of flat  $G=\mathrm{PSL}(2,\mathbb{R})$  connections on the surface. Higher Teichmüller space extends these notions to appropriate higher rank classical Lie groups  $G$ , and  $N=1$  super Teichmüller space likewise studies the extension to the super Lie group  $G=\mathrm{OSp}(1|2)$ . In this talk, I will review the solution to the problem of producing Penner-type coordinates on super-Teichmüller space and its higher analogs. I will also talk about some applications of these coordinates.

12:00pm **Workshop: Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop: Simone Noja - SCGP 102**

**Speaker:** Simone Noja

**Title:** Nilpotence Varieties, Pure Spinors Superfields and Supersymmetry

**Abstract:** In this talk I will introduce a (super-)mathematical perspective on the pure spinor superfield formalism, showing how to recover supersymmetry multiplets from geometric data related to the nilpotence variety of a certain Poincaré superalgebra. After discussing some lower dimensional examples, I will focus on the case of supersymmetry in six dimensions, where the nilpotence variety is a Segre manifold, and I will hint at a generalization of the formalism in the direction of derived algebraic geometry. If time permits, I will discuss how nilpotence varieties of some classical Lie superalgebras are related to the superconformal field theories.

3:30pm **Workshop: Tea Time - SCGP Cafe**

**Title:** Tea Time

4:00pm **Workshop: Charles Wang - SCGP 102**

**Speaker:** Charles Wang

**Title:** On the Relationship Between Super-Riemann Surfaces and PCOs

**Abstract:** We will discuss the relationship between the two formulations of superstring perturbation theory: the SRS formalism, based on integration over the supermoduli space of super-Riemann Surfaces, and the PCO formalism, based on integration over bosonic moduli space with insertions of picture changing operators and vertical integration. I will show that the picture changing operator approach arises from a specific construction of the supermoduli integration contour. If time permits, I will also discuss an explicit construction in genus two.

**Tuesday, March 28th**

9:00am **Workshop: Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop: Daniel Hernandez Ruiperez (ZOOM) - SCGP 102/ZOOM**

**Speaker:** Daniel Hernandez Ruiperez

**Title:** Stable supermaps and SUSY Nori motives

**Abstract:** We define stable supercurves and stable supermaps, and based on these notions we develop a theory of Nori motives for the category of stable supermaps of SUSY curves with punctures. This will require several preliminary constructions, including the development of a basic theory of supercycles.

10:30am **Workshop: Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop: Oliver Schlotterer - SCGP 102**

**Speaker:** Oliver Schlotterer

**Title:** Worldsheet supersymmetry and uniform transcendentality of string amplitudes

**Abstract:** The low-energy expansion of string scattering amplitudes involves special numbers and functions such as polylogarithms and multiple zeta values which are informally assigned a notion of transcendental weight. For superstring tree amplitudes and certain building blocks of one-loop amplitudes, the order in the inverse string tension  $\alpha'$  matches the transcendental weight of the accompanying (elliptic) zeta values. This property is known as uniform transcendentality (UT) and also occurs in the context of Feynman integrals and perturbative field-theory computations. Since amplitudes of bosonic and heterotic string theories violate UT, one may wonder if UT originates from worldsheet supersymmetry which is supported by the short-distance properties of worldsheet-supersymmetric vertex operators. This talk aims to give a gentle introduction into the above ideas to a mixed audience of mathematicians and physicists.

12:00pm **Workshop: Lunch - SCGP Cafe**

**Title:** Lunch

1:15pm **SCGP Weekly Talk & Workshop Speaker: Eric D'Hoker - SCGP 102/ZOOM**

**Speaker:** Eric D'Hoker (zoom)

**Title:** Supermoduli and superstring amplitudes

**Abstract:** Perturbative superstring amplitudes are given by integrals over the moduli spaces of super Riemann surfaces, summed over genera. We concentrate on the case of genus two which is the smallest genus at which supermoduli enter superstring amplitudes in a non-trivial way. Using the special properties of genus-two supermoduli space, we will show that superstring amplitudes may be evaluated for an arbitrary number of Neveu-Schwarz string states, thanks to new identities between modular tensors of  $Sp(4, \mathbb{Z})$ .

2:30pm **Workshop: Katherine Maxwell - SCGP 102**

**Speaker:** Katherine Maxwell

**Title:** Extended super Mumford form on the Sato Grassmannian

**Abstract:** The super Mumford form is a section over the moduli space of super Riemann surfaces, characterized by invariance under the action of the Neveu-Schwarz algebra action. In light of difficulties in performing integrals in superstring theory arising from the super Mumford form, it was suggested in the 80s that the relationship of the moduli space of super Riemann surfaces to the super Sato Grassmannian may be fruitful. Based on joint work with A. Voronov, I will discuss possible approaches to extending the super Mumford form, including our results on the proposed formula by A. Schwarz.

