Compactification, Model Building, and Fluxes

Angel M. Uranga
IFT, Madrid, and CERN, Geneva

Lecture 2. Model building in type IIB string theory

We review the construction of chiral four-dimensional compactifications of type IIB string theory with B-type branes. These models are mirror to type IIA compactifications with A-type branes, namely intersecting brane worlds studied in lecture 1. But it is interesting to consider them directly. We describe the construction of two large classes of models, namely magnetised D-branes on toroidal compactifications, and D-branes at singularities.

1 Introduction

In the previous lecture we have studied interesting compactifications of type IIA string theory on Calabi-Yau threefolds, with A-type D-branes, namely D6-branes wrapped on intersecting 3-cycles. Mirror symmetry exchanges type IIA and type IIB string compactifications, and maps A-type branes to B-type branes. Hence, it should be possible to obtain interesting compactifications in type IIB string theory on Calabi-Yau threefolds with B-type branes. This lecture is devoted to studying these compatifications (and their mirror relation to intersecting brane models).

B-type branes correspond to D-branes wrapped on holomorphic cycles of the Calabi-Yau threefold, and carrying holomorphic (and stable) world-volume gauge bundles. Namely, we should consider D3-brane sitting at points, D5- and D7- branes wrapped on 2- and 4-cycles, respectively, and D9-branes wrapped on the entire Calabi-Yau. On the wrapped volumes, one can turn on a topologically non-trivial background for the world-volume gauge field.

As a jargon-related comments, in the literature the different branes are described in a unified language, by describing them as 'coherent sheaves' on the CY threefold. Sheaves are generalizations of gauge bundles, which generalize to backgrounds with support on lower-dimensional subspaces. Thus once can describe a D7-brane wrapped on a 4-cycle as a sheaf on a CY, with support on the corresponding 4-cycle. We will see a more explicit realization of this idea in a class of examples below.

Although the discussion of the construction could be carried out quite far in this general language, it is more pedagogical to center on particular simple classes of this general formalism. We consider two such classes. The first is magnetised D-branes on toroidal compactifications (and quotients thereof). Namely, D-branes wrapped on products of 2-tori in $\mathbf{T^6}$, carrying constant $U(1)$ magnetic fields on their world-volume. The simple properties of the torus make the description of the bundles very easy. Regarding the models as containing B-type D9-branes, they can be described as a toroidal compactification of a 10d theory with a non-trivial gauge background. This description, reminiscent of heterotic compactifications, suggests that 4d chiral matter arises form a non-trivial index of the Dirac operator for 10d fermions charged under the gauge background. We will see how this arises in detail. Also, the mirror relation to toroidal intersecting brane models is manifest, and helps in understanding the construction.

The second class corresponds to D-branes at singularities, which can be regarded as a limit where the B-bype branes are wrapping cycles which are collapsed at a singularity of the CY threefold. The connection can be carried out quite explicitly by blowing up the singularity and taking the large volume limit of the cycles. Nevertheless, for orbifold singularities, there is a particularly simple and practical way to describe the system directly at the orbifold configuration. In this description, 4d chiral fermions arise from the orbifold projection.

Clearly, more general constructions are possible, on more general Calabi-Yau threefolds. In fact, the recent developments on the construction of holomorphic stable bundles (usually applied for heterotic models) could be exploited in B-type model building. Nevertheless, we prefer to skip these more technical constructions, and hope that the two above classes suffice to illustrate the conceptual issues on this kind of construction.

This lecture is organized as follows. In section 2 we describe magnetised D-branes and their physics in compactifications, both in toroidal models and orientifold and $\mathbf{Z_2}\times\mathbf{Z_2}$ quotients thereof. In section 3 we describe the application to building MSSM-like models. In section 4 we construct models of branes at singularities. Finally, section 5 contains our final remarks. Appendix A provides some details on the quantization of open strings in magnetised D-brane configurations.

2 Magnetised D-branes

In this section we review configurations of magnetised D9-branes in toroidal models. Useful references for this discussion are [1, 2] and [3, 4, 5, 6]. We first consider the case of toroidal compactifications, and subsequently incorporate orientifold projections and orbifold projections. The models are T-dual/mirror to the models of intersecting D6-branes in the previous lecture, in toroidal compactifications [5], toroidal orientifolds

[4] and orbifolds [7].

2.1 Magnetised branes on $\mathbf{T^2}$

The computation of the boundary conditions for open strings stretched between D-branes with constant world-volume magnetic fields in flat 10d space is carried out in appendix A. The open string spectrum is easily obtained by relating the question to the T-dual side, where it is mapped to the spectrum of open strings between two D-branes at relative angle $\theta_{ab} = \tan^{-1} F_b - \tan^{-1} F_a$.

There are some additional features when considering the D-branes to wrap on a $\mathbf{T^2}$. These are manifest when regarded in the T-dual picture, of D-branes wrapped on 1-cycles on the dual $\mathbf{T^2}$. For simplicty we may center on rectangular 2-tori, with vanishing NSNS 2-form, generalization to tilted tori and non-zero B-field are easy, but not essential.

The general configuration we are interested in consists of IIB D($2p$)-branes (labeled by an index a) multiply wrapped (with multiplicity m_a) on the $\mathbf{T^2}$, and carrying n_a units of world-volume $U(1)$ magnetic flux. Namely, we have

$$m_a \frac{1}{2\pi} \int_{\mathbf{T^2}} F_a = n_a \tag{1}$$

Notice that the magnetic flux is quantized in order to have a well-defined path integral for charged states. Taking $m = 1$ for simplicity, the argument is as follows. Consider the contribution to the path integral of an open string endpoint charged under the corresponding $U(1)$, running around a small topologically trivial closed loop C in the $\mathbf{T^2}$. The contribution is roughly $e^{i\int_C} A_1$. However, the gauge potential A_1 is not globally well-defined, so it is more appropriate to define the contribution as follows. Picking a 2d surface Σ whose boundary is C, e.g. the small 'inside' of C, the contribution can be written $Z = e^{i\int_\Sigma F_2}$. Now, there is another possible choice of surface Σ' with boundary C, namely the 'outside' of C, leading to a contribution $Z' e^{i\int_\Sigma' F_2}$. Since $\Sigma - \Sigma' = \mathbf{T^2}$ in homology (where the minus sign is due to a change of orientation to allow for the glueing), the result is independent of the choice if $\int_{\mathbf{T^2}} \in 2\pi\mathbf{Z}$ (since then the ratio of both contributions $Z/Z' = e^{i\int_{\Sigma-\Sigma'} F}$ is 1). For $m \neq 1$, the result follows from realizing that the gauge group is $U(m)$, broken to the diagonal $U(m)$ by the gauge background, and that this results in a effective charge of $1/n$ for the open string endpoints.

Notice that because of the CS couplings of the D($2p$)-branes, the worldvolume magnetic field induces m units of D($2p-2$)-brane charge. This is an alternative way to understand quantization of world-volume magnetic fluxes. Hence our configuration is a bound state of n units of D($2p$)-brane charge and m units of D($2p-2$)-brane charge.

