

Mathematical Aspects of $N=4$ Super-Yang-Mills Theory: February 26-March 1, 2024

Events for:
Monday, February 26th - Friday, March 1st

Monday, February 26th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Gang Yang - SCGP 102**

Speaker: Gang Yang

Title: Form Factor of the stress-tensor supermultiplet in $N=4$ SYM

Abstract: In this talk, I will give an overview of the form factor (FF) of the stress-tensor supermultiplet in $N=4$ SYM, as well as its conjectured duality with a periodic Wilson line (WL). I will then discuss recent progress in calculating two-loop corrections for the four-point FF. Special attention is given to the lightlike limit where the operator momentum q becomes null. I will demonstrate the presence of an exact dual conformal symmetry in this limit, which provides a strong verification of the FF-WL duality conjecture. Furthermore, I will introduce the non-perturbative lightlike FFOPE framework, which features a novel lightlike transition and naturally encodes the dual conformal symmetry.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Christian Zickert - SCGP 102**

Speaker: Christian Zickert

Title: Holomorphic polylogarithms, motivic complexes, and differential forms

Abstract: The classical weight n polylogarithm has a holomorphic variant defined modulo $(2\pi i)^n$ on the universal abelian cover of $\mathbb{C} \setminus \{0, 1\}$. We discuss its functional relations, and its relationship to Goncharov's Bloch complexes and motivic cohomology. The derivative of the holomorphic polylogarithm is a holomorphic 1-form. More generally, we associate to each multiple polylogarithm a holomorphic 1-form with several remarkable properties related to Hodge structures and the motivic Lie coalgebra. The latter is joint work with Greenberg, Kaufman and Li.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Johannes Broedel - SCGP 102**

Speaker: Johannes Broedel

Title: Towards a Kronecker function at genus two

Abstract: Polylogarithms can be represented as iterated integrals on Riemann surfaces of various genera. For genus-one surfaces, the so-called Kronecker function generates all necessary differentials for the construction of those homotopy-invariant iterated integrals. For genus two and higher, an analogue of the Kronecker function delivering the differentials for the construction of hyperelliptic polylogarithms is not known explicitly. In the talk, I am going to set the stage by reviewing properties for the Kronecker function at genus one and its relation to elliptic polylogarithms. Afterwards I am going to discuss, which generalizations have to be taken and which difficulties arise, when stepping up from genus one to genus two. Using Abel's map and the so-called Schottky-cover formulation, the way for an analytic characterization of a Kronecker function at genus two is going to be paved.

2:30pm **Workshop: Frank Coronado - SCGP 102**

Speaker: Frank Coronado

Title: Determinant operators and massive amplitudes in $N=4$ SYM

Abstract: In planar $N=4$ SYM, massless scattering amplitudes are dual to null polygonal Wilson loops (T-duality) or the same as the four-dimensional null limit of stress-tensor correlators. I will present a (conjectured) generalization of this duality which equates correlators of determinant operators, in a special ten-dimensional null limit, with massive scattering amplitudes in the Coulomb branch of $N=4$. This determinant operator is a generating function of all half-BPS single-traces operators. By taming it on twistor space I will show its correlators have ten dimensional poles which combine space-time and R-charge distances.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Hugh Thomas - SCGP 102**

Speaker: Hugh Thomas

Title: String-like amplitudes for surfaces beyond the disk

Abstract: In 1969, Koba and Nielsen found some equations (now known as u-equations or non-crossing equations) whose solutions can be described as cross-ratios of n points on a line. The tree string amplitude, or generalized Veneziano amplitude, can be defined as an integral over the non-negative solutions to the u-equations. This is a function of the Mandelstam variables and has interesting properties: it does not diverge as the Mandelstam variables get large, and it exhibits factorization when one of the variables approaches zero. One should think of these functions as being associated to the disk with marked points on the boundary. I will report on ongoing work with Nima Arkani-Hamed, Hadleigh Frost, Pierre-Guy Plamondon, and Giulio Salvatori, in which we replace the disk by other oriented surfaces. (The first two papers in what will be a longer series are available online at [arXiv:2309.15913](https://arxiv.org/abs/2309.15913), [arXiv:2311.09284](https://arxiv.org/abs/2311.09284).) I will emphasize the combinatorial aspects of our approach.

