# Supersymmetry at TeV scales

Kiwoon Choi (KAIST)

Graduate Summer School on String Phenomenology

July (2012), Simons Center for Geometry and Physics

# Plan

- **Lectures A: Introduction to TeV scale SUSY**

  * Motivations:

    Hierarchy problem and Natural electroweak symmetry breaking

  * Supersymmetric Standard Model

    i) Basics of $4D\ N = 1$ SUSY and soft SUSY breaking

    ii) Constraints on Supersymmetric Standard Model from flavor, CP, baryon number, and/or lepton number violations

    iii) Electroweak symmetry breaking and Higgs boson mass in Supersymmetric Standard Model

    iv) Implications of a Standard Model-like 125 GeV Higgs boson

  * SUSY signatures at the LHC

    i) Missing transverse momentum, displaced vertex, ...

    ii) Implications of the recent LHC results

- **Lecture B: Theory of soft terms**

  SUSY breakdown and its mediation

  * Generic features

  * Simple examples:
    1) Gravity mediation
    2) Gauge mediation
    3) Anomaly mediation
    4) D-term contribution

  * Multiple mediation with string moduli stabilization

    KKLT, Large Volume Scenario, G2 Scenario

## Introduction to TeV scale SUSY

The standard model (SM) of particle physics has been enormously successful to explain most of the observed particle physics phenomena at energy scales below TeV.

However still there are numerous fundamental questions not answered by the SM:

* Origin of the electroweak symmetry breaking

* Dark matter, Matter-antimatter asymmetry in the Universe

* Origin of the flavor structure

* Strong CP problem, Cosmic inflation, Grand unification, Quantum gravity, ...

So we have many compelling reasons to anticipate new physics beyond the SM at high energy scales.

**Then what is the energy scale where new physics appears first?**

**Standard Model:**

Effective field theory for the strong, weak and electromagnetic forces with a priori unknown cutoff scale $\Lambda_{\text{SM}}$ which can be identified as the scale where new physics beyond the SM appears first.

In principle, $\Lambda_{\text{SM}}$ can be anywhere between TeV and $M_{\text{Planck}} \sim 10^{18}$ GeV.

**Light degrees of freedom in the model:**

- spin = 1 gauge bosons for $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry:

$$G_\mu = (8,1)_0, \;\; W_\mu = (1,3)_0, \;\; B_\mu = (1,1)_0$$

- 3-generations of spin = 1/2 quarks and leptons:

$$q_i = (3,2)_{\frac{1}{6}}, \;\; u_i^c = (\bar{3},1)_{-\frac{2}{3}}, \;\; d_i^c = (\bar{3},1)_{\frac{1}{3}},$$

$$\ell_i = (1,2)_{-\frac{1}{2}}, \;\; e_i^c = (1,1)_1 \qquad (i = 1,2,3)$$

- spin = 0 Higgs boson: $\;H = (1,2)_{\frac{1}{2}}$

**Attractive features of the SM:**

* Local masses of gauge bosons, quarks and leptons are forbidden by $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry, so can be generated only through a spontaneous symmetry breaking, which allows them naturally light compared to the cutoff scale $\Lambda_{\rm SM}$.

  (A light point-like degree of freedom is in fact something special in QFT as its mass can receive a potentially large self-energy contribution from UV physics around the cutoff scale. The only known way to make it natural is to have a symmetry which forbids non-zero mass in the symmetric limit.)

* Baryon and lepton numbers ($B$ & $L$) are good accidental symmetries of the renormalizable part of the model, which nicely explains why protons are long-lived and neutrinos are light.

* Flavor violation occurs only through the charged-current weak interactions, which nicely explains the suppression of FCNC effects in the light meson system .

**Incomplete or unattractive features of the SM:**

* Many fundamental questions not answered by the SM:

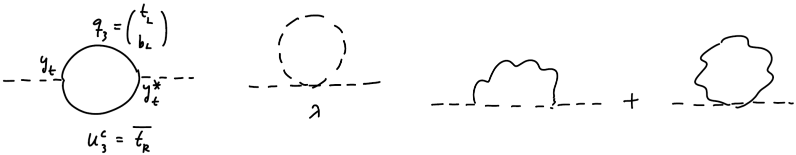
Dark matter, Matter-antimatter asymmetry, Flavor, Strong CP problem, Cosmic inflation, Grand unification, Quantum gravity, ...

* **Hierarchy problem:**

Higgs boson mass is not protected by any symmetry, so it receives a large quantum correction from UV physics around $\Lambda_{\rm SM}$:

$$\mathcal{L}_{\rm higgs} = D_\mu H^\dagger D^\mu H - m_H^2 |H|^2 - \frac{1}{4}\lambda |H|^4 + y_t H q_3 u_3^c + ...$$

$$\Rightarrow \quad \delta m_H^2 = \left[ -3y_t^2 + 3\lambda + \frac{9g_2^2 + 3g_1^2}{8} + ... \right] \frac{\Lambda_{\rm SM}^2}{16\pi^2}$$



On the other hand, we know thar

$$|m_H^2| = |m_{\text{bare}}^2 + \delta m_H^2| \sim (100\ \text{GeV})^2$$

$$\left(\delta m_H^2 = \left[-3y_t^2 + 3\lambda + \frac{9g_2^2 + 3g_1^2}{8} + ...\right] \frac{\Lambda_{\text{SM}}^2}{16\pi^2}\right)$$

So if $\Lambda_{\text{SM}} \gg 1$ TeV, we need a fine tuning of $\mathcal{O}\left((\text{TeV}/\Lambda_{\text{SM}})^2\right)$ to have the correct electroweak symmetry breaking at the weak scale.

To avoid this fine tuning, SM should be modified at scales around TeV in a way to regulate the quadratically divergent Higgs boson mass.

If it is indeed true, the 1st new physics that appear at the lowest scale is likely to be the one to regulate the top-quark-loop contribution to the Higgs boson mass.

So the hierarchy problem implies that new physics beyond the SM (BSM physics) is likely to be around the TeV scale, and **SUSY** is the prime candidate for such new physics regulating the quadratically divergent Higgs boson mass.

In fact, SUSY does not only solve the hierarchy problem, but also provide an attractive theoretical framework to address many other fundamental questions such as dark matter, baryogenesis, grand unification and quantum gravity:

* Lightest SUSY particle is a good dark matter candidate.

* Some squark or slepton fields can have a nontrivial cosmological evolution which would generate baryon or lepton asymmetry in the early Universe.

* With SUSY around the TeV scale, the three gauge couplings of $SU(3)_c \times SU(2)_L \times U(1)_Y$ are successfully unified at $M_{\rm GUT} \sim 10^{16}$ GeV.

* SUSY is an essential component of string/M theory.

## Some basics of $4D$ $N=1$ SUSY in $N=1$ superspace

* $N=1$ SUSY algebra:

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, \quad \{Q_\alpha, Q_\beta\} = 0$$

* SUSY algbra realized as a translation in superspace $(x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$

$$\delta\theta^\alpha = \xi^\alpha, \quad \delta\bar{\theta}_{\dot{\alpha}} = \bar{\xi}_{\dot{\alpha}}, \quad \delta x^\mu = i\theta\sigma^\mu\bar{\xi} - i\xi\sigma^\mu\bar{\theta}$$

$$\Rightarrow \quad iQ_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu, \quad i\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\sigma^\mu_{\beta\dot{\alpha}}\theta^\beta\partial_\mu$$

* SUSY algebra realized as a transformation of superfield:

$$\delta_{\text{SUSY}}\Omega(x,\theta,\bar{\theta}) = i\left(\xi^\alpha Q_\alpha + \bar{\xi}^{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}\right)\Omega(x,\theta,\bar{\theta})$$

$$\left(\text{Dimensional analysis: mass-dim}(Q_\alpha) = -\,\text{mass-dim}(\theta_\alpha) = \frac{1}{2}\right)$$

* SUSY-invariant operators useful to construct irreducible superfield:

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu, \quad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\sigma^\mu_{\beta\dot\alpha}\theta^\beta$$

$$\left([P_\mu, D_\alpha] = \{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, D_\beta\} = 0\right)$$

**Irreducible superfields relevant for particle physics:**

* Chiral superfield including matter fermion or Higgs boson:

$$\bar D_{\dot\alpha}\Phi = 0 \ \ (\delta_{\text{Lorentz}}\Phi = 0)$$

$$\Rightarrow \quad \Phi(x,\theta,\bar\theta) \ = \ \Phi(y,\theta) \ = \ \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

$$(y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta)$$

* Vector (or Real) superfield including gauge boson:

$$V = V^* \ \ (\delta_{\text{Lorentz}} V = 0)$$

$$\Rightarrow \quad V = \Phi + \Phi^* + V_{\text{WZ}}$$

$$V_{\text{WZ}} = -\theta\sigma^\mu\bar\theta A_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\theta\theta\bar\theta\bar\theta\, D$$

## Supersymmetric gauge theory

* Vector superfields $V = V^a T_a$ ($[T_a, T_b] = if_{abc} T_c$), for gauge bosons and chiral superfields $\Phi$ for charged matter, which transform as

$$\delta\Phi(y,\theta) = i\epsilon(y,\theta)\Phi(y,\theta), \quad e^{V+\delta V} = e^{-i\epsilon^\dagger} e^V e^{i\epsilon}$$

under infinitesimal gauge transformation parametrized by chiral superfield $\epsilon(y,\theta) = \epsilon^a(y,\theta)T_a$.