Upon T-duality in the vertical direction, the configuration maps to a IIA D(2p-1)-brane wrapped on the (n, m) 1-cycle of the dual $\mathbf{T^2}$. That is, it wraps n times in the horizontal direction and m times in the vertical one. Notice that in the T-dual picture it is possible to consider the case of branes with numbers $(0, 1)$, namely wrapping just in the vertical direction. Its interpretation in the original picture of IIB magnetised D-branes deserves some discussion. Carrying out the T-duality directly, we obtain a IIB D($2p-2$)-brane sitting at a point in the $\mathbf{T^2}$. Applying the general language IIB description for (n, m)-branes, we can regard of the $(0, 1)$ D-branes (namely D($2p-2$)-branes) as gauge bundles with support just at a point in $\mathbf{T^2}$ (and hence with zero wrapping). This is a layman's description of the mathematical objects know as sheaves, mentioned above. The bottomline is that one can work with labels (n, m) even in these extreme case.

2.2 Magnetised D-branes in toroidal compactifications

We start with the simple case of toroidal compactification, with no orientifold projection. Consider the compactification of type IIB theory on $\mathbf{T^6}$, assumed factorizable [1]

We consider sets of N_a D9-branes, labelled D9_a-branes, wrapped m_a^i times on the i^{th} 2-torus $(\mathbf{T}^2)_i$ in $\mathbf{T^6}$, and with n_a^i units of magnetic flux on $(\mathbf{T^2})_\mathbf{i}$. Namely, we turn on a world-volume magnetic field F_a for the center of mass $U(1)_a$ gauge factor, such that

$$m_a^i \, \frac{1}{2\pi} \int_{\mathbf{T^2_i}} F_a^i \, = \, n_a^i \tag{2}$$

Hence the topological information about the D-branes is encoded in the numbers N_a and the pairs (m_a^i, n_a^i) [2]

We can include other kinds of lower dimensional D-branes using this description. For instance, a D7-brane (denoted D$7_{(i)}$) sitting and a point in $\mathbf{T^2_i}$ and wrapped on the two remaining two-tori (with generic wrapping and magnetic flux quanta) is described

[1]Without orbifold projections, this requires a constrained choice of fluxes, stabilizing moduli at values corresponding to a factorized geometry. In our orbifolds below, such moduli are projected out by the orbifold, and hence are simply absent.

[2]Notice the change of roles of n and m as compared with other references. This however facilitates the translation of models in the literature to our language.

by $(m^i, n^i) = (0,1)$ (and arbitrary (m^j, n^j) for $j \neq i$); similarly, a D5-brane (denoted $D5_{(i)}$) wrapped on $\mathbf{T^2_i}$ (with generic wrapping and magnetic flux quanta) and at a point in the remaining two 2-tori is described by $(m^j, n^j) = (0,1)$ for $j \neq i$; finally, a D3-brane sitting at a point in $\mathbf{T^6}$ is described by $(m^i, n^i) = (0,1)$ for $i = 1,2,3$. This is easily derived by noticing that the boundary conditions for an open string ending on a D-brane wrapped on a two-torus with magnetic flux become Dirichlet for (formally) infinite magnetic field.

D9-branes with world-volume magnetic fluxes are sources for the RR even-degree forms, due to their worldvolume couplings

$$\int_{D9_a} C_{10} \quad ; \quad \int_{D9_a} C_8 \wedge \mathrm{tr}\, F_a \quad ; \quad \int_{D9_a} C_6 \wedge \mathrm{tr}\, F_a^2 \quad ; \quad \int_{D9_a} C_4 \wedge \mathrm{tr}\, F_a^3 \tag{3}$$

Consistency of the configuration requires RR tadpoles to cancel. Following the discussion in [5], leads to the conditions

$$\begin{array}{ll} \sum_a N_a n_a^1 n_a^2 n_a^3 = 0 & \\ \sum_a N_a m_a^1 n_a^2 n_a^3 = 0 & \text{and permutations of } 1,2,3 \\ \sum_a N_a m_a^1 m_a^2 n_a^3 = 0 & \text{and permutations of } 1,2,3 \\ \sum_a N_a m_a^1 m_a^2 m_a^3 = 0 & \end{array} \tag{4}$$

Which amounts to cancelling the D9-brane charge as well as the induced D7-, D5- and D3-brane charges.

Introducing for the i^{th} 2-torus the even homology classes $[\mathbf{0}]_i$ and $[\mathbf{T^2}]_i$ of the point and the two-torus, the vector of RR charges of the one D9-brane in the a^{th} stack is

$$[\mathbf{Q}_a] \;=\; \prod_{i=1}^{3} (m_a^i [\mathbf{T}^2]_i + n_a^i [\mathbf{0}]_i) \tag{5}$$

The RR tadpole cancellation conditions read

$$\sum_a N_a [\mathbf{Q}_a] \;=\; 0 \tag{6}$$

The conditions that two sets of D9-branes with worldvolume magnetic fields F_a^i, F_b^i preserve some common supersymmetry can be derived from [8]. Indeed, it is possible to compute the spectrum of open strings stretched between them and verify that it is supersymmetric if

$$\Delta_{ab}^1 \pm \Delta_{ab}^2 \pm \Delta_{ab}^3 = 0 \tag{7}$$

for some choice of signs. Here

$$\Delta_i = \arctan\left[(F_a^i)^{-1}\right] - \arctan\left[(F_b^i)^{-1}\right] \tag{8}$$

and

$$F_a^i = \frac{n_a^i}{m_a^i R_{x_i} R_{y_i}} \tag{9}$$

which follows from (2).

The spectrum of massless states is easy to obtain. The sector of open strings in the aa sector leads to $U(N_a)$ gauge bosons and superpartners with respect to the 16 supersymmetries unbroken by the D-branes. In the $ab + ba$ sector, the spectrum is given by I_{ab} chiral fermions in the representation $(N_a, \overline{N}_b)$, where

$$I_{ab} = [\mathbf{Q}_a] \cdot [\mathbf{Q}_b] = \prod_{i=1}^{3} (n_a^i m_b^i - m_a^i n_b^i) \tag{10}$$

is the intersection product of the charge classes, which on the basic classes $[\mathbf{0}]_i$ and $[\mathbf{T}^2]_i$ is given by the bilinear form

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{11}$$

The above multiplicity can be computed using the α'-exact boundary states for these D-branes [4], or from T-duality with configurations of intersecting D6-branes. We now provide an alternative derivation which remains valid in more complicated situations where the worldsheet theory is not exactly solvable. Consider for simplicity a single two-torus. We consider two stacks of N_a and N_b branes wrapped m_a and m_b times, and with n_a, n_b monopole quanta. Consider the regime where the two-torus is large, so that the magnetic fields are diluted and can be considered a small perturbation around the vacuum configuration. In the vacuum configuration, open strings within each stack lead to a gauge group $U(N_a m_a)$ and $U(N_b m_b)$ respectively, which is subsequently broken down to $U(N_a) \times U(N_b)$ by the monopole background, via the branching

$$U(N_a m_a) \times U(N_b m_b) \to U(N_a)^{m_a} \times U(N_b)^{m_b} \to U(N_a) \times U(N_b) \tag{12}$$

Open ab strings lead to a chiral 10d fermion transforming in the bifundamental $(\Box_a, \overline{\Box}_b)$ of the original $U(N_a m_a) \times U(N_b m_b)$ group. Under the decomposition (12) the representation splits as

$$(\Box_a, \overline{\Box}_b) \to (\underline{\Box_a, \ldots}; \underline{\overline{\Box}_b, \ldots}) \to m_a m_b (\Box_a, \overline{\Box}_b) \tag{13}$$

The 8d theory contains chiral fermions arising from these, because of the existence of a nonzero index for the internal Dirac operator (coupled to the magnetic field background). The index is given by the first Chern class of the gauge bundle to which the corresponding fermions couples. Since it has charges $(+1, -1)$ under the a^{th} and b^{th} $U(1)$'s, the index is

$$\mathrm{ind}\, \not{D}_{ab} = \int_{\mathbf{T^2}} (F_a - F_b) = \frac{n_a}{m_a} - \frac{n_b}{m_b} \tag{14}$$

Because of the branching (13), a single zero mode of the Dirac operator gives rise to $m_a m_b$ 8d chiral fermions in the $(\square_a, \overline{\square}_b)$ of $U(N_a) \times U(N_b)$. The number of chiral fermions in the 8d theory in the representation $(\square_a, \overline{\square}_b)$ of the final group is given by $m_a m_b$ times the index, namely

$$I_{ab} = m_a m_b \int_{\mathbf{T^2}} (F_a - F_b) = n_a m_b - m_a n_b \tag{15}$$

The result (10) is a simple generalization for the case of compactification on three two-tori.