Tuesday, February 27th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Alex Tumanov (Zoom) - SCGP 102/Zoom**

Speaker: Alex Tumanov (Zoom)

Title: Form Factor OPE program and its underlying dual Wilson loop description

Abstract: In recent years, the Form Factor Operator Product Expansion (FFOPE) approach has been developed in order to compute the form factors of all protected $1/2$ -BPS operators in $N=4$ SYM theory non-perturbatively in the near-collinear limit. This approach is based on the Wilson loop OPE program developed earlier for scattering amplitudes, but relies on several important generalizations. Firstly, I will build the underlying dual description of $1/2$ -BPS form factors in terms of matrix elements of infinite periodic Wilson loops with states made out of zero-momentum scalars. Based on this description, I will introduce the FFOPE expansion, which, apart from standard pentagon transitions, also involves a new type of building block: form factor transitions. I will show how these objects can be bootstrapped non-perturbatively, leading to the emergence of the octagon kernel in their finite coupling behavior.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Melissa Sherman-Bennett - SCGP 102**

Speaker: Melissa Sherman-Bennett

Title: Cluster adjacency and tilings for the $m=4$ amplituhedron

Abstract: I'll discuss recent developments on tilings of the $m=4$ amplituhedron, which is joint work with Even-Zohar, Lakrec, Parisi, Tessler and Williams. First, we prove that all ways of running the BCFW recursion produces tilings of the amplituhedron, generalizing a result of Even-Zohar, Lakrec and Tessler. Second, we show that each tile in a BCFW tiling is exactly the region where a collection of compatible cluster variables for the Grassmannian have a specified sign. This confirms cluster adjacency conjectures of Drummond-Foster-Gurdogan. I will focus primarily on the second result. After a reminder on the cluster structure of the Grassmannian, I will try to give an idea of how it is related to the BCFW recursion. This talk is complementary to the talk of Lauren Williams later in the week.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

12:00pm **Workshop: Group Photo - SCGP Lobby**

Title: Group Photo

1:15pm **Workshop: Johannes Henn - SCGP 102**

Speaker: Johannes Henn

Title: Coulomb Branch Amplitudes from a Deformed Amplituhedron Geometry

Abstract: The Amplituhedron provides, via geometric means, the all-loop integrand of scattering amplitudes in maximally supersymmetric Yang-Mills theory. Unfortunately, dimensional regularization, used conventionally for integration, breaks the beautiful geometric picture. This motivates us to propose a 'deformed' Amplituhedron. Focusing on the four-particle amplitude, we introduce two deformation parameters, which can be interpreted as particle masses. We provide evidence that the mass pattern corresponds to a specific choice of vacuum expectation values on the Coulomb branch. The deformed amplitude is infrared finite, making the answer well-defined in four dimensions. Leveraging four-dimensional integration techniques based on differential equations, we compute the amplitude up to two loops. In the limit where the deformation parameters are taken to zero, we recover the known Bern-Dixon-Smirnov amplitude. In the limit where only one deformation parameter is taken to zero, we find a connection to the angle-dependent cusp anomalous dimension.

2:30pm **Workshop: David Kosower - SCGP 102**

Speaker: David Kosower

Title: Finite Feynman Integrals

Abstract: I describe how to construct bases of numerators that give rise to finite Feynman integrals. I classify all configurations of loop momenta giving rise to infrared divergences using Landau singularities. I use this classification to construct numerators canceling all of them, to build locally finite integrals along with generators for the corresponding ideal. As a by product we also classify so-called evanescent integrals. This classification is a first step to organizing integral bases by infrared properties, which may simplify expressions for scattering amplitudes.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Livia Ferro - SCGP 102**

Speaker: Livia Ferro

Title: Null-cone geometry for $N=4$ scattering amplitudes

Abstract: In recent years it has become clear that positive geometries underlie various observables in quantum field theories. In this talk I will focus on amplituhedra, i.e. geometries encoding scattering amplitudes, for maximally supersymmetric Yang-Mills theory. After a broad review of the main ingredients involved, I will discuss the momentum amplituhedron in the four-dimensional split-signature space of dual momenta. The null structure of this kinematic space allows to define geometries whose canonical differential forms produce loop-amplitude integrands. In particular, I will describe the one-loop geometry, which is a curvy version of a simple polytope, whose vertices are the maximal cuts of the amplitude. I will provide a simple formula for all one-loop integrands originating from this geometry. Finally, I will discuss some of the questions which remain open in this framework.