* Supersymmetric action:

As SUSY corresponds to a translation in superspace, super-translation invariant integral of gauge-invariant Lorentz scalar superfield is invariant under SUSY, Poincare and gauge transformations.

$$S = \int d^4x\, d^2\theta d^2\bar\theta\, K(x,\theta,\bar\theta) + \int d^4x\, d^2\theta\, \Gamma(y,\theta) + \text{c.c}$$

$$K = \text{generic real gauge-invariant superfield}$$

$$\Gamma = \text{generic gauge-invariant chiral superfield}$$

**Supersymmetric lagrangian of gauge and charged matter fields:**

$$\mathcal{L}_{\rm SUSY} = \int d^2\theta d^2\bar{\theta}\, K(\Phi, \Phi^*, V) + \left(\int d^2\theta\, \Gamma(\Phi, \mathcal{W}) + {\rm c.c}\right)$$

$$\begin{aligned}
K &= Z_{\bar{I}J}\Phi_I^* e^{-V}\Phi_J + \ldots = \text{Real Kähler potential} \\
\Gamma &= \frac{1}{4} f_a \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + W(\Phi) + \ldots \\
W &= \frac{1}{2}\mu_{IJ}\Phi_I\Phi_J + \frac{1}{6}y_{IJK}\Phi_I\Phi_J\Phi_K + .. = \text{Holomorphic superpotential} \\
f_a &= \frac{1}{g_a^2} + i\frac{\theta_{\rm vac}}{8\pi^2} = \text{Holomorphic gauge coupling}
\end{aligned}$$

Chiral field strength superfield: $\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2(e^{-V}D_\alpha e^V) = \mathcal{W}^a_\alpha T_a$

$$\mathcal{W}^a_\alpha = -i\lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F^a_{\mu\nu} + \theta^2\sigma^\mu D_\mu\bar{\lambda}^a_\alpha$$

* Rules of Grassmann integration:

$$\int d\theta f(\theta) = \frac{\partial}{\partial\theta}f(\theta), \quad \int d\theta\, \frac{\partial}{\partial\theta}f(\theta) = 0 \quad \Rightarrow \quad \int d\theta\, \theta = 1, \quad \int d\theta = 0$$

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta_\alpha = -\frac{1}{2}d\theta_1 d\theta_2, \quad \theta^2 = \theta^\alpha\theta_\alpha = 2\theta_1\theta_2 \quad \Rightarrow \quad \int d^2\theta\, \theta^2 = 1$$

$$\Rightarrow \quad \mathcal{L}_{\rm SUSY} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Yukawa} - V + \text{higher dimensional operators}$$

$$\begin{aligned}
\mathcal{L}_{\rm kinetic} &= -\frac{1}{4g_a^2} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i}{g_a^2} \bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a + Z_{\bar{I}J} \left( D_\mu \phi^{I*} D^\mu \phi^J - i\bar{\psi}^I \bar{\sigma}^\mu D_\mu \psi^J \right) \\
\mathcal{L}_{\rm Yukawa} &= i\sqrt{2} Z_{\bar{I}J} \phi^{I*} T^a \psi^J \lambda^a - \frac{1}{2} \partial_I \partial_J W \psi^I \psi^J + \text{c.c} \\
V &= V_F + V_D \; = \; Z_{\bar{I}J} F^{I*} F^J + \frac{1}{2} g_a^2 D_a^2
\end{aligned}$$

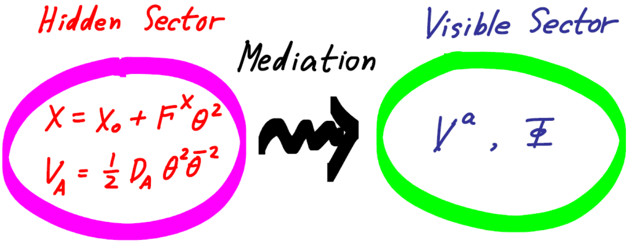
$$\left( F^I = -Z^{I\bar{J}} \partial_{\bar{J}} W^*, \quad D^a = \left. \frac{\partial K}{\partial V_a} \right|_{V=0} = Z_{\bar{I}J} \phi^{I*} T^a \phi^J \right)$$

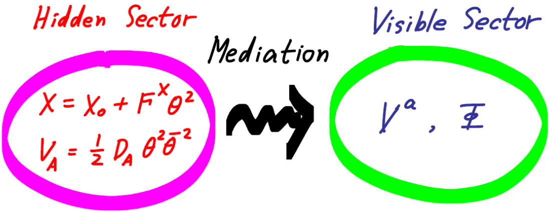
As we all know, none of the superpartners of the known particles is discovered yet, so SUSY must be a broken symmetry.

Typically SUSY is spontaneously broken by Poincare-invariant $\theta$-dependent vacuum values of chiral and/or vector superfields in a hidden sector: (Recall that SUSY is a translation in Grassmann coordinate direction.)

$$\langle X \rangle = X_0 + F^X \theta^2, \qquad \langle V_A \rangle = \frac{1}{2} D_A \theta^2 \bar{\theta}^2$$

Then this SUSY breaking is transmitted to the visible sector:






whose low energy consequences can be encoded in the effective local interactions between the visible sector gauge and matter fields and the SUSY breaking fields, e.g.

$$\int d^4\theta \left( \frac{X^* X}{M_{\rm mess}^2} + \frac{X}{M_{\rm mess}} + \frac{X^*}{M_{\rm mess}} + V_A \right) \Phi^* \Phi$$

$$+ \quad \int d^2\theta \frac{X}{M_{\rm mess}} \left( \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + \mu' \Phi^2 + y' \Phi^3 \right) + {\rm c.c}$$

In this setup, **mediation mechanism** refers to the underlying physics to generate these effective interactions at a mass scale $\mathbf{M_{mess}}$ which is called **the messenger scale**.

* Generic supersymmetric lagrangian:

$$\begin{aligned}\mathcal{L}_{\rm SUSY} &= \int d^2\theta d^2\bar{\theta}\left(Z_{\bar{I}J}\Phi_I^* e^{-V}\Phi_J + ...\right)\\ &+ \left[\int d^2\theta\,\left(\frac{1}{4}f_a\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + \frac{1}{2}\mu_{IJ}\Phi_I\Phi_J + \frac{1}{6}y_{IJK}\Phi_I\Phi_J\Phi_K + ...\right) + \text{c.c}\right]\end{aligned}$$

* Effective interactions generated by mediation mechanism:

$$\begin{aligned}&\int d^4\theta\,\left(\frac{X^*X}{M^2_{\rm mess}} + \frac{X}{M_{\rm mess}} + \frac{X^*}{M_{\rm mess}} + V_A\right)\Phi^*\Phi\\ &+ \int d^2\theta\frac{X}{M_{\rm mess}}\left(\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + \mu'\Phi^2 + y'\Phi^3\right) + \text{c.c}\end{aligned}$$

Including the effective local interactions between SUSY breaking fields and the visible sector fields, the supersymmetric couplings $Z_{\bar{I}J}$, $f_a$, $\mu_{IJ}$, $y_{IJK}$ of the visible sector fields can be regarded as $X$ and/or $V_A$-dependent superfields, e.g.

$$Z_{\bar{I}J} = \delta_{\bar{I}J} \;\;\rightarrow\;\; Z_{\bar{I}J}(X, X^*, V_A) = \delta_{\bar{I}J} + \frac{XX^*}{M^2_{\rm mess}} + \frac{X}{M_{\rm mess}} + \frac{X^*}{M_{\rm mess}} + V_A$$

$$f_a = \frac{1}{g_a^2} + i\frac{\theta_{\rm vac}}{8\pi^2} \;\;\rightarrow\;\; f_a(X) = \frac{1}{g_a^2} + i\frac{\theta_{\rm vac}}{8\pi^2} + \frac{X}{M_{\rm mess}}$$

After replacing $X$ and $V_A$ with their vacuum values, SUSY appears to be explicitly but softly broken by the $\theta$-dependent couplings.