An important property about these chiral fields is that they are localized at points in the internal space. From the string theory viewpoint this follows because boundary conditions for open strings with endpoints on D-branes with different magnetic fields require the absense of center of mass zero mode in the worldsheet mode expansion. From the low energy effective theory viewpoint, this follows because such strings behave as charged particles in a magnetic field. From elementary quantum mechanics, such particles feel a harmonic oscillator potential and are localized in the internal space. Excited states in the harmonic oscillator system (Landau levels) correspond to stringy oscillator (gonions [5] in T-dual picture).

Notice that the field theory argument to obtain the spectrum is valid only in the large volume limit. However, the chirality of the resulting multiplets protects the result, which can therefore be extended to arbitrarily small volumes. This kind of argument will be quite useful in the more involved situation with closed string field strength fluxes, where we do not have a stringy derivation of the results.

It is a simple exercise to verify that the above formula remains valid in situations where the open strings under consideration end on lower-dimensional D-branes. The result from directly quantizing open strings in these configurations is exactly reproduced by formally replacing the entires (n, m) associated to the transverse directions to the brane by the value $(0, 1)$. This should be interpreted as 'zero wrapping, delta function magnetic field', which is a possible description for a localized D-brane (a laymans version of the skyscraper (or delta-function) sheaf).

The relation of magnetised D-brane models to intersecting D-brane models is clear, by performing three T-dualities along say the vertical directions. This relation facilitates the computations of diverse results in the magnetised D-brane picture by translating them from the more geometric and intuitive intersecting D-brane picture. This relation will actually permeate the discussion in this lecture. It is useful nevertheless to rederive several results directly from the magnetised D-brane picture. For instance, the discussion of anomaly cancellation, as we do in the following.

Cancellation of anomalies

Following [8, 9], the gauge anomaly induced by each localized chiral fermion is cancelled by an anomaly inflow mechanism associated to the branes. Namely, the violation of charge induced by the anomaly is compensated by a charge inflow from the bulk of the intersecting branes. This explanation is sufficient in situations where the branes are infinitely extended. In the compact context, however, within a single brane the charge 'inflowing' into an intersection must be compensated by charge 'outflowing' from other intersections. Consistency of anomaly inflow in a compact manifold imposes global constraints on the configuration.

From the point of view of the compactified four-dimensional effective field theory, which does not resolve the localization of the different chiral fermions, these global constraints correspond to cancellation of triangle gauge anomalies in the usual sense. In fact, the cancellation of cubic non-abelian anomalies for the gauge factor $SU(N_a)$ is

$$\sum_{b=1}^{K} I_{ab}\, N_b = 0 \tag{16}$$

Thus tadpole cancellation conditions imply the cancellation of cubic non-abelian anomalies. Namely, string theory consistency conditions imply consistency of the low-energy effective theory.

Mixed $U(1)$ anomaly cancellation

Mixed $U(1)$ anomalies are proportional to $A_{ab} = N_a I_{ab}$, and cancel by a Green-Schwarz mechanism, in analogy with intersecting brane models. We describe it directly in the picture of D9-branes with magnetic fluxes. The couplings on the world-volume of D9-branes to bulk RR fields are of the form (wedge products implied)

$$\begin{array}{ccccc} \int_{D9_a} C_0\, F_a^5 & ; & \int_{D9_a} C_2\, F_a^4 & ; & \int_{D9_a} C_4\, F_a^3 \\ \int_{D9_a} C_6\, F_a^2 & ; & \int_{D9_a} C_8\, F_a & ; & \int_{D9_a} C_{10} \end{array} \tag{17}$$

In order to obtain the four-dimensional version of these couplings, we define

$$\begin{array}{rclcrcl} C_2^I & = & \int_{(\mathbf{T^2})_\mathbf{I}} C_4 & ; & C_0^I & = & \int_{(\mathbf{T^2})_\mathbf{I}} C_2 \\ B_2^I & = & \int_{(\mathbf{T^2})_\mathbf{J}\times(\mathbf{T^2})_\mathbf{K}} C_6 & ; & B_0^I & = & \int_{(\mathbf{T^2})_\mathbf{J}\times(\mathbf{T^2})_\mathbf{K}} C_4 \\ B_2 & = & \int_{(\mathbf{T^2})_\mathbf{1}\times(\mathbf{T^2})_\mathbf{2}\times(\mathbf{T^2})_\mathbf{2}} C_8 & ; & B_0 & = & \int_{(\mathbf{T^2})_\mathbf{1}\times(\mathbf{T^2})_\mathbf{2}\times(\mathbf{T^2})_\mathbf{3}} C_6 \end{array}$$

where $I \neq J \neq K \neq I$ in second row. The fields C_2 and C_6, and also C_0 and C_8 are Hodge duals, while C_4 is self-dual. In four dimensions, the duality relations are

$$\begin{array}{rclcrcl} dC_0 & = & *dB_2 & ; & dB_0^I & = & *dC_2^I \\ dC_0^I & = & -*dB_2^I & ; & dB_0 & = & -*dC_2 \end{array}$$

In the dimensional reduction, one should take into account that integration of F_a along the I^{th} two-torus yields a factor m_a^I. Also, integrating the pullback of the RR forms on the (multiply wrapped) D9_a-brane over the I^{th} two-torus yields a factor n_a^I. We obtain the couplings

$$\begin{array}{ccc} N_a\, n_a^1\, n_a^2\, n_a^3 \int_{M_4} C_2 \wedge F_a & ; & m_b^1\, m_b^2\, m_b^3 \int_{M_4} B_0 \wedge F_b \wedge F_b \\ N_a\, m_a^I\, n_a^J\, n_a^K \int_{M_4} C_2^I \wedge F_a & ; & m_b^J\, m_b^K\, n_b^I \int_{M_4} B_0^I \wedge F_b \wedge F_b \\ N_a\, m_a^J\, m_a^K\, n_a^I \int_{M_4} B_2^I \wedge F_a & ; & m_b^I\, n_b^J\, n_b^K \int_{M_4} C_0^I \wedge F_b \wedge F_b \\ N_a\, m_a^1\, m_a^2\, m_a^3 \int_{M_4} B_2 \wedge F_a & ; & n_b^1\, n_b^2\, n_b^3 \int_{M_4} C_0 \wedge F_b \wedge F_b \end{array}$$

As usual, the N_a prefactors arise from $U(1)_a$ normalization.