Wednesday, February 28th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Dani Kaufman (Zoom) - SCGP 102/Zoom**

Speaker: Dani Kaufman (Zoom)

Title: Grassmannian Cluster Algebras Beyond Finite Type

Abstract: The structure of the finite-type Grassmannian cluster algebras is well understood. However, beyond this case very little is known, including for example the set of cluster variables, the cluster modular group, and the topology of the cluster complex. In work joint with Zachary Greenberg we studied the cluster modular group of the mutation-finite grassmannians $\text{Gr}(4,8)$ and $\text{Gr}(3,9)$. Our calculations make use of a special seed of the algebra which plays the role of a Dynkin-diagram in these cases. From this special seed it is easy to classify all of the sub algebra types and count all of the cluster variables up to the action of the cluster modular group. Recently, I have also studied a new generalised ‘folding’ operation on cluster algebras. Many Grassmannian cluster algebras can be folded using this operation producing unexpected results. For example, the Grassmannian $\text{Gr}(4,9)$ can be folded with the resulting algebra being a new kind of finite type cluster algebra. In this talk I will give a bit of background on the progress on understanding the cluster structure of the non-finite type grassmannians including some ideas towards finding new Dynkin seeds, and give an introduction and examples of this new folding operation.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Michael Green - SCGP 102**

Speaker: Michael Green

Title: Novel features of modular forms arising in correlators of $N=4$ SYM

Abstract: This talk will describe properties of certain integrated four-point correlators of superconformal primary operators of $SU(N)$ $N = 4$ supersymmetric Yang-Mills theory. We will consider two specific integration measures for which the integrated correlators are determined by supersymmetric localisation. These integrated correlators are novel modular functions of the complex coupling τ and functions of N . The simpler example (the 'first correlator') can be represented by a two-dimensional lattice sum that can formally be expressed as the sum of an infinite series of non-holomorphic Eisenstein series. This has a number of intriguing properties at finite N and in the large- N expansion. Notably, it satisfies an interesting Laplace equation that relates correlators with different values of N . The 'second correlator' has a richer structure. Although its exact form is not completely understood, its large- N expansion has modular invariant coefficients that are sums of generalised Eisenstein series, which can be expressed as four-dimensional lattice sums and are familiar from the low-energy expansion of type IIB scattering amplitudes. These are closely related to modular graph functions (which arise, for example, in the analysis of the low energy expansion of genus-one type II superstring amplitudes). This talk will describe some intriguing properties of these coefficients, such as the manner in which various holomorphic cusp forms that are generically present, gratifyingly cancel in all the cases of interest in the context of SYM/IIB string theory.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Lara Bossinger - SCGP 102**

Speaker: Lara Bossinger

Title: The Gröbner theory of Scattering Amplitudes

Abstract: Cluster algebras are known to have an intimate relationship with the symbol alphabet as cluster variables (in certain cluster algebras) are letters and adjacency of these letters can be read from the g -vector fan of the cluster algebra. In this talk I will explain how and when the g -vector fan can be obtained as the T_+ positive part of the tropicalization of a certain ideal presenting the cluster algebra. As T_+ sits inside the Gröbner fan of the ideal it can be computed even in cases the cluster structure is not or only partially known from the ideal and its initial ideals. In joint work with James Drummond and Ross Glew we have successfully applied these methods to e.g. the kinematic ideal associated with non-dual conformal massless scattering written in terms of spinor helicity variables for 5 particles (JHEP11(2023)002). The mathematical context is based on arXiv:2208.01723.

2:30pm **Workshop: Gregory Korchemsky - SCGP 102**

Speaker: Gregory Korchemsky

Title: Tracy-Widom distribution in supersymmetric gauge theories

Abstract: It has recently been recognized that various observables in different four-dimensional supersymmetric gauge theories can be computed as determinants of certain semi-infinite matrices. It turns out that these determinants are closely related to the celebrated Tracy-Widom distribution, specifically its finite temperature generalization, which describes level-spacing distributions in matrix models. This relationship enables the computation of observables for any 't Hooft coupling.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Jianrong Li - SCGP 102**