Whatever it is the underlying mediation mechanism, its low energy consequence can be parameterized by the following set of **soft SUSY breaking parameters:**

$$
\begin{aligned}
Z_{\bar{I}J}(\langle X\rangle, \langle X^*\rangle, \langle V_A\rangle) &= \delta_{\bar{I}J} - \Delta A_{\bar{I}J}\theta^2 - \Delta A^*_{I\bar{J}}\bar{\theta}^2 - m^2_{\bar{I}J}\theta^2\bar{\theta}^2, \\
f_a(\langle X\rangle) &= \frac{1}{g_a^2}\left(1 - M_a\theta^2\right), \quad \mu_{IJ}(\langle X\rangle) = \mu_{IJ}\left(1 - B_{IJ}\theta^2\right), \\
y_{IJK}(\langle X\rangle) &= y_{IJK}\left(1 - A_{IJK}\theta^2\right)
\end{aligned}
$$

Note: i) $Z_{\bar{I}J}$ = real superfields,

$\{f_a, \mu_{IJ}, y_{IJK}\}$ = chiral superfields

ii) $\Delta A_{\bar{I}J}$ can be rotated away by a holomorphic field redefinition $\Phi^I \to \Phi^I + \Delta A_{\bar{I}J}\theta^2\Phi^J$.

iii) Once the mediation mechanism of SUSY breaking is identified, the soft parameters at $M_{\text{mess}}$ can be computed.

* Generic renormalizable theory with softly broken SUSY:

$$\int d^4\theta(\delta_{\bar{I}J} - m^2_{\bar{I}J}\theta^2\bar{\theta}^2)\Phi^{I*}e^{-V}\Phi^J - \left(\int d^2\theta\, \frac{1}{4g_a^2}\left(1 - M_a\theta^2\right)\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + \text{c.c}\right)$$
$$+ \left(\int d^2\theta\, \frac{\mu_{IJ}}{2}(1 - B_{IJ}\theta^2)\Phi^I\Phi^J + \frac{y_{IJK}}{6}(1 - A_{IJK}\theta^2)\Phi^I\Phi^J\Phi^K + \text{c.c}\right)$$

$$\Rightarrow \;\mathcal{L}_{\text{soft}} = -m^2_{\bar{I}J}\phi^{I*}\phi^J - \left(\frac{1}{2}M_a\lambda^a\lambda^a + \text{c.c}\right) - \left(\frac{1}{2}b_{IJ}\phi^I\phi^J + \frac{1}{6}a_{IJK}\phi^I\phi^J\phi^K + \text{c.c}\right)$$

$$\left(b_{IJ} = \mu_{IJ}B_{IJ}, \quad a_{IJK} = y_{IJK}A_{IJK}\right)$$

So we have four-types of SUSY breaking soft masses parametrizing the low energy consequences of SUSY breaking in a hidden sector, which is transmitted to the visible sector by certain mediation mechanism:

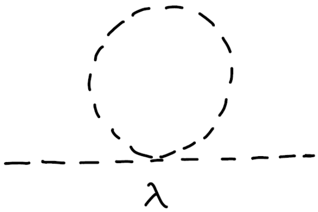
Soft scalar masses $= m^2_{\bar{I}J}$, Gaugino masses $= M_a$,

$B$-parameters $= b_{IJ} \equiv \mu_{IJ}B_{IJ}$, $A$-parameters $= a_{IJK} \equiv y_{IJK}A_{IJK}$

**SUSY eliminates the quadratic divergence in scalar boson mass.**

* Simple scalar theory:

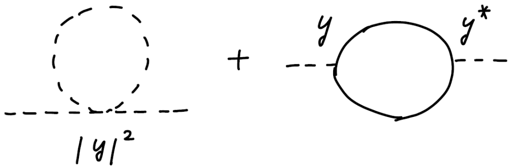
$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \mu^2 |\phi|^2 - \frac{\lambda}{4} |\phi|^4$$




$$\Rightarrow \quad \delta\mu^2 = \frac{\lambda}{16\pi^2}\Lambda^2 + \mathcal{O}\left(\frac{\mu^2 \ln \Lambda}{16\pi^2}\right)$$

* SUSY:

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\, \Phi^*\Phi + \left[\int d^2\theta\, \left(\frac{\mu}{2}\Phi^2 + \frac{y}{6}\Phi^3\right) + \text{c.c}\right] \\ &= \partial_\mu\phi^*\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - \left|\mu\phi + \frac{y}{2}\phi^2\right|^2 + \left[\frac{1}{2}\left(\mu + y\phi\right)\psi\psi + \text{c.c}\right]\end{aligned}$$




$$\Rightarrow \quad \delta\mu^2 = \left(\frac{y^2}{16\pi^2}\Lambda^2 - \frac{y^2}{16\pi^2}\Lambda^2\right) + \mathcal{O}\left(\frac{\mu^2\ln\Lambda}{16\pi^2}\right) = \mathcal{O}\left(\frac{\mu^2\ln\Lambda}{16\pi^2}\right)$$

This cancellation of quadratic divergence is not an artifact of one-loop approximation, but a consequence of symmetries, so should be valid even when higher order quantum corrections are taken into account.

**Symmetries and selection rules in SUSY model:**

$$\mathcal{L} = \int d^4\theta\, \Phi^*\Phi + \left[\int d^2\theta \left(\frac{\mu}{2}\Phi^2 + \frac{y}{6}\Phi^3\right) + \text{c.c}\right]$$

i) SUSY: $m_\phi = m_\psi = \mu$

ii) Chiral (spurion) symmetry for superfield $\Phi$:

$$U(1)_\Phi : \Phi \to e^{i\alpha}\Phi, \quad \mu \to e^{-2i\alpha}\mu, \quad y \to e^{-3i\alpha}y, \quad \Lambda \to \Lambda$$

$U(1)_\Phi$-selection rule (= $U(1)_\Phi$-covariance) assures that there can not be any power-law divergent radiative correction to the mass parameter $\mu$, while there can be logarithmically divergent one:

$$16\pi^2\delta\mu \sim 0 \times \Lambda + \mu \ln \Lambda,$$

so $\mu$ can be hierarchically lighter than the cutoff scale without fine tuning.

**Note:** Here we are considering the canonically normalized mass $\mu$, not a holomorphic mass, so we do not require $\delta\mu$ to be a holomorphic function. For holomorphic $\mu$, symmetry and selection rules (or non-renormalization theorem for superpotential) assure that even the coefficient of $\ln \Lambda$ is zero.

**UV divergence structure of mass parameters in softly broken SUSY**

$$\int d^4\theta(1-m_0^2\theta^2\bar{\theta}^2)\Phi^* e^{q_\Phi V}\Phi - \frac{1}{4}\int d^2\theta\left(\frac{1}{g_a^2}-M_a\theta^2\right)\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha$$
$$+\int d^2\theta\left[\frac{\mu}{2}(1-B\theta^2)\Phi^2+\frac{y}{6}(1-A\theta^2)\Phi^3\right]$$

* Symmetries and selection rules:

i) SUSY: Invariant masses $= \{\Lambda,\, \mu\}$,
SUSY-breaking masses $= \{m_0^2,\, b=\mu B,\, a_y = yA,\, M_a\}$

ii) $U(1)_\Phi$ : $\Phi \to e^{i\alpha}\Phi,\;\; \Lambda\to\Lambda,\;\; \mu\to e^{-2i\alpha}\mu,\;\; y\to e^{-3i\alpha}y, \ldots$

iii) $U(1)_R$ : $\theta^2 \to e^{i\beta}\theta^2,\;\; \Lambda\to\Lambda,\;\; (M_a,\, b,\, a_y)\to e^{-i\beta}(M_a,\, b,\, a_y), \ldots$

* Pattern of radiative corrections to mass parameters (= parameter mixing through the RG evolution) consistent with symmetries and selection rules:

$$\begin{aligned}
16\pi^2\delta\mu &\sim \mu\ln\Lambda \\
16\pi^2\delta M_a &\sim M_a\ln\Lambda, \\
16\pi^2\delta a_y &\sim (a_y + yM_a)\ln\Lambda \\
16\pi^2\delta b &\sim (b+\mu y^* a_y + \mu M_a)\ln\Lambda \\
16\pi^2\delta m_0^2 &\sim \left(m_0^2 + |a_y|^2 + |M_a|^2\right)\ln\Lambda
\end{aligned}$$

$$
\begin{aligned}
16\pi^2 \delta\mu &\sim \mu \ln \Lambda \\
16\pi^2 \delta M_a &\sim M_a \ln \Lambda \\
16\pi^2 \delta a_y &\sim (a_y + y M_a) \ln \Lambda \\
16\pi^2 \delta b &\sim (b + \mu y^* a_y + \mu M_a) \ln \Lambda \\
16\pi^2 \delta m_0^2 &\sim \left(m_0^2 + |a_y|^2 + |M_a|^2\right) \ln \Lambda
\end{aligned}
$$

i) No power-law divergence, so all mass parameters can be hierarchically lighter than the cutoff scale without fine tuning:

$$\mu,\ m_{\rm soft} = \{M_a,\, A \equiv a_y/y,\, B \equiv b/\mu,\, m_0\} \ll \Lambda$$

ii) The RG mixing between the four type of SUSY breaking soft masses $m_{\rm soft} = \{M_a,\, A \equiv a_y/y,\, B \equiv b/\mu,\, m_0\}$ often limits the possible hierarchical structure of soft masses, so can have interesting phenomenological implications as we will discuss later.