The GS amplitude where $U(1)_a$ couples to one untwisted field which propagates and couples to two $SU(N_b)$ gauge bosons is proportional to

$$\begin{array}{r} -N_a\;\, n_a^1 n_a^2 n_a^3 m_b^1 m_b^2 m_b^3 + N_a \sum_I m_a^I n_a^J n_a^K m_b^J m_b^K m_b^I - N_a \sum_I m_a^I m_a^J n_a^K m_b^K n_b^I n_b^J + \\ N_a\, m_a^1 m_a^2 n_a^3 n_b^1 n_b^2 n_b^3 \;= N_a \prod_I (m_a^I n_b^I - n_a^I m_b^I) = N_a I_{ab} \end{array} \quad (18)$$

as required to cancel the residual mixed $U(1)$ anomaly.

Similarly to our discussion for intersecting brane models, the linear combinations of $U(1)$ gauge bosons with non-trivial $B \wedge F$ couplings become massive and disappear from the low energy dynamics.

2.3 Magnetised D-branes in toroidal orientifolds

We are interested in adding orientifold planes into this picture, since they are required to obtain supersymmetric fluxes. Consider type IIB on $\mathbf{T^6}$ (with zero NSNS B-field) modded out by ΩR, with $R : x_m \to -x_m$. This introduces 64 O3-planes, which we take to be all $O3^-$. It also requires the D9-brane configuration to be $\mathbf{Z_2}$ invariant. Namely, for the N_a D9_a-brane with topological numbers (m_a^i, n_a^i) we need to introduce their N_a ΩR images D$9_{a'}$ with numbers $(-m_a^i, n_a^i)$.

The RR tadpole cancellation conditions read

$$\sum_a N_a[\mathbf{Q}_a] + \sum_a N_a[\mathbf{Q}_{a'}] - 32\,[\mathbf{Q}_{O3}] = 0 \tag{19}$$

with $[\mathbf{Q}_{O3}] = [\mathbf{0}]_1 \times [\mathbf{0}]_2 \times [\mathbf{0}]_3$. More explicitly

$$\begin{aligned} &\textstyle\sum_a N_a m_a^1 m_a^2 n_a^3 = 0 \quad \text{and permutations of } 1,2,3 \\ &\textstyle\sum_a N_a n_a^1 n_a^2 n_a^3 = 16 \end{aligned} \tag{20}$$

Namely, cancellation of induced D7- and D3-brane charge. Notice that there is no net D9- or D5-brane charge, in agreement with the fact that the orientifold projection eliminates the corresponding RR fields

There is also an additional discrete constraint, which we would like to point out. It follows from a careful analysis of K-theory D-brane charge in the presence of orientifold planes. Following [10], the charge of D5-branes wrapped on some $\mathbf{T^2}$ in the presence of O3-planes is classified by a real K-theory group which is $\mathbf{Z_2}$. This statement is T-dual to the fact that D7-brane charge is $\mathbf{Z_2}$ valued in type I theory. Following [11] RR tadpole cancellation requires cancellation of the K-theory D-brane charge. Hence the total induced D5-brane charge on the D9_a-branes (without images) must be even in the above configurations. This amounts to the condition

$$\sum_a N_a m_a^1 n_a^2 n_a^3 = \text{even and permutations of } 1,2,3 \tag{21}$$

The condition is non-trivial, and models satisfying RR tadpole conditions in homology, but violating RR tadpole conditions in K-theory can be constructed [12]. Such models are inconsistent, as can be made manifest by introducing a D7-brane probe, on which world-volume the inconsistency manifests as a global gauge anomaly [11]. The condition is however happily satisfied by models in the literature, and also in our examples below.

The rules to obtain the spectrum are similar to the above ones, with the additional requirement of imposing the ΩR projections. This requires a precise knowledge of the ΩR action of the different zero mode sectors (in field theory language, on the harmonic oscillator groundstates for chiral fermions). The analysis is simplest in terms of the T-dual description, where it amounts to the geometric action of the orientifold on the intersection points of the D-branes. The result, which is in any case derivable in our magnetised brane picture, can be taken from [4].

The aa sector is mapped to the $a'a'$ sector, hence suffers no projection [3]. We obtain a 4d $U(N_a)$ gauge group, and superpartners with respect to the $\mathcal{N}=4$ supersymmetry unbroken by the brane.

The $ab+ba$ sector is mapped to the $b'a'+a'b'$ sector, hence does not suffer a projection. We obtain I_{ab} 4d chiral fermions in the representation $(\square_a, \overline{\square}_b)$. Plus additional scalars which are massless in the susy case, and tachyonic or massive otherwise.

The $ab'+b'a$ sector is mapped to the $ba'+a'b$. It leads to $I_{ab'}$ 4d chiral fermions in the representation $(\square_a, \square_b)$ (plus additional scalars).

The $aa'+a'a$ sector is invariant under ΩR, so suffers a projection. The result is $n_{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ and $n_{\square\square}$ 4d chiral fermions in the $\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$, $\square\square_a$ representations, resp, with

$$
\begin{aligned}
n_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} &= \frac{1}{2}(I_{aa'} + 8 I_{a,O3}) \,=\, -4\, m_a^1 m_a^2 m_a^3\, (n_a^1 n_a^2 n_a^3 + 1) \\
n_{\square\square} &= \frac{1}{2}(I_{aa'} - 8 I_{a,O3}) \,=\, -4\, m_a^1 m_a^2 m_a^3\, (n_a^1 n_a^2 n_a^3 - 1)
\end{aligned}
\tag{22}
$$

where $I_{a,O3} = [\mathbf{Q}_a]\cdot[\mathbf{Q}_{O3}]$.

2.4 Magnetised D-branes in the $\mathbf{T^6}/(\mathbf{Z_2}\times\mathbf{Z_2})$ orbifold

Finally, we will be interested in models with orbifold and orientifold actions. In particular, consider type IIB on the orbifold $\mathbf{T^6}/(\mathbf{Z_2}\times\mathbf{Z_2})$, modded out by ΩR. The model contains 64 O3-planes (with $-1/2$ units of D3-brane charge), and 4 O7$_i$-planes (with -8 units of D7$_i$-brane charge), transverse to the i^{th} two-torus. Their total charges are given by -32 times the classes

$$
\begin{aligned}
[\mathbf{Q}_{O3}] &= [\mathbf{0}_1]\times[\mathbf{0}_2]\times[\mathbf{0}_3] \quad ; \qquad [\mathbf{Q}_{O7_1}] = -\,[\mathbf{0}_1]\times[(\mathbf{T}^2)_2]\times[(\mathbf{T}^2)_3] \\
[\mathbf{Q}_{O7_2}] &= -\,[(\mathbf{T}^2)_1]\times[\mathbf{0}_2]\times[(\mathbf{T}^2)_3] \quad ; \qquad [\mathbf{Q}_{O7_3}] = -\,[(\mathbf{T}^2)_1]\times[(\mathbf{T}^2)_2]\times[\mathbf{0}_3]
\end{aligned}
\tag{23}
$$

where the signs are related to the specific signs in the definition of the $\mathbf{Z_2}\times\mathbf{Z_2}$ action. We define $[\mathbf{Q}_{Op}] = [\mathbf{Q}_{O3}] + [\mathbf{Q}_{O7_1}] + [\mathbf{Q}_{O7_2}] + [\mathbf{Q}_{O7_3}]$. The RR charge is cancelled using magnetised D9-branes and their orientifold images (the orbifold projection maps each stack of D9-branes to itself), which carry just induced D7$_i$- and D3-brane charges. The RR tadpole conditions read

$$
\sum_a N_a[\mathbf{Q}_a] \,+\, \sum_a N_a[\mathbf{Q}_{a'}] \,-\, 32\,[\mathbf{Q}_{Op}] \,=\, 0 \tag{24}
$$

[3] We do not consider branes for which $a=a'$ here; they will be taken care of explicitly in the examples below.