Speaker: Jianrong Li

Title: Quantum affine algebras and their applications to scattering amplitudes

Abstract: Quantum affine algebra $U_q(\widehat{\mathfrak{g}})$ is a Hopf algebra that is a q -deformation of the universal enveloping algebra of an affine Lie algebra $\widehat{\mathfrak{g}}$. Hernandez and Leclerc in 2010 introduced a certain subcategory C_{ℓ} of the category of finite dimensional $U_q(\widehat{\mathfrak{g}})$ -modules. They proved that $K_0(C_{\ell})$ has a cluster algebra structure and in the case of $\mathfrak{g}=\mathfrak{sl}_k$, $K_0(C_{\ell})$ is isomorphic to a quotient of Grassmannian cluster algebra. In joint work with Wen Chang, Bing Duan, and Chris Fraser, we proved that the dual canonical basis of a Grassmannian cluster algebra is parametrized by semistandard Young tableaux. Using results in representations of p -adic groups and quantum affine algebras, we gave a formula to compute elements in the dual canonical basis of a Grassmannian cluster algebra. In this talk, I will talk about joint work with Nick Early about a construction of prime modules of quantum affine algebras using Newton polytopes. We apply the results of prime modules to construct u -variables for Grassmannian cluster algebras which are useful in scattering amplitudes. I will also talk about joint work with James Drummond and Ömer Gürdöğ?an about tropicalization of quasi-automorphisms of cluster algebras. Using tropicalization, we study fixed points and orbits of Chris Fraser's braid group actions on Grassmannian cluster algebras.

6:00pm **Workshop: Banquet Dinner - SCGP Cafe**

Title: Banquet Dinner

Thursday, February 29th

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Lorenzo Tancredi - SCGP 102**

Speaker: Lorenzo Tancredi

Title: Feynman integrals and Feynman Amplitudes beyond polylogarithms

Abstract: I will elaborate on the generalization of some of the concepts routinely used to make sense of analytic properties of polylogarithmic scattering amplitudes, beyond the polylogarithmic case. Among these are leading singularities and canonical integrals, the concept of transcendental weight and its connection to the expected singularities of the corresponding amplitudes and integrals. Building upon the insights provided by the differential equations method, I will attempt to highlight which concepts can be easily exported, and which ones require extension or possibly a radical redefinition.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Omer Gurdogan - SCGP 102**

Speaker: Omer Gurdogan

Title: Antipodal dualities: reading form factors backwards

Abstract: Our understanding of scattering amplitudes is in perpetual refinement where new results lead to new insights, which in turn enable even more ambitious calculations. I will report on a recent observation that certain amplitudes and form factors are mapped to each other under the antipode of the Hopf algebra of multiple polylogarithms. I will discuss the two known instances of this duality which lead to constraining relations between four- and three-particle form factors and six-particle amplitudes.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Lauren Williams - SCGP 102**

Speaker: Lauren Williams

Title: A cluster of results on amplituhedron tiles

Abstract: I will explain some results on tiles and tilings of the $m=4$ amplituhedron, illustrated by concrete examples. In particular, I'll explain how to use a chord diagram to obtain a BCFW tile, identify the facets of the tile, and build the corresponding cluster quiver. I'll also discuss a tiling of the amplituhedron that does not come from the BCFW recurrence and involves the spurion. This talk is based on joint work with Even-Zohar, Lakrec, Parisi, Sherman-Bennett, and Tessler, and will be complementary to the talk of Melissa Sherman-Bennett.

2:30pm **Workshop: Steven Charlton - SCGP 102**

Speaker: Steven Charlton

Title: Symmetries of weight 6 multiple polylogarithms, and Goncharov's depth conjecture

Abstract: As part of a programme to tackle Zagier's Polylogarithm Conjecture and understand the structure of multiple polylogarithms, Goncharov proposed an ambitious Depth Conjecture giving an exact criterion, in terms of the motivic cobracket, to determine when a linear combination of MPL's has a certain depth. In particular, this explains why all weight 2 and 3 multiple polylogarithms can be expressed via depth 1; it was also one of the main catalysts for simplifying the 2-loop 6-point remainder function $R_6^{(2)}$, and expressing it purely via classical polylogarithms. In weight 6 depth 3, Goncharov's Depth Conjecture predicts that $Li_{\{3; 1,1,1\}}(x,y,z)$ [closely related to $Li_{\{4,1,1\}}(x y z, 1/x, 1/y)$] should satisfy dilogarithm functional equations in argument, modulo terms of depth 2. Using the quadrangular polylogarithm relation, Matveikin and Rudenko showed the 5-term part of this holds, but only by working modulo the 6-fold dilogarithm symmetries $Li_{\{3; 1,1,1\}}(x,y,z) + Li_{\{3; 1,1,1\}}(1-x,y,z)$, and $Li_{\{3; 1,1,1\}}(x,y,z) + Li_{\{3; 1,1,1\}}(1/x,y,z)$ which they assumed would reduce to depth 2. I will explain how to show that $Li_{\{3; 1,1,1\}}(x,y,z)$ satisfies these 6-fold symmetries, by systematically understanding how the quadrangular polylogarithm relation degenerates to boundary components of (the compactification of) $M_{\{0,9\}}$. Together with Matveikin and Rudenko's proof of the 5-term part, this means Goncharov's Depth Conjecture holds in weight 6 depth 3. Finally, I can try to indicate some expectations and future directions for investigating the Depth Conjecture.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Stefan Weinzierl - SCGP 102**