# Minimal Supersymmetric Standard Model (MSSM)

## Field contents:

- $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge multiplets:

$$V_a = -\theta\sigma^\mu\bar{\theta}A^a_\mu + i\theta\theta\bar{\theta}\bar{\lambda}^a - i\bar{\theta}\bar{\theta}\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2 D^a$$

$$(A^a_\mu, \lambda_a) = (G_\mu, \tilde{g}),\ (W_\mu, \tilde{W}),\ (B_\mu, \tilde{B})$$

- 3 generations of quark and lepton multiplets:

$$\Phi^I = \phi^I + \sqrt{2}\theta\psi^I + \theta^2 F^I \equiv (\phi^I, \psi^I)$$

$$Q_i = (\tilde{q}_i, q_i), \quad U^c_i = (\tilde{u}^c_i, u^c_i), \quad D^c_i = (\tilde{d}^c_i, d^c_i),$$
$$L_i = (\tilde{\ell}_i, \ell_i), \quad E^c_i = (\tilde{e}^c_i, e^c_i) = (1,1)_1$$

- Higgs multiplets:

$$H_u = (H_u, \tilde{H}_u) = (1,2)_{\frac{1}{2}}, \quad H_d = (H_d, \tilde{H}_d) = (1,2)_{-\frac{1}{2}}$$

**Lagrangian:**

$$\int d^4\theta \, Z_{\bar{I}J} \Phi^{I*} e^V \Phi^J + \left[ \int d^2\theta \left( \frac{1}{4} f_a \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + W \right) + \text{c.c} \right]$$

$$Z_{\bar{I}J} = \delta_{\bar{I}J} - m^2_{\bar{I}J} \theta^2 \bar{\theta}^2, \qquad f_a = \frac{1}{g_a^2} \left(1 - M_a \theta^2\right)$$

$$W = W_{\rm MSSM} + \Delta W$$

$$\begin{aligned} W_{\rm MSSM} &= \mu(1 - B\theta^2) H_u H_d + y^u_{ij}(1 - A^u_{ij}\theta^2) H_u Q_i U^c_j \\ &\quad + y^d_{ij}(1 - A^d_{ij}\theta^2) H_d Q_i D^c_j + y^\ell_{ij}(1 - A^\ell_{ij}\theta^2) H_d L_i E^c_j \end{aligned}$$

$$\begin{aligned} \Delta W &= \mu'_i L_i H_u + \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k \\ &\quad + \frac{\gamma_{ijkl}}{M_{\rm Planck}} Q_i Q_j Q_k L_l + \frac{\gamma'_{ijkl}}{M_{\rm Planck}} U^c_i U^c_j D^c_k E^c_l + \ldots \end{aligned}$$

$W_{\rm MSSM} =$ $B$ and $L$ conserving renormalizable superpotential including the associated soft SUSY breaking terms

$\Delta W =$ Potentially dangerous $B$ and/or $L$ violating superpotential

Compared to the SM, the MSSM includes bunch of new interactions which can induce dangerous **flavor, CP, $B$ or $L$ violating processes**, which are severely constrained by low energy data.

$$
\begin{aligned}
\mathcal{L}_{\rm soft} \;=\;& -\frac{1}{2}\left(M_{\tilde{g}}\tilde{g}\tilde{g} + M_{\tilde{W}}\tilde{W}\tilde{W} + M_{\tilde{B}}\tilde{B}\tilde{B} + \text{c.c.}\right) \\
& - (B\mu H_u H_d + \text{c.c.}) - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 \\
& - (m_{\tilde{q}}^2)_{ij}\tilde{q}_i^*\tilde{q}_j - (m_{\tilde{u}}^2)_{ij}\tilde{u}_i^{c*}\tilde{u}_j^c - (m_{\tilde{d}}^2)_{ij}\tilde{d}_i^{c*}\tilde{d}_j^c \\
& - (m_{\tilde{\ell}}^2)_{ij}\tilde{\ell}_i^*\tilde{\ell}_j - (m_{\tilde{e}}^2)_{ij}\tilde{e}_i^{c*}\tilde{e}_j^c \\
& - \left(A_{ij}^u y_{ij}^u H_u \tilde{q}_i \tilde{u}_j^c + A_{ij}^d y_{ij}^d H_d \tilde{q}_i \tilde{d}_j^c + A_{ij}^\ell y_{ij}^\ell H_d \tilde{\ell}_i \tilde{e}_j^c + \text{c.c.}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta W \;=\;& \mu_i' L_i H_u + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \\
& + \frac{\gamma_{ijkl}}{M_{\rm Planck}} Q_i Q_j Q_k L_l + \frac{\gamma'_{ijkl}}{M_{\rm Planck}} U_i^c U_j^c D_k^c E_l^c + \ldots
\end{aligned}
$$

The price for solving the hierarchy problem with SUSY is not so cheap!

We lose the two nice features of the SM: (i) automatic $B/L$ conservation at renormalizable level, and (ii) GIM suppression of flavor violation.

This might not be a problem, but an opportunity to understand the underlying physics as SUSY model have quite different phenomenological features depending upon how to make the model compatible with the constraints from $B$, $L$ and/or flavor violations.

**Constraints from B/L violation**

$$\begin{aligned}\Delta W &= \mu'_i L_i H_u + \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \lambda''_{ijk} U^c_i D^c_j D^c_k \\ &\quad + \frac{\gamma_{ijkl}}{M_{\rm Planck}} Q_i Q_j Q_k L_l + \frac{\gamma'_{ijkl}}{M_{\rm Planck}} U^c_i U^c_j D^c_k E^c_l + \ldots\end{aligned}$$

$$\begin{array}{ccccccccc} & & \mu' & \lambda & \lambda' & \lambda'' & \gamma & \gamma' \\ (|\Delta B|, |\Delta L|) & = & (0,1) & (0,1) & (0,1) & (1,0) & (1,1) & (1,1)\end{array}$$

* proton decay: $|\Delta B| = |\Delta L| = 1$

  $\Rightarrow$ Constraints on $(\frac{\mu'}{\mu} + \lambda + \lambda') \times \lambda''$, $\quad \gamma$, $\quad \gamma'$

* neutrino masses: $|\Delta L| = 2$

  $\Rightarrow$ Constraints on $(\frac{\mu'}{\mu} + \lambda + \lambda') \times (\frac{\mu'}{\mu} + \lambda + \lambda')$

* $n$-$\bar{n}$ oscillation: $|\Delta B| = 2$

  $\Rightarrow$ Constraints on $\lambda'' \times \lambda''$

**Constraints from proton decay:**

$$\frac{\mu_i'}{\mu}\lambda''_{112} \lesssim 10^{-21}\left(\frac{m_{\rm soft}}{\rm TeV}\right)^2 \ (i=1,2),$$
$$\lambda'_{i1k}\lambda''_{11k} \lesssim 10^{-24}\left(\frac{m_{\rm soft}}{\rm TeV}\right)^2 \ (i=1,2),$$
$$\lambda_{33i}\lambda''_{112} \lesssim (10^{-16}-10^{-21})\left(\frac{m_{\rm soft}}{\rm TeV}\right)^2 \ (i=1,2,3)$$
$$\gamma_{112i} \lesssim 10^{-8}\left(\frac{m_{\rm soft}}{\rm TeV}\right) \ (i=1,2,3),$$
$$\gamma'_{12ij} \lesssim 10^{-7}\left(\frac{m_{\rm soft}}{\rm TeV}\right) \ (i,j=1,2),$$

so we need some symmetry to suppress these B/L violating couplings!

**Symmetries to suppress the B/L violating couplings:**

## A. Symmetry involving an exact $R$-parity $= (-1)^{3(B-L)} e^{2\pi i J_z}$:

A-1) Matter parity $Z_2 = (-1)^{3(B-L)}$

$$\Rightarrow \quad \mu' = \lambda = \lambda' = \lambda'' = 0$$

(But matter parity alone does not explain why $\gamma$ and $\gamma'$ are so small.)