The models with magnetised D9-branes in this orientifold are T-dual to those in [7], whose main features are easily translated. The spectrum can be computed using the above techniques, taking care of the additional orbifold projections on the spectrum, or equivalently translated from [7]. The result is shown in table 1, where $I_{a,Op} = [Q_a] \cdot [Q_{Op}]$.

Sector	**Representation**
aa	$U(N_a/2)$ vector multiplet 3 Adj. chiral multiplets
$ab+ba$	I_{ab} $(\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}$ $(\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - 4I_{a,Op})$ $\square\!\square$ fermions $\frac{1}{2}(I_{aa'} + 4I_{a,Op})$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

Table 1: General chiral spectrum on generic magnetised $D9_a$-branes in the ΩR orientifold of $\mathbf{T^6}/(\mathbf{Z_2}\times\mathbf{Z_2})$. The models may contain additional non-chiral pieces which we ignore here. In supersymmetric situations, scalars combine with the fermions given above to form chiral supermultiplets.

The discrete conditions arising from cancellation of K-theory torsion charges was carried out in [13] following ideas in [11]. Here we simply quote the result

$$\begin{aligned} \textstyle\sum_\alpha N_\alpha m_\alpha^1 m_\alpha^2 m_\alpha^3 \ &\in \ 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha n_\alpha^1 n_\alpha^2 m_\alpha^3 \ &\in \ 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha n_\alpha^1 m_\alpha^2 n_\alpha^3 \ &\in \ 4\mathbf{Z}, \\ \textstyle\sum_\alpha N_\alpha m_\alpha^1 n_\alpha^2 n_\alpha^3 \ &\in \ 4\mathbf{Z}. \end{aligned} \tag{25}$$

Notice that these are actually $\mathbf{Z_2}$ charge constraints, since N_α are already even integers. In K-theory language, we are imposing the global cancellation of $\mathbf{Z_2}$ RR charges, carried by fractional $D5_i - \overline{D5}_i$ and $D9 - \overline{D9}$ pairs.

Second, we would like to construct models free of NSNS tadpoles, that is, such that the tensions of the objects in the configuration do also cancel. In a magnetised D-brane configuration with vanishing RR tadpoles, this can be achieved by requiring that every set of D-branes preserves the same $\mathcal{N}=1$ supersymmetry unbroken by the orientifold. This usually implies a condition on the Kähler parameters, which in the present context reads[4]

$$\sum_i \tan^{-1}\left(\frac{m_a^i A_i}{n_a^i}\right) = 0, \tag{26}$$

where A_i is the area of $(\mathbf{T}^2)_i$ in α' units. A small deviation from this condition can be understood as a non-vanishing FI-term in the $D = 4$ effective theory [7, 14].

3 MSSM-like models

The most practical way to deal with the model building applications of magnetised D-brane models is to translate them from the similar discussion for intersecting D-branes (as is done in the literature). Hence, here we simply recover the models in lecture 1.

Again, the idea is to embed in a globally consistent way a local structure of D-branes leading to a MSSM-like structure. Here we now interpret the integers (n, m) as wrapping numbers and magnetic monopole quanta of the corresponding D-branes. Again, our general arguments on the computation of the spectrum guarantee that any model containing such subsector will reproduce a gauge theory with MSSM like chiral spectrum (plus additional exotics, whose structure depends on the detailed set of additonal branes in the model).

To provide one example, we simply present a generalization of the example studied in the previous lecture [13]. In table 3 we present a magnetised D-brane model which satisfies the necessary requirements to accommodate both the MSSM local model of the previous section and non-trivial 3-form fluxes, while still satisfying RR and NSNS tadpole conditions. Indeed, it is easy to check that these magnetic numbers satisfy the tadpole conditions, by simply imposing $g^2 + N_f = 14$. Notice that this give us an upper bound for the number of generations, namely $g \leq 3$.

The gauge group of this model is

$$SU(3) \times SU(2) \times SU(2) \times U(1)_{B-L} \times \left[U(1)' \times USp(8N_f)\right], \tag{27}$$

where $U(1)' = [U(1)_a + U(1)_d] - 2g\,[U(1)_{h_1} - U(1)_{h_2}]$ is the only Abelian factor that, besides $U(1)_{B-L}$, survives the generalised Green-Schwarz mechanism. The $USp(8N_f)$ gauge group will only remain as such when all the D3-branes are placed on top of an orientifold singularity. Eventually, by moving them away it can be Higgsed down to $U(1)^{2N_f}$. Of course, the new D-brane sectors will also imply new chiral matter, some of it charged under the Left-Right MSSM gauge group. We will explain below how to deal with these chiral exotics.

[4] This formula is actually only valid for the case $n_a^i \geq 0$. See below for some other important cases.

N_α	(n^1_α, m^1_α)	(n^2_α, m^2_α)	(n^3_α, m^3_α)
$N_a = 6$	$(1, 0)$	$(g, 1)$	$(g, -1)$
$N_b = 2$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 2$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_d = 2$	$(1, 0)$	$(g, 1)$	$(g, -1)$
$N_{h_1} = 2$	$(-2, 1)$	$(-3, 1)$	$(-4, 1)$
$N_{h_2} = 2$	$(-2, 1)$	$(-4, 1)$	$(-3, 1)$
$8N_f$	$(1, 0)$	$(1, 0)$	$(1, 0)$

Table 2: D-brane magnetic numbers giving rise to an $\mathcal{N} = 1$ MSSM-like model.

In order to satisfy the RR tadpole cancellation conditions, we need

$$g^2 + N_f = 14. \tag{28}$$

Clearly the most interesting solution is $n = 0$, $g = 3$, $N_f = 5$, on which we center in what follows.

Also, in order to satisfy the $N = 1$ susy conditions, we need

$$\begin{gathered} A_2 = A_3 \\ \tan^{-1}(A_1/2) + \tan^{-1}(A_2/3) + \tan^{-1}(A_3/4) = \pi \end{gathered} \tag{29}$$

These fix the Kähler parameters A_i in terms of the overall volume $A_1A_2A_3$. This 'fixing' of moduli should not be thought of as a dynamical stabilisation process. As explained in [7], changes in the Kahler moduli lead to Fayet-Illiopoulos terms on the D-branes. They force some of the charged scalars in ab sector to acquire a vev and break gauge symmetry, but preserving $N = 1$ supersymmetry. This corresponds to recombining some of the branes in teh model into bound states. Hence, the condition (29) correspond simply to imposing that the model is supersymmetric with branes as they stand in table (3).