3:30pm **Workshop: Tea Time - SCGP Cafe**

**Title:** Tea Time

**Wednesday, March 29th**

9:00am **Workshop: Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop: Vera Serganova - SCGP 102**

**Speaker:** Vera Serganova

**Title:** Localization of integrals on supermanifolds with application to representation theory of supergroups.

**Abstract:** We compute volumes of supergrassmannians and odd symmetric grassmannians using Schwarz-Zaboronsky localization formula which expresses a Berezin integral as a sum of local contributions at all singular points of an odd vector field. To generalize this computation to other classical supermanifolds, we need a CS analogue of localization, since manyclassical supergroups don't have compact real forms. We prove an analogue of the Schwarz-Zaboronsky localization formula for complex smooth supermanifolds. Let  $X$  be a compact CS manifold and  $Q$  an odd vector field on  $X$  such that  $[Q, Q]$  is compact. Assume that  $Q$  has isolated singular points on  $X$  and preserves a volume form  $w$ . Then the integral of  $w$  over  $X$  equals the sum of local contribution at all singular points. We apply the localization formula in the case of homogeneous superspace  $X=G/K$  which admits a  $G$  invariant volume form. For specific choices of  $G$  and  $K$  we show that the integral of  $w$  over  $X$  is not zero. This allows us to use the unitary trick and show that  $K$  is a splitting subgroup of  $G$ , i.e. the restriction functor from  $\text{Rep } G$  to  $\text{Rep } K$  induces injection of Ext groups. In particular, we prove that a defect subgroup is splitting in the case when  $\text{Lie } G$  is any basic classical or exceptional superalgebra. This has several applications in support theory for supergroups. For example, we show that the DS associated variety detects projectivity in the category of finite-dimensional  $G$ -modules. The talk is based on joint papers with A. Sherman and D. Vaintrob.

10:30am **Workshop: Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop: Rita Fioresi - SCGP 102**

**Speaker:** Rita Fiorese

**Title:** Generalized Root System

**Abstract:** In this talk (joint work with I. Dimitrov, Queens U.) we introduce the category of generalized root systems. The notion of ordinary root systems is the key to understand Lie theory and its many generalizations (contragredient superalgebras, affine, Kac-Moody (super) algebras etc). However, such notion is "rigid", it does not behave reasonably under quotients and moreover lacks of a unified treatment, that is definitions and results are usually confined to the realm of application. The rigidity of ordinary root systems stems from their invariance under the action of the Weyl group. Once we abandon the notion of Weyl group as we know it, we can look for another definition of root systems that is able to take into account all examples mentioned above and more. For example, the systems stemming from the eigenspace decomposition with respect to a non maximal toral subalgebra (Kostant root systems). They play a key role in the classification of the complex structures on the symmetric space  $G/K$ , for  $K$  and non maximal torus. This is a generalization of the hermitian symmetric spaces theory. In this talk we give an effective way to compute bases for generalized root systems, which are quotients of Lie algebra ones and we classify all root systems of rank two up to combinatorial equivalence finding 16 such.

12:00pm **Workshop: Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop: John Huerta - SCGP 102**

**Speaker:** John Huerta

**Title:** Body and soul decompositions in supergeometry

**Abstract:** Given a supermanifold with a projection map to the reduced manifold, elements of the structure sheaf decompose into a "body", a function on the reduced manifold, and a complementary part called the "soul". This induces, in turn, a decomposition of various kinds of geometric structure one could put on a supermanifold, including de Rham forms, line bundles with connection, as well as higher degree classes in Deligne cohomology.

3:30pm **Workshop: Tea Time - SCGP Cafe**

**Title:** Tea Time

4:00pm **Workshop & Algebraic Geometry Seminar Speaker: Gavril Farkas - SCGP 102**

**Speaker:** Gabriel Farkas

**Title:** The birational geometry of the moduli of curves: geometric and tropical aspects.

**Abstract:** It is one of the landmark results in algebraic geometry of the 20th century that the moduli space  $M_g$  of curves of genus  $g$  is a variety of general type when  $g > 23$ . I will discuss joint work with Jensen and Payne proving that both moduli spaces  $M_{22}$  and  $M_{23}$  are of general type, highlighting both the geometrical and the novel tropical aspects related to this circle of ideas. Time permitting, I will also discuss how novel ideas from non-abelian Brill-Noether theory can be used to prove that the moduli space of genus 16 is uniruled.

## Thursday, March 30th

9:00am **Workshop: Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop: Slava Pimenov (ZOOM) - SCGP 102/ZOOM**

**Speaker:** Slava Pimenov

**Title:** Gelfand-Fuchs cohomology for super-manifolds

**Abstract:** Let  $X$  be a smooth super-manifold of super-dimension  $(m, n)$  and consider the Lie super-algebra  $V_X$  of vector fields on  $X$ . We are interested in the Lie algebra cohomology of  $V_X$  with trivial coefficients. Using a local to global construction the question can be reduced to calculation of cohomology of Lie algebra of super-vector fields in a formal neighborhood of a point on an affine super-space  $\mathbb{A}^{m,n}$ . Previously known results covered the classical case  $\mathbb{A}^m = \mathbb{A}^{m,0}$  as well as the case  $\mathbb{A}^{m,n}$  with  $m \leq n$ . I will speak about my recent work where I extended these results to the case of super-dimension  $(m, 1)$ . I will also propose a conjectural answer in the general case and speculate on how one might try to prove it.