A-2) Proton hexality $Z_6 = (-1)^{2B}(-1)^{3(B-L)}$

$$\Rightarrow \quad \mu' = \lambda = \lambda' = \lambda'' = \gamma = \gamma' = 0$$

## B. Symmetry not involving an exact $R$-parity:

B-1) Baryon triality $Z_3 = (-1)^{2B}$

$$\Rightarrow \quad \lambda'' = \gamma = \gamma' = 0$$

(Still need to explain why $\mu'/\mu$, $\lambda$ and $\lambda'$ are small.)

B-2) Spontaneoulsy broken discrete $R$-symmetry

$\Rightarrow$ $B$ or $L$ violating couplings $\propto (m_{3/2}/M_{\rm Planck})^{\Delta}$

($\Delta = R$-charge dependent rational numbers)

**Why $R$-parity ($\equiv$ matter parity) is special?**

* All known ordinary particles $\ni$ quarks, leptons, gauge bosons, Higgs bosons, gravition (also axion if exists): Even under $R$-parity

* All superpartners $\ni$ squarks, sleptons, gauginos, Higgsinos, gravitino, (axino): Odd under $R$-parity

$\Rightarrow$ If $R$-parity is an exact symmetry, the lightest superpartner (LSP) is stable, which has important implications for cosmology, e.g. LSP as dark matter, and also for SUSY signatures at colliders, e.g. missing energy carried away by invisible LSPs.

**Constraints from flavor or CP violations**

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} &= -\frac{1}{2}\left(M_{\tilde{g}}\tilde{g}\tilde{g} + M_{\tilde{W}}\tilde{W}\tilde{W} + M_{\tilde{B}}\tilde{B}\tilde{B} + \text{c.c.}\right) \\
&\quad - \left(B\mu H_u H_d + \text{c.c.}\right) - m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2 \\
&\quad - (m_{\tilde{q}}^2)_{ij}\tilde{q}_i^*\tilde{q}_j - (m_{\tilde{u}}^2)_{ij}\tilde{u}_i^{c*}\tilde{u}_j^c - (m_{\tilde{d}}^2)_{ij}\tilde{d}_i^{c*}\tilde{d}_j^c \\
&\quad - (m_{\tilde{\ell}}^2)_{ij}\tilde{\ell}_i^*\tilde{\ell}_j - (m_{\tilde{e}}^2)_{ij}\tilde{e}_i^{c*}\tilde{e}_j^c \\
&\quad - \left(A_{ij}^u y_{ij}^u H_u\tilde{q}_i\tilde{u}_j^c + A_{ij}^d y_{ij}^d H_d\tilde{q}_i\tilde{d}_j^c + A_{ij}^\ell y_{ij}^\ell H_d\tilde{\ell}_i\tilde{e}_j^c + \text{c.c.}\right)
\end{aligned}
$$

Since the present data implies that flavor violation beyond the SM should be quite suppressed, it is convenient to decompose flavor-violating soft masses into two parts:

soft mass = flavor-universal part + flavor-non-universal part

$$
\begin{aligned}
\left(m_\phi^2\right)_{ij} &= m_\phi^2\delta_{ij} + \left(\Delta m_\phi^2\right)_{ij} \quad \left(\phi = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{\ell}, \tilde{e}\right) \\
A_{ij}^x &= A_x + \Delta A_{ij}^x \quad \left(x = u, d, \ell\right)
\end{aligned}
$$

Note: This decomposition is not unique, and is just for an order of magnitude estimate of the flavor constraints.

**Some bounds on soft masses at TeV scale from FCNC and CPV:**

- $K$-$\bar{K}$ mass difference and $\epsilon_K$ :

$$\sqrt{\left(\mathrm{Re},\mathrm{Im}\right)\left(\frac{(\Delta m^2_{\tilde{q}})_{12}(\Delta m^2_{\tilde{d}})_{12}}{m^2_{\tilde{q}}m^2_{\tilde{d}}}\right)} \leq \left(5\times 10^{-3},\, 4\times 10^{-4}\right)\left(\frac{m_{\tilde{q},\tilde{d}}}{1\ \mathrm{TeV}}\right)$$

- $\mu \to e\gamma$:

$$\frac{(\Delta A^{\ell})_{12}}{m_{\tilde{\ell}}} \leq 2\times 10^{-2}\left(\frac{m_{\tilde{\ell}}}{100\ \mathrm{GeV}}\right)\left(\frac{M_{\tilde{W}}}{100\ \mathrm{GeV}}\right)$$

- EDMs:

$$\mathrm{Arg}\left(\frac{M_a}{M_b},\frac{M_a}{A_x},\frac{M_a}{B}\right) \leq \left(10^{-2}-10^{-3}\right)\times\left(\frac{m_{\tilde{q},\tilde{\ell}}}{100\ \mathrm{GeV}}\right)^2$$

$\left(m_{\tilde{q},\tilde{d},\tilde{\ell}} = \text{1st or 2nd generation squark and slepton masses}\right)$

Flavor and CP constraints imply

**A. Universality:** Soft masses are nearly flavor-universal (at least for the 1st and 2nd generations) and CP conserving:

$$\frac{\Delta m_\phi^2}{m_\phi^2}, \ \frac{\Delta A^x}{m_\phi}, \ \mathrm{Arg}\left(\frac{M_a}{M_b}, \frac{M_a}{A_x}, \frac{M_a}{B}\right) \ \text{are small enough}$$

**or**

**B. Decoupling:** Sfermion masses (at least for the 1st and 2nd generations) are heavy enough:

$$m_{\tilde{q},\tilde{d}} \gtrsim \mathcal{O}(10-100)\ \text{TeV}, \quad m_{\tilde{\ell}} \gtrsim \text{few TeV}$$

**Is it natural to have such particular structure of soft masses?**

* Is it stable against RG evolution?

* Do heavy sfermions cause a fine tuning in the electroweak symmetry breaking?

For the option A of flavor-universal soft masses, the approximate $U(2)$-flavor symmetries for the 1st and 2nd generations:

$$U(2)^5 \equiv U(2)_Q \times U(2)_{U^c} \times U(2)_{D^c} \times U(2)_L \times U(2)_{E^c},$$

which result from small Yukawa couplings $y^{u,d,\ell}$ of the 1st and 2nd generations, assure that a flavor-universal pattern of soft masses of the 1st and 2nd generations is stable against the RG evolution.

In other words, if the bare soft masses at the messenger scale are $U(2)^5$-invariant, the resulting low energy soft masses at TeV scales are nearly $U(2)^5$-invariant also, and therefore $\Delta m_\phi^2$ and $\Delta A^x$ at TeV scale are small enough.

Then, the next step is to search for a mediation of SUSY breaking giving flavor-universal and CP-conserving soft masses at the messenger scale.

As we will see, if we wish to have electroweak symmetry breaking without severe fine tuning worse than $\mathcal{O}(1)$ %, we need $m_{\tilde{t}}$ not heavier than 1 TeV.

Then, the option B of heavy 1st and 2nd generation sfermions means an inverted sfermion mass spectrum at TeV scale:

$$m_{\tilde{q}} \gtrsim \mathcal{O}(10-100)\ \text{TeV}, \quad m_{\tilde{t}} \lesssim 1\ \text{TeV},$$

$$\left(m_{\tilde{q}} = \text{1st and 2nd generation squark masses}\right)$$

and one needs to check if this inverted hierarchy of sfermion masses at TeV scale can be achieved without a severe fine tuning of soft masses at the messenger scale.

Let us first examine the effects of the RG evolution of relevant soft masses on the electroweak symmetry breaking (EWSB) in the MSSM.