Speaker: Stefan Weinzierl

Title: ?Trading a Calabi-Yau three-fold for a curve of genus two

Abstract: TBA

Friday, March 1st

8:30am **Workshop: Breakfast - SCGP Cafe**

Title: Breakfast

9:30am **Workshop: Vladimir Kazakov (Zoom) - SCGP 102/Zoom**

Speaker: Vladimir Kazakov (Zoom)

Title: Fishnet CFT's and integrable Feynman graphs

Abstract: After a short review of Fishnet CFT's and their integrability properties, I will speak on recent progress in this subject: Yangian symmetry of fishnet "amplitudes" and construction of the most general integrable "Loom" Fishnet CFT's, following our recent works arXiv:2212.09732 and arXiv:2304.04654. I will also consider, following the paper arXiv:2311.14608, an interesting special case of such Fishnet CFT's - the Checkerboard CFT, where we study the spectra of certain operators and the properties of "diamond" and Basso-Dixon-type Feynman diagrams.

10:30am **Workshop: Coffee Break - SCGP Cafe**

Title: Coffee Break

11:00am **Workshop: Albrecht Klemm - SCGP 102**

Speaker: Albrecht Klemm

Title: Fishnet Theories, Yangian Symmetries and Calabi-Yau volumes

Abstract: 2d Fishnet Theories exhibit Yangian integrable symmetries which constrain their amplitudes. We show that the latter are calibrated volumes of associated Calabi-Yau varieties. These volumes are given by the Griffiths non-holomorphic period bilinears. The corresponding periods are solutions of the flat Gauss-Manin connection or equivalent the Picard-Fuchs differential ideal in a special integral monodromy representation. The Picard-Fuchs differential ideal is equivalent to the semi direct product of the Yangian over the conformal algebra and the automorphism group of the underlying graph. We discuss graphs associated to hexagonal, quadratic and triangular tilings of the plane, the corresponding Calabi-Yau varieties and discuss how their geometries are related under additional symmetries of the theory. The formalism leads also to an efficient analytic evaluation of multi-parameter fishnet integrals.

12:00pm **Workshop: Lunch - SCGP Cafe**

Title: Lunch

1:15pm **Workshop: Anne Spiering - SCGP 102**

Speaker: Anne Spiering

Title: Towards solving two-loop scattering in planar $N=4$ SYM theory

Abstract: The penta-box and double-pentagon Feynman integrals, together with the double box, take on a central role in planar $N=4$ SYM theory in that they are basis elements for solving two-loop scattering. Their direct integration is complicated due to the presence of several elliptic curves. Dealing with this challenge requires alternative means of organising the computation. In the case of the double-box integral, recent progress has been made based on its known relation to the one-loop hexagon integral in 6 dimensions. I will present similar relations for the penta-box and double pentagon, and will also mention progress on finding their symbols.

2:30pm **Workshop: Jacob Bourjaily - SCGP 102**

Speaker: Jacob Bourjaily

Title: The Geometric, Algebraic, and Transcendental Structure of Perturbative QFT

Abstract: I will address the question of: what are the mathematical/functional forms of predictions made using perturbative QFT? Beyond one loop, very little about this question has been known until quite recently, in part due to the appearance of non-polylogarithmic functions (often, if not always, involving Calabi-Yau geometries) which remain to be well understood. I will review the role and importance of such geometric structures, their complicating implications for the use of differential equations to "perform" loop integration, and describe how unitarity can be used construct (dramatically) improved bases of so-called "master integrals" in diverse applications.

3:30pm **Workshop: Tea Time - SCGP Cafe**

Title: Tea Time

4:00pm **Workshop: Shruti Paranjape - SCGP 102**

Speaker: Shruti Paranjape

Title: Loops in a loop expansion

Abstract: In a paper by Arkani-Hamed, Henn and Trnka, it was shown that the amplituhedron construction of $N=4$ SYM can be recast in terms of negative geometries with a certain hierarchy of loops (closed cycles) in the space of loop momenta. Furthermore, using differential equation methods, it was possible to calculate and resum integrated expressions and obtain strong coupling results. In this talk, we provide a more general framework for the loops of loops expansion and outline a powerful method for the determination of differential forms for higher-order geometries. In particular, we will focus on the case of 1 closed cycle in loop space and select integrated results.