As mentioned before, the addition of the D-brane sectors h_1, h_2 and f, which are necessary to embed the MSSM local model into a global $\mathcal{N} = 1$ compactification, add new gauge groups as well as chiral matter. In general, some of this additional chiral matter will be charged under the MSSM gauge group, and hence will introduce chiral exotics in our spectrum. Nevertheless, we can get rid of most of these exotics by taking appropriate scalar flat directions. In the present context, such flat directions can be engineered from the D-brane perspective by the process of D-brane recombination. We now consider one such example. Consider that stacks a and d are on top of each other

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times [USp(40)]$	Q_a	Q_{h_1}	Q_{h_2}	Q'
(ab)	F_L	$3(4,2,1)$	1	0	0	1/3
(ac)	F_R	$3(\bar{4},1,2)$	-1	0	0	$-1/3$
(bc)	H	$(1,2,2)$	0	0	0	0
(ah_1')		$6(\bar{4},1,1)$	-1	-1	0	5/3
(ah_2)		$6(4,1,1)$	1	0	-1	$-5/3$
(bh_1)		$8(1,2,1)$	0	-1	0	2
(bh_2)		$6(1,2,1)$	0	0	-1	-2
(ch_1)		$6(1,1,2)$	0	-1	0	2
(ch_2)		$8(1,1,2)$	0	0	-1	-2
(h_1h_1')		$23(1,1,1)$	0	-2	0	4
(h_2h_2')		$23(1,1,1)$	0	0	-2	-4
(h_1h_2')		$196(1,1,1)$	0	1	1	0
(fh_1)		$(1,1,1) \times [40]$	0	-1	0	2
(fh_2)		$(1,1,1) \times [40]$	0	0	-1	-2

Table 3: Chiral spectrum of the three generation Pati-Salam $\mathcal{N}=1$ chiral model of table 3. The Abelian generator of the unique massless $U(1)$ is given by $Q' = \frac{1}{3}Q_a - 2(Q_{h_1} - Q_{h_2})$.

and hence we have a Pati-Salam gauge group. This will hardly affect the discussion, but will render our expressions more compact. Also assume that all the D3-branes are at the origin, and hence our gauge group includes a $USp(40)$ factor.

The total chiral spectrum of this model is displayed in table 3, including the charges of the chiral matter under the only $U(1)$ factor which is massless. This $U(1)$ is given by the combination $U(1)' = \frac{1}{3}U(1)_a - 2\,[U(1)_{h_1} - U(1)_{h_2}]$, and almost all the chiral matter is charged under it. The two exceptions are the Higgs multiplet and the 196 singlets in the h_1h_2' sector of the theory. The latter are of particular interest, since they parametrise a subspace of flat directions in the $\mathcal{N}=1$ effective theory. Indeed, we can give a non-vanishing v.e.v. to a particular combination of the scalar fields in the 196 chiral multiplets without breaking supersymmetry. In terms of D-brane physics, this is nothing but the D9-brane recombination

$$h_1 + h_2' \to h. \tag{30}$$

More precisely, it amounts to deforming the gauge bundle on the D9-branes, from a direct sum of the Abelian bundles h_1 and h_2' to a non-Abelian bundle given by h. As usual, the magnetic charges of the new bundle will be given by $[\mathbf{Q}_h] = [\mathbf{Q}_{h_1}] + \Omega[\mathbf{Q}_{h_2}]$.

Sector	Matter	$SU(4) \times SU(2) \times SU(2) \times [USp(40)]$	Q_a	Q_h	Q'
(ab)	F_L	$3(4, 2, 1)$	1	0	1/3
(ac)	F_R	$3(\bar{4}, 1, 2)$	-1	0	$-1/3$
(bc)	H	$(1, 2, 2)$	0	0	0
(bh)		$2(1, 2, 1)$	0	-1	2
(ch)		$2(1, 1, 2)$	0	+1	-2

Table 4: $\mathcal{N} = 1$ spectrum derived from the D-brane content of table 3 after D-brane recombination. There is no chiral matter arising from ah, ah', hh' or charged under $USp(40)$. The generator of $U(1)'$ is now given by $Q' = \frac{1}{3}Q_a - 2Q_h$.

This Higgsing does not affect the Pati-Salam gauge group. It does, however, have an important effect on the chiral spectrum of the theory. Indeed, we can compute the chiral spectrum after (30) with the charge vector $[\mathbf{Q}_h]$ and the topological formulae of table 1, finding that the final theory has the extremely simple chiral content of Table 3.

Generically expected in CY's.

4 D-branes at singularities

As mentioned in the introduction, D-branes at orbifold singularities provide another very tractable class of B-type brane model. We describe some of its main features in this section. As mentioned the orbifold configuration does not really correspond to a large volume regime (since there are collapsed cycles, which make α' corrections important). Hence, the system should be studied by directly quantizing open strings in the orbifold configuration. This is easily done by applying the techniques developed in [15]. It is however important to point out that certain topological and protected quantities (like the chiral spectrum and the world-volume superpotential) can be computed in the large volume limit and reliably extrapolated to the orbifold configuration, in the spirit of [16]. For $\mathbf{C^3}/\mathbf{Z_3}$ this analysis has been carried out in [17], where the identification of the appropriate large volume bundles for the involved B-type branes was carried out. We skip this interesting discussion, and work directly at the orbifold point.

For concreteness, let us center of a stack of n D3-branes sitting at the Origin of a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity. These models were first Considered in [15]. The $\mathbf{Z_N}$

generator θ acts on the three complex coordinates of $\mathbf{C^3}$ as follows

$$(z_1, z_2, z_3) \to (e^{2\pi I\, a_1/N} z_1, e^{2\pi i\, a_2/N} z_2, e^{2\pi i\, a_3/N} z_3) \tag{31}$$

where the $a_i \in \mathbf{Z}$ in order to have and order N action [5]. We will center on orbifolds that preserve some supersymmetry, hence their holonomy must be in $SU(3)$ and thus we require $a_1 \pm a_2 \pm a_3 = 0$ mod N, for some choice of signs.

The closed string spectrum in the configuration can be easily obtained. However, this sector is uncharged under the gauge group on the D-brane world-volume, so we skip its discussion.

Concerning the open string sector, the main observation is that there are no twisted sectors. This follows because the definition of twisted sectors in closed strings made use of the periodicity in the worldsheet direction σ, and this is not allowed in open strings. Hence, the spectrum of open strings on a set of D3-branes at a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity is simply obtained by considering the open string spectrum on D3-branes in flat space $\mathbf{C^3}$, and keeping the $\mathbf{Z_N}$-invariant ones. Each open string state on D3-branes in flat space is given by a set of oscillators acting on the vacuum, and an $n \times n$ Chan-Paton matrix λ encoding the $U(n)$ gauge degrees of freedom. The action of θ on one such open string state is determined by the action on the corresponding set of oscillators and the action on the Chan-Paton matrix. For concreteness,let us center on massless states. The eigenvalues of the different sets of oscillators for these states are

Sector	State	θ eigenvalue
NS	$(0,0,0,\pm)$ 1	
	$(\underline{+,0,0},0)$	$e^{2\pi i\, a_i/N}$
	$(\underline{-,0,0},0)$	$e^{-2\pi i\, a_i/N}$
R	$\pm\frac{1}{2}(+,+,+,-)$	1
	$\frac{1}{2}(\underline{-,+,+},+)$	$e^{2\pi i\, a_i/N}$
	$\frac{1}{2}(\underline{+,-,-},-)$	$e^{-2\pi i\, a_i/N}$

The eigenvalues can be described as $e^{2\pi i\, r\cdot v}$, where r is The $SO(8)$ weight and $v = (a_1, a_2, a_3, 0)/N$. The above action can easily be understood by decomponsing the $SO(8)$ representation with respect to the $SU(3)$ subgroup in which the $\mathbf{Z_N}$ is embedded. In fact we have $8_V = 3+\overline{3}+1+1$, and $8_C = 3+\overline{3}+1+1$, and noticing that (31) defines the action on the representation 3. Notice that the fact that bosons and fermions have

[5]One also needs $N\sum_i a_i$ = even (so that the quotient is a spin manifold, i.e. allows spinors to be defined).

the same eigenvalues reflects the fact that the orbifold preserves $\mathcal{N}=1$ supersymmetry on the D-brane world-volume theory. In fact we see that the different states group into a vector multiplet V, with eigenvalue 1, and three chiral multiplets, Φ_i with eigenvalue $e^{2\pi i\, a_i/N}$.