We first recall the fine tuning problem of the EWSB in the SM.

$$V_{\rm SM} = m_H^2 |H|^2 + \frac{\lambda}{4}|H|^4$$

$$\begin{aligned}
\Rightarrow \quad \frac{M_Z^2}{2} &= \frac{g_1^2+g_2^2}{4}\langle |H|^2\rangle = -\frac{g_1^2+g_2^2}{4}\left(\frac{2m_H^2}{\lambda}\right) \\
&= -\left(\frac{g_1^2+g_2^2}{2\lambda}\right)\left(m_{H,\rm bare}^2 + \delta m_H^2\right) \\
&= -\left(\frac{g_1^2+g_2^2}{2\lambda}\right)\left[m_{H,\rm bare}^2 - \frac{\Lambda_{\rm SM}^2}{16\pi^2}\left(3y_t^2 - 3\lambda - \frac{3g_1^2+8g_2^2}{8} + ...\right)\right]
\end{aligned}$$

$\Rightarrow \quad \Lambda_{\rm SM} \lesssim \mathcal{O}(1)$ TeV to avoid a severe fine tuning

**Electroweak symmetry breaking in the MSSM:**

$$\begin{aligned} V_{\rm MSSM} &= (m_{H_u}^2+|\mu|^2)|H_u|^2+(m_{H_d}^2+|\mu|^2)|H_d|^2-(B\mu H_u H_d+\text{c.c}) \\ &+ \left(\frac{g_1^2+g_2^2}{8}\left(|H_u|^2-|H_d|^2\right)^2+\frac{g_2^2}{2}|H_u^\dagger H_d|^2\right) \end{aligned}$$

$$\frac{\partial V}{\partial H_u}=\frac{\partial V}{\partial H_d}=0$$

$$\begin{aligned} \Rightarrow \quad i)\ \frac{M_Z^2}{2} &= \frac{g_1^2+g_2^2}{2}\langle\, |H_u|^2+|H_d|^2\rangle = \frac{m_{H_d}^2-m_{H_u}^2\tan^2\beta}{\tan^2\beta-1}-|\mu|^2 \\ &\simeq -m_{H_u}^2-|\mu|^2+\frac{m_{H_d}^2}{\tan^2\beta} \qquad \left(\tan\beta=\frac{\langle|H_u|\rangle}{\langle|H_d|\rangle}\right) \end{aligned}$$

$$ii)\ \frac{2|B\mu|}{\sin 2\beta} = m_{H_u}^2+m_{H_d}^2+2|\mu|^2$$

**Some RG mixings between soft masses:**

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{H_u}^2 = 6y_t^2\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2\right) + \ldots$$

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{\tilde{t}_{L,R}}^2 = -\frac{32}{3} g_3^2 M_{\tilde{g}}^2 + \frac{8}{3\pi^2} g_3^4 m_{\tilde{q}}^2 + \ldots$$

$$y_t = y_{33}^u, \quad m_{\tilde{t}_L}^2 = (m_{\tilde{q}}^2)_{33}, \quad m_{\tilde{t}_R}^2 = (m_{\tilde{u}}^2)_{33},$$

$m_{\tilde{q}}$ = 1st and 2nd generation squark masses which are presumed to be comparable to each other

$$\Rightarrow \quad m_{H_u}^2 = m_{H_u,\text{bare}}^2 - \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{M_\text{mess}}{m_{\tilde{t}}}\right) - \frac{2y_t^2}{\pi^2}\frac{g_3^2}{4\pi^2} M_{\tilde{g}}^2 \left(\ln\left(\frac{M_\text{mess}}{M_{\tilde{g}}}\right)\right)^2 + \ldots$$

$$m_{\tilde{t}}^2 = m_{\tilde{t},\text{bare}}^2 + \frac{2g_3^2}{3\pi^2} M_{\tilde{g}}^2 \ln\left(\frac{M_\text{mess}}{M_{\tilde{g}}}\right) - \frac{1}{6}\left(\frac{g_3^2}{\pi^2}\right)^2 m_{\tilde{q}}^2 \ln\left(\frac{M_\text{mess}}{m_{\tilde{q}}}\right) + \ldots$$

$$\left( m_{\tilde{t}}^2 = \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2}, \quad m_{\phi,\text{bare}} = m_\phi(M_\text{mess}) \ \text{ for } \ M_\text{mess} = \text{ messenger scale} \right)$$

**Potential fine tuning problem in the MSSM:**

$$\frac{M_Z^2}{2} \simeq -m_{H_u}^2 - |\mu|^2 + \frac{m_{H_d}^2}{\tan^2\beta}$$

$$= \left[-m_{H_u,\text{bare}}^2 + \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{M_{\text{mess}}}{m_{\tilde{t}}}\right) + \frac{2y_t^2}{\pi^2}\frac{g_3^2}{4\pi^2} M_{\tilde{g}}^2 \left(\ln\left(\frac{M_{\text{mess}}}{M_{\tilde{g}}}\right)\right)^2 + \ldots\right]$$

$$- |\mu|^2 + \frac{m_{H_d}^2}{\tan^2\beta}$$

* EWSB is only logarithmically sensitive to $M_{\text{mess}}$, so the messenger scale (= UV cutoff scale for soft masses) can be close to $M_{\text{Planck}}$ without causing a severe fine tuning problem.

(Except for gauge mediation, most of the known mediation schemes have $M_{\text{mess}}$ close to the GUT scale or the Planck scale.)

* However EWSB in the MSSM is quadratically sensitive to $m_{\text{soft}}$, in fact most sensitive to $m_{\tilde{t}}$ and $M_{\tilde{g}}$ due to the large top-quark Yukawa coupling and QCD coupling.

As a result, if $m_{\tilde{t}}$ or $M_{\tilde{g}}$ is far above $M_Z$, EWSB requires a fine tuning of $\mathcal{O}(M_Z^2/m_{\tilde{t}}^2)$ or of $\mathcal{O}(M_Z^2/M_{\tilde{g}}^2)$. (Note: $m_{\text{soft}} \sim \Lambda_{\text{SM}}$)

## A more quantitative estimate of the naturalness condition

$$m_{\tilde{t}} \lesssim 0.5 \left(\frac{10\,\%}{\epsilon_{\rm tuning}}\right)^{1/2} \left(\frac{3}{\ln(M_{\rm mess}/m_{\tilde{t}})}\right)^{1/2} \text{ TeV}$$

$$M_{\tilde{g}} \lesssim 1.3 \left(\frac{10\,\%}{\epsilon_{\rm tuning}}\right)^{1/2} \left(\frac{3}{\ln(M_{\rm mess}/M_{\tilde{g}})}\right) \text{ TeV}$$

$$\mu \lesssim 0.2 \left(\frac{10\,\%}{\epsilon_{\rm tuning}}\right)^{1/2} \text{ TeV}$$

* **Fully natural ($\epsilon_{\rm tuning} > 10\%$) EWSB (for $M_{\rm mess} \sim M_{\rm GUT}$) if**

$$m_{\tilde{t}} \sim M_{\tilde{g}} \sim \mu \sim m_{H_u} \sim M_Z.$$

However unfortunately Nature does not take this natural scenario.

* **Not-so-unnatural EWSB with acceptable fine tuning:**

We may accept $\epsilon_{\rm tuning} = \mathcal{O}(1)\%$ for $M_{\rm mess} \sim M_{\rm GUT}$.
$\big($or $\epsilon_{\rm tuning} = \mathcal{O}(10)\%$ for $M_{\rm mess} \sim 10$ TeV$\big)$

$$\Rightarrow \quad m_{\tilde{t}} \lesssim 0.5 \text{ TeV}, \quad M_{\tilde{g}} \lesssim 1.3 \text{ TeV}, \quad \mu \lesssim 0.2 - 0.7 \text{ TeV}$$

*** Not-so-unnatural EWSB with acceptable fine tuning:**

We may accept $\epsilon_{\rm tuning} = \mathcal{O}(1)\%$ for $M_{\rm mess} \sim M_{\rm GUT}$.
$\big($or $\epsilon_{\rm tuning} = \mathcal{O}(10)\%$ for $M_{\rm mess} \sim$ 10 TeV$\big)$

$$\Rightarrow \quad m_{\tilde{t}} \lesssim 0.5 \text{ TeV}, \quad M_{\tilde{g}} \lesssim 1.3 \text{ TeV}, \quad \mu \lesssim 0.2 - 0.7 \text{ TeV}$$

In these days, people call this "Natural SUSY".

**Note:**

**i) This is not a constraint, but just a favoured range of soft masses in view of the naturalness.**

**ii) In many cases, such a light stop favored by natural EWSB is in conflict with the Higgs boson mass $m_h \simeq 125$ GeV.**

$$m_{\tilde{t}}^2 \;=\; m_{\tilde{t},\mathrm{bare}}^2 + \frac{2g_3^2}{3\pi^2} M_{\tilde{g}}^2 \ln\left(\frac{\Lambda}{M_{\tilde{g}}}\right) - \frac{1}{6}\left(\frac{g_3^2}{\pi^2}\right)^2 m_{\tilde{q}}^2 \ln\left(\frac{\Lambda}{m_{\tilde{q}}}\right) + \ldots$$

$$\big(m_{\tilde{q}} = \text{1st and 2nd generation sfermion masses}\big)$$

Again, if we wish to avoid a fine tuning worse than $\mathcal{O}(1-10)$ %,

$$m_{\tilde{q}} \lesssim \mathcal{O}(10\, M_{\tilde{g}}) \;\text{ or }\; \mathcal{O}(10\, m_{\tilde{t}})$$

This implies that **an inverted sfermion mass spectrum** with heavy 1st and 2nd generation sfermions masses,

$$m_{\tilde{q}} \;=\; \mathcal{O}(10)\ \text{TeV}, \quad m_{\tilde{t}} \sim M_{\tilde{g}} \sim 1\ \text{TeV}$$

can be achieved without causing a severe fine tuning.