10:30am **Workshop: Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop: Nadia Ott - SCGP 102**

**Speaker:** Nadia Ott

**Title:** TBA

**Abstract:** Artin's theorems on approximation and algebraization of formal deformations and stacks give general criteria for functors to be, in various senses, described by algebraic objects. It has long been expected that analogous results hold in supergeometry, however proofs of the full suite of Artin theorems have remained absent from the supergeometry literature. One reason for this may be the sense that establishing the Artin theorems in supergeometry would require the tedious repetition of various difficult arguments in commutative algebra, deformation theory, and algebraic geometry. I have recently proved the Artin theorems in the super case. Moreover, at several key points I was able to reduce to the (known) bosonic case by an argument which is significantly simpler than the original bosonic argument. In my talk I will show how to use the Artin theorem to construct some moduli spaces of interest in supergeometry, e.g., the Picard (super)stack.

12:00pm **Workshop: Lunch - SCGP Cafe**

**Title:** Lunch

1:15pm **Workshop: Enno Kessler - SCGP 102**

**Speaker:** Enno Kessler

**Title:** Odd torus actions on moduli spaces of super stable maps of genus zero

**Abstract:** Super stable maps are supergeometric generalizations of stable maps from a Riemann surface in an almost Kähler manifold and appear naturally in the compactification of the moduli space of super J-holomorphic curves. Super J-holomorphic curves are maps from a super Riemann surface to an almost Kähler manifold satisfying a Cauchy-Riemann equation. In this talk we will explain the construction of moduli spaces of super stable maps of genus zero of fixed tree type and show that they carry a torus action that leaves the even directions invariant. The invariant manifolds are then the corresponding moduli spaces of stable maps and the normal bundles are described as holomorphic sections of twisted spinor bundles. Based on joint work with Artan Shesmani and Shing-Tung Yau

2:30pm **Workshop: Artan Shesmani - SCGP 102**

**Speaker:** Artan Sheshmani

**Title:** Genus zero super Gromov Witten invariants via odd torus localization

**Abstract:** A major challenge in construction of supergeometric analogue of Gromov-Witten invariants is the suitable generalization of intersection theory. We propose to circumvent this difficulty by assuming a virtual torus localization theorem for the odd directions. That is, we construct a super virtual normal bundle to the torus-fixed loci of the moduli space of super stable maps, and compute the super Gromov-Witten invariants, via dividing by the equivariant Euler class of the super virtual normal bundle and intersecting with the virtual class of the torus fixed superstable maps. We define the super Gromov-Witten invariants of genus zero which satisfy generalized Kontsevich-Manin axioms. Furthermore, we present a recipe for calculation of super Gromov-Witten invariants of projective space. Based on joint work with Enno Keßler and Shing-Tung Yau.

3:30pm **Workshop: Tea Time - SCGP Cafe**

**Title:** Tea Time

4:00pm **Workshop: Jeffrey Rabin - SCGP 102**

**Speaker:** Jeffrey Rabin

**Title:** Cohomology and Combinatorics for Supertori

**Abstract:** A supertorus (“elliptic supercurve”) of dimension  $1|1$  is a simple example showing that sheaf cohomology groups of supermanifolds are generally non-free modules over the ring of Grassmann constants. I will generalize this example to supertori of dimension  $1|n$ , computing  $H_0(X, \mathcal{O})$  and  $H_1(X, \mathcal{O})$ . These groups simply reflect combinatorial invariant-theoretic properties of Grassmann algebras, and exhibit Serre duality and Lefschetz properties. This is a first step toward a general theory of Abelian supervarieties and Jacobians of supercurves.

6:00pm **Workshop: Banquet - SCGP Cafe**

**Title:** Banquet

**Friday, March 31st**

9:00am **Workshop: Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop: Sheldon Katz - SCGP 102**

**Speaker:** Sheldon Katz

**Title:** TBA

**Abstract:** TBA

10:30am **Workshop: Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop: Ron Donagi - SCGP 102**

**Speaker:** Ron Donagi

**Title:** Ramond punctures

**Abstract:** Ramond punctures on super Riemann surfaces, and their moduli, exhibit many unexpected features. I will describe some of these. (Partially based on joint work with Nadia Ott.)

12:00pm **Workshop: Lunch - SCGP Cafe**

**Title:** Lunch

3:30pm **Workshop: Tea Time - SCGP Cafe**

**Title:** Tea Time