On the other hand, the action of θ on the Chan-Paton degrees of freedom corresponds to a $U(n)$ gauge transformation. This is defined by a unitary order N matrix $\gamma_{\theta,3}$, which without loss of generality we can diagonalize and write in the general form

$$\gamma_{\theta,3} = \text{diag}\,(1_{n_0}, e^{2\pi i/N} 1_{n_1}, \ldots, e^{2\pi i(N-1)/N} 1_{n_{N-1}}) \tag{32}$$

with $\sum_{a=0}^{N-1} n_a = n$. The action on the Chan-Paton wavefunction (which transforms in the adjoint representation) is

$$\lambda \to \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{33}$$

We now have to keep states invariant under the combined action of θ on the oscillator and Chan-Paton piece. For states in the $\mathcal{N}=1$ vector multiplet, the action on the oscillators is trivial, hence the surviving states correspond to Chan-Paton matrices satisfying the condition

$$\lambda = \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{34}$$

The surviving states correspond to a block diagonal matrix. The gauge group is easily seen to be

$$U(n_0) \times \ldots \times U(n_{N-1}) \tag{35}$$

For the i^{th} chiral multiplet Φ_i, the oscillator part picks up a factor of $e^{2\pi i\, a_i/N}$. So surviving states have Chan-Paton wavefunction must satisfy

$$\lambda = e^{2\pi i\, a_i/N} \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{36}$$

The surviving multiplets correspond to matrices with entries in a diagonal shifted by a_i blocks. It is easy to see that the surviving multiplets transform in the representation

$$\sum_{i=1}^{3} \sum_{a=0}^{N-1} (\square_a, \overline{\square}_{a+a_i}) \tag{37}$$

We clearly see that in general the spectrum is chiral, so we have achieved the construction of D-brane configurations with non-abelian gauge symmetries and charged chiral fermions. Moreover, we see that in general the different fermions have different quantum numbers. The only way to obtain a replication of the fermion spectrum (i.e. a structure of families, like in the Standard Model), we need some of the a_i to be equal (modulo N). The most interesting example is obtained for the $\mathbf{C^3/Z_3}$ singularity, with $v = (1,1,-2)/3$. The spectrum on the D3-brane world-volume is given by

$$\begin{array}{rl} \mathcal{N}=1 \text{ Vect.Mult.} & U(n_0)\times U(n_1)\times U(n_2) \\ \mathcal{N}=1 \text{ Ch.Mult.} & 3\,[\,(n_0,\overline{n}_1,1)+(1,n_1,\overline{n}_2)+(\overline{n}_0,n_1,1) \end{array} \quad (38)$$

we see there is a triplication of the chiral fermion spectrum. Hence in this setup the number of families is given by the number of complex planes with equal eigenvalue.

We would like to point out that, as usual in models with open strings, there exist some consistency conditions, known as cancellation of RR tadpoles. Namely, there exist disk diagrams, see figure 1, which lead to the coupling of D-branes at singularities to RR fields in the θ^k twisted sector. When the θ^k twist has the origin as the only fixed point, the corresponding RR fields do not propagate over any dimension transverse to the D-brane. This implies that they have compact support, and Gauss law will impose the corresponding charges must vanish, namely that the corresponding disk diagrams cancel. The coefficient of the disk diagram is easy to obtain: from the figure, we see that any worldsheet degree of freedom must suffer the action of θ^k as it goes around the closed string insertion. In particular it means that the Chan-Paton degrees of freedom suffer the action of $\gamma^k_{\theta^k,3=(\gamma_{\theta,3}}$ as they go around the boundary. Hence the disk amplitude is proportional to $\mathrm{tr}\,\gamma_{\theta^k,3}$, and the RR tadpole condition reads

$$\mathrm{Tr}\,\gamma_{\theta^k,3}=0 \quad ,\text{for } ka_i \neq 0 \bmod N \quad (39)$$

For instance, for the above $\mathbf{Z_3}$ model these constraint require $n_0 = n_1 = n_2$ In general, the above constrains ensure that the 4d chiral gauge field theory on the volume of the D3-branes is free of anomalies.

Clearly the above model is not realistic. However, more involved models of this kind, with additional branes (like D7-branes, also passing through thesingularity), can lead to models much closer to the Standard Model, see [18], also [19].

Following the general arguments in section 3.1, the strategy to obtain a field theory with standard model gauge group from the $\mathbf{Z_3}$ singularity is to choose a D3-brane Chan-Paton embedding

$$\gamma_{\theta,3} \;=\; \mathrm{diag}\,(\mathrm{I}_3, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_1) \quad (40)$$

The simplest way to satisfy the tadpole conditions is to introduce only one set of D7-branes, e.g. $D7_3$-branes, with Chan-Paton embedding $u_0^3 = 0$, $u_0^1 = 3$, $u_0^2 = 6$.

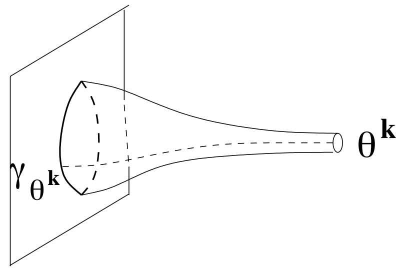

Figure 1: D3-branes at singularities are charged under RR forms in the θ^k twisted sector, via a disk diagram. Worldsheet degrees of freedom suffer the action of θ^k as they go around the cut, shown as a dashed line. The amplitude is proportional to $\mathrm{tr}\, \gamma_{\theta^k}$.