Still, for any mediation scheme yielding such an inverted sfermion mass spectrum, one needs a careful examination of the low energy stop mass to make sure that it has a phenomenologically viable value.

**Higgs boson mass in the MSSM:**

MSSM Higgs sector: $H_u = (H_u^+, H_u^0),\ H_d = (H_d^0, H_d^-)$

* 3 Goldstone bosons for the longitudinal components of $W^\pm, Z$
* 2 CP-even neutral Higgs bosons
* 1 CP-odd neutral Higgs boson
* 1 charged Higgs boson

As the recent experimental hint of SM-like Higgs boson with $m_h \simeq 125$ GeV is a hot issue, here we focus on the lightest CP-even Higgs boson which behaves like the SM Higgs boson in most cases.

For simplicity, we take the limit that all Higgs bosons other than the lightest CP-even Higgs are heavy enough, and consider the effective theory of the light Higgs boson after the heavy Higgs bosons are integrated out.

$$
\begin{aligned}
V_{\rm MSSM} &= (m_{H_u}^2 + |\mu|^2)|H_u|^2 + (m_{H_d}^2 + |\mu|^2)|H_d|^2 - (B\mu H_u H_d + \text{c.c}) \\
&+ \left( \frac{g_1^2 + g_2^2}{8} \left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2}|H_u^\dagger H_d|^2 \right)
\end{aligned}
$$

$$
H_u^0 = \frac{h \sin\beta}{\sqrt{2}}, \quad H_d^0 = \frac{h\cos\beta}{\sqrt{2}} \quad \left(h = \text{light CP-even neutral Higgs boson}\right)
$$

$$
\begin{aligned}
\Rightarrow \quad V_{\rm higgs} &= -m^2 h^2 + \frac{\lambda}{16} h^4 \qquad \left(\lambda = \frac{g_1^2 + g_2^2}{2}\cos^2 2\beta\right) \\
m_h^2 &= \left.\frac{\partial^2 V}{\partial h^2}\right|_{h=\langle h\rangle} = M_Z^2 \cos^2 2\beta
\end{aligned}
$$

$$
\left(M_Z^2 = \frac{g_1^2 + g_2^2}{2}\langle\, |H_u|^2 + |H_d|^2\rangle = \frac{g_1^2 + g_2^2}{4}\langle h^2\rangle\right)
$$

In generic case, this value of $m_h$ corresponds to the upper bound on the tree level mass of the lightest CP-even Higgs boson in the MSSM.

So, at tree level, the MSSM predicts a Higgs boson lighter than $M_Z$, which has been excluded a long time ago.

But there are important radiative corrections which saves the life of the MSSM Higgs boson.

$$\Delta V = \frac{1}{64\pi^2} \mathrm{Str}\, \mathcal{M}^4 \ln\left(\frac{\mathcal{M}^2}{Q^2}\right) \qquad (Q = \text{renormalization point})$$
$$= \frac{3}{32\pi^2}\left[\sum_{i=1,2} m_{\tilde{t}_i}^4 \ln\left(\frac{m_{\tilde{t}_i}^2}{Q^2}\right) - 2m_t^4 \ln\left(\frac{m_t^2}{Q^2}\right) + \ldots\right]$$

For simplicity, let us consider the limit

$$\tan\beta \gg 1 \quad \left(\Rightarrow\ H_u^0 \simeq \frac{h}{\sqrt{2}}\right), \qquad \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2} \gg y_t A_t h \gg \frac{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|}{2}$$

in which a sizable $A_t \equiv A_{33}^u$ is helpful for raising up the Higgs boson mass:

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + y_t^2 h^2/2 & y_t A_t h/\sqrt{2} \\ y_t A_t h/\sqrt{2} & m_{\tilde{t}_R}^2 + y_t^2 h^2/2 \end{pmatrix}, \qquad m_t = y_t h/\sqrt{2}$$

$$\Rightarrow \quad m_{\tilde{t}_i}^2 \simeq m_{\tilde{t}}^2 + \frac{y_t^2 h^2}{2} \pm \frac{y_t A_t h}{\sqrt{2}} \quad \left(m_{\tilde{t}}^2 \equiv \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2}\right) \quad (i = 1, 2)$$

$$\Rightarrow \qquad \Delta V = \frac{\Delta\lambda}{16} h^4 + \ldots \qquad \left( \Delta\lambda \simeq \frac{3y_t^4}{4\pi^2} \left[ \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{A_t^4}{m_{\tilde{t}}^4} \right] \right)$$

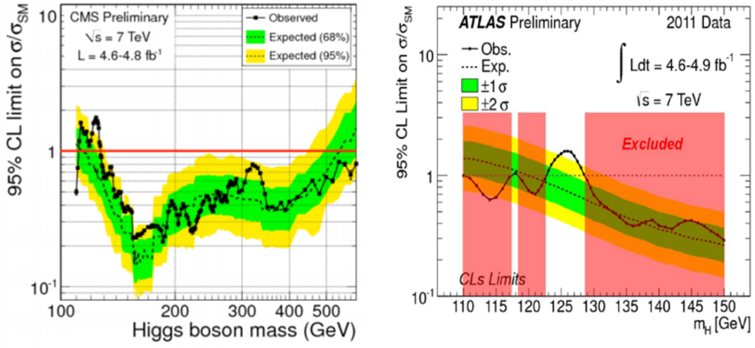
$$\Rightarrow \qquad \Delta m_h^2 \simeq \frac{3y_t^2 m_t^2}{4\pi^2} \left[ \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{A_t^4}{m_{\tilde{t}}^4} \right]$$

In more general situation including the case with small $\tan\beta$, we have

$$\left(m_h^2\right)_{\mathrm{MSSM}} \simeq M_Z^2 \cos^2 2\beta + \frac{3y_t^2 m_t^2}{4\pi^2} \left[ \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{|X_t|^4}{m_{\tilde{t}}^4} \right]$$

$$\left(X_t = A_t - \mu \cot\beta\right)$$

**Implications of SM-like Higgs boson with $m_h \simeq 125$ GeV:**



**125 GeV Higgs in the MSSM:**

$$\begin{aligned}(m_h^2)_{\rm MSSM} &\simeq M_Z^2\cos^2 2\beta + \frac{3y_t^2 m_t^2}{4\pi^2}\left[\ln\left(\frac{m_{\tilde t}^2}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde t}^2} - \frac{1}{12}\frac{|X_t|^4}{m_{\tilde t}^4}\right] \\ &= (91\ {\rm GeV})^2 + (89\ {\rm GeV})^2 = (125\ {\rm GeV})^2 \qquad (\tan\beta \gg 1) \\ &\quad \left(X_t = A_t - \mu\cot\beta\right)\end{aligned}$$

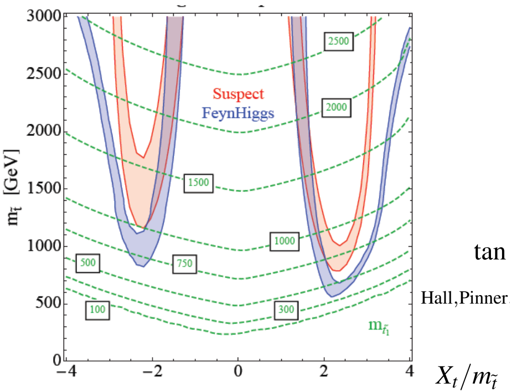
To have such large radiative correction, we need heavy stop and/or a large stop mixing $X_t = A_t - \mu\cot\beta \simeq \sqrt{6}m_{\tilde t}$!

But this is precisely what EWSB does not like: more fine tuning!
(**Little hierarchy problem**)

$$16\pi^2\frac{dm_{H_u}^2}{d\ln\Lambda} = 12y_t^2\left(m_{\tilde t}^2 + \frac{|A_t|^2}{2}\right) + \ldots$$

$$\Rightarrow \quad \sqrt{m_{\tilde t}^2 + \frac{|A_t|^2}{2}} \lesssim 0.5\left(\frac{10\,\%}{\epsilon_{\rm tuning}}\right)^{1/2}\left(\frac{3}{\ln(M_{\rm mess}/m_{\tilde t})}\right)^{1/2}\ {\rm TeV}$$

## 125 GeV Higgs in the MSSM:




$\tan\beta = 20$

Hall,Pinner,Ruderman (2012)

* Unless $|X_t/m_{\tilde{t}}| \simeq \sqrt{6}$, we need $m_{\tilde{t}} =$ few $- \mathcal{O}(10)$ TeV.

* For $M_{\rm mess} \sim M_{GUT}$ ($M_{\rm mess} \sim 10$ TeV), the required degree of fine tuning is at least $\epsilon_{\rm tuning} = \mathcal{O}(0.1)\%$ ($\mathcal{O}(1)$ %) even for nearly maximal mixing $|X_t/m_{\tilde{t}}| \simeq \sqrt{6}$, and becomes significantly worse for other values of $X_t/m_{\tilde{t}}$.

**Although a bit heavier than what we have hoped (anticipated), still $m_h \simeq 125$ GeV is not bad news for low scale SUSY.**

To see this, let the SUSY masses $\{m_\phi^2, M_a, a_y = yA, b = B\mu, \mu\}$ free from the condition of natural EWSB, but just obey the conditions for correct EWSB

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - |\mu|^2, \quad \frac{2|b|}{\sin 2\beta} = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2$$

and follow the the RG evolution:

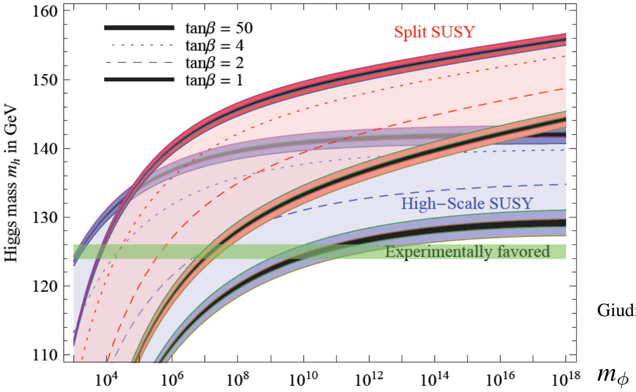
$$\begin{aligned}
16\pi^2 \frac{d\mu}{d\ln\Lambda} &\sim \mu \\
16\pi^2 \frac{dM_a}{d\ln\Lambda} &\sim M_a \\
16\pi^2 \frac{da_y}{d\ln\Lambda} &\sim (a_y + yM_a) \\
16\pi^2 \frac{db}{d\ln\Lambda} &\sim (b + \mu y^* a_y + \mu M_a) \\
16\pi^2 \frac{dm_\phi^2}{d\ln\Lambda} &\sim \left(m_\phi^2 + |a_y|^2 + |M_a|^2\right)
\end{aligned}$$

If we don't mind fine tuning, $m_\phi$ (including $m_{\tilde{t}}$) can take any value between TeV and $M_{\rm Planck}$, and we can consider the following two scenarios

$$\text{High-Scale SUSY:} \quad m_\phi \sim \sqrt{b} \sim M_a \sim A \sim \mu \gg 1 \text{ TeV}$$

$$\text{Split SUSY:} \quad m_\phi \sim \sqrt{b} \gg M_a \sim A \sim \mu \sim 1 \text{ TeV}$$

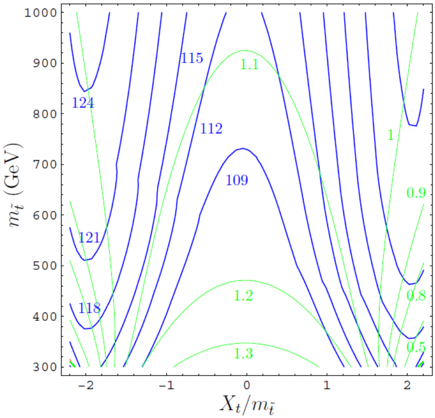
both of which are consistent with the correct EWSB and the RG evolution.




Giudice,Strumia (2011)

**Unless** $\tan\beta$ **is quite small,** $m_h \simeq 125$ **GeV implies that** $m_\phi$ **is rather close to the lower end (**$\sim$ **TeV) which is favored by the naturalness argument.**

However within the MSSM, a relatively light stop favored by natural EWSB, e.g. $m_{\tilde{t}} \lesssim 0.5$ TeV, indicates a higgs boson mass somewhat lighter than 125 GeV:




Dermisek,Low (2007)

This motivates an extension of the MSSM in the direction to have additional contribution to the Higgs boson mass other than the top-stop loops.

**Simple extensions of the MSSM gving such additional contribution to the Higgs boson mass:**

There are quite simple extensions of the MSSM which can give $m_h \simeq 125$ GeV even with sub-TeV stop mass.

* **Next to Minimal Supersymmetric Standard Model (NMSSM):**

Perhaps the simplest extension of the MSSM providing with additional Higgs boson mass contribution is the NMSSM including a singlet $S$ with

$$\Delta W = \kappa S H_u H_d$$

$$\Rightarrow \quad \Delta V = \left| \frac{\partial W}{\partial S} \right|^2 = \kappa^2 |H_u H_d|^2 = \frac{\kappa^2 \sin^2 2\beta}{16} h^4$$

$$\left( H_u^0 = \frac{h \sin \beta}{\sqrt{2}}, \quad H_d^0 = \frac{h \cos \beta}{\sqrt{2}} \right)$$

$$\Rightarrow \quad \text{Higgs quartic coupling :} \quad \lambda_{\text{NMSSM}} = \lambda_{\text{MSSM}} + \kappa^2 \sin^2 2\beta$$

$$\Rightarrow \quad (m_h^2)_{\text{NMSSM}} \simeq (m_h^2)_{\text{MSSM}} + \frac{2\kappa^2 \sin^2 2\beta}{g_1^2 + g_2^2} M_Z^2$$

Additional Higgs boson mass can be sizable in the small $\tan \beta$ limit.

*** Models with extra $U(1)$:**

Models with extra $U(1)$ gauge symmetry under which the Higgs bosons are charged can provide with additional Higgs quartic coupling through the $D$-term potential

However, we have a strong lower bound on the extra $U(1)$ gauge boson, e.g $M_{Z'} \gtrsim$ few TeVs, implying that the extra $U(1)$ vector superfiled is integrated out at the Higgs boson mass scale while leaving some effects suppressed by $1/M_{Z'}^2$:

$$\int d^4\theta \left(1 - m_{\rm soft}^2 \theta^2 \bar{\theta}^2\right) \left(-\frac{1}{2} M_{Z'}^2 V'^2 + g' V' \left(|H_u|^2 - |H_d|^2 + ...\right) + ...\right)$$

$$\Rightarrow \quad M_{Z'}^2 V' = g' \left(|H_u|^2 - |H_d|^2 + ...\right) \qquad (q'_{H_u} = -q'_{H_d})$$

$$\Rightarrow \quad \Delta\mathcal{L}_{\rm eff} = \int d^4\theta \left(1 - m_{\rm soft}^2 \theta^2 \bar{\theta}^2\right) \frac{g'^2}{2M_{Z'}^2} \left(|H_u|^2 - |H_d|^2 + ...\right)^2$$

$$\Rightarrow \quad \Delta V = \frac{g'^2 m_{\rm soft}^2}{2M_{Z'}^2} \left(|H_u|^2 - |H_d|^2\right)^2 = \frac{g'^2 m_{\rm soft}^2}{M_{Z'}^2} \frac{\cos^2 2\beta}{8} h^4$$

$$\Rightarrow \quad \Delta m_h^2 = \left(\frac{4 g'^2 m_{\rm soft}^2}{g_1^2 + g_2^2} M_{Z'}^2\right) M_Z^2 \cos^2 2\beta$$

Can be sizable in the large $\tan\beta$ limit and $M_{Z'}$ is near its lower bound.

## Summary

* SUSY at TeV scale has been introduced to avoid the fine tuning for electroweak symmetry breaking (EWSB) by regulating the quadratically divergent Higgs boson mass, so natural EWSB has been an important factor to be taken into account for SUSY model building.

* There is a significant tension between natural EWSB and the Higgs boson mass: natural EWSB favors $m_{\tilde{t}}^2 + \frac{1}{2}A_t^2$ in sub-TeV region, while $m_h \simeq 125$ GeV within the MSSM requires it to be a in multi-TeV region, and this might enforce us to abandon the idea of naturalness.

* Taking into account these point together with the constraints from B/L/flavor/CP violations, we can consider various different possibilities:

i) Natural or unnatural EWSB ?

(sub)TeV stop & gluino or multi-TeV stop & gluino

ii) Universal sfermion masses or inverted sfermion masses?

iii) Conserved $R$-parity or broken $R$-parity?

iv) MSSM Higgs or an extension to raise up the Higgs boson mass?

**Hopefully LHC will provide us with guidelines to answer these questions.**