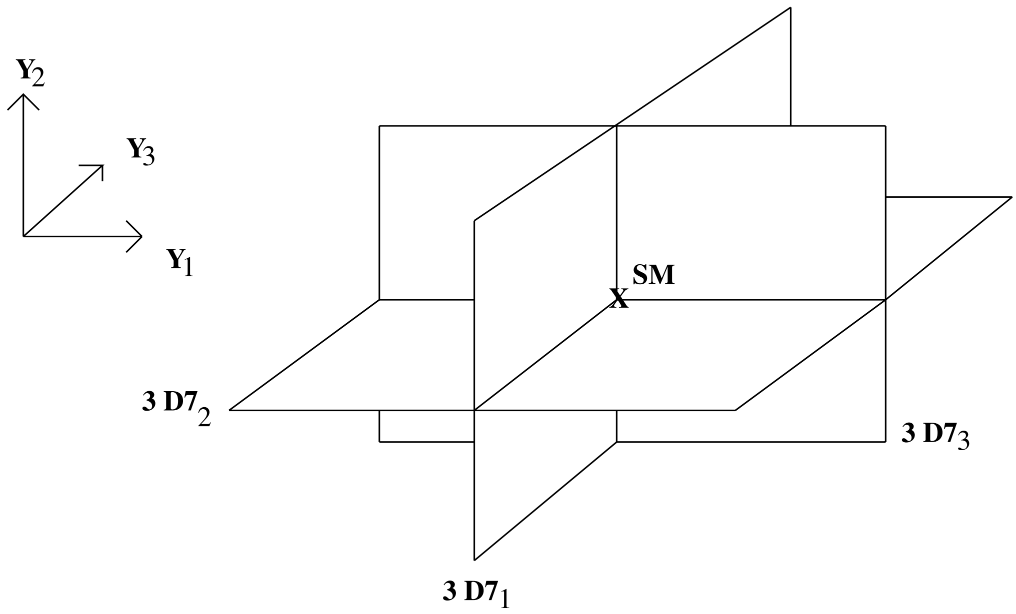

Figure 2: A non-compact Type IIB $\mathbf{Z_3}$ orbifold singularity yielding SM spectrum. Six D3 branes sit on top of a $\mathbf{Z_3}$ singularity at the origin. Tadpoles are canceled by the presence of intersecting D7-branes with their worldvolumes transverse to different complex planes.

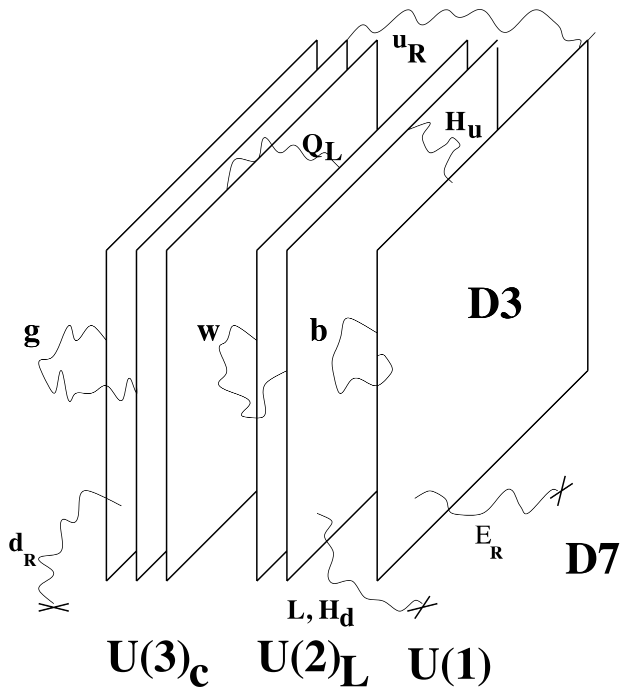

Figure 3: D-brane configuration of a SM $\mathbf{Z_3}$ orbifold model. Six D3-branes (with worldvolume spanning Minkowski space) are located on a $\mathbf{Z_3}$ singularity and the symmetry is broken to $U(3) \times U(2) \times U(1)$. For the sake of visualization the D3-branes are depicted at different locations, even though they are in fact on top of each other. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting in one set and ending on different sets originate the left-handed quarks, right-handed U-quarks and one set of Higgs fields. Leptons, and right-handed D-quarks correspond to open strings starting on some D3-branes and ending on the D7-branes (with world-volume filling the whole figure).

The gauge group on the D3-branes is $U(3) \times U(2) \times U(1)$, whereas in the $D7_3$-branes is $U(3) \times U(6)$ on each. Note that, before compactification, the latter behave as global symmetries in the worldvolume of the D3-branes. The $D7_3$-branes group can be further broken by global effects, since the corresponding branes are extended along some internal dimensions.

An alternative procedure to obtain a smaller group on the D7-branes is to use all three kinds of D7-branes, as depicted in Figure 2. For instance, a very symmetrical choice consistent with (40) is $u_0^r = 0$, $u_1^r = 1$, $u_1^r = 2$, for $r = 1, 2, 3$. Each kind of D7-brane then carries a $U(1) \times U(2)$ group.

The spectrum for this latter model is given in table 5 (for later convenience we have also included states in the 7_r7_r sectors; their computation is analogous to the computation of the 33 sector). In the last column we give the charges under the anomaly-free combination

$$Y = -\left(\frac{1}{3}\, Q_3 \, + \, \frac{1}{2}\, Q_2 \, + \, Q_1 \right) \tag{41}$$

As promised, it gives the correct hypercharge assignments for standard model fields. A pictorial representation of this type of models is given in Figure 3.

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	1/6
$3(\bar{3},1)$	-1	0	1	0	0	-2/3
$3(1,2)$	0	1	-1	0	0	1/2
$\mathbf{37}_r$ sector						
$(3,1)$	1	0	0	-1	0	-1/3
$(\bar{3},1;2')$	-1	0	0	0	1	1/3
$(1,2;2')$	0	1	0	0	-1	-1/2
$(1,1;1')$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1;2)'$	0	0	0	1	-1	0

Table 5: Spectrum of $SU(3)\times SU(2)\times U(1)$ model. We present the quantum numbers under the $U(1)^9$ groups. The first three $U(1)$'s come from the D3-brane sector. The next two come from the D7_r-brane sectors, written as a single column with the understanding that e.g. fields in the $\mathbf{37}_r$ sector are charged under the $U(1)$ in the $\mathbf{7_r 7_r}$ sector.

We find it remarkable that such a simple configuration produces a spectrum so close to that of the standard model. In particular, we find encouraging the elegant appearance of hypercharge within this framework, as the only linear combination of $U(1)$ generators which is naturally free of anomalies in systems of D3-branes at orbifold singularities.

The model constructed above, once embedded in a global context, may provide the simplest semirealistic string compactifications ever built. Indeed, in Section 4 we will provide explicit compact examples of this kind. Let us once again emphasize that, however, many properties of the resulting theory will be independent of the particular global structure used to achieve the compactification, and can be studied in the non-compact version presented above, as we do in Section 5.

One may wonder about the mirror version of this construction, which should be in terms of intersecting D6-branes in the mirror geometry. This has been worked out in [20, 21], to which we refer the reader for details.

5 Final remarks

We have described a new class of type IIB compactification leading to interesting chiral physics in four dimensions. They moreover have a beautiful relation to intersecting brane models via mirror symmetry. Notice that, despite the equivalence in string theory of both kinds of constructions, very often one side is far simpler than the other, and allows for more efficient discussion of the physics. For instance, in toroidal models the discussion in terms of intersecting branes can be considered more intuitive and pedagogial (and that is why they went first in these lectures). However, in general Calabi-Yaus it is far simpler to construct holomorphic stable bundles than to construct special lagrangian submanifolds, hence the discussion of model building in terms of type IIB theory is more practical.

Another important point is that the equivalence of Calabi-Yau compactifications with A- and B- branes does not hold (in this form) in the presence of fluxes, to be introduced in the coming lecture. Hence, it is extremely useful to have a well-developed intuition about each of these two pictures independently in order to address further developments.

A Boundary conditions for open strings ending on D-branes with magnetic fields

In this section we describe the quantization of open strings stretching between D-branes with different constant $U(1)$ magnetic fields on their world-volume. We also make manifest the connection via T-duality with D-branes at angles. Some early references on this kind of system are [22].

Consider the world-sheet action for an open string stretching between two coincident D-branes, labeled a and b, carrying constant world-volume $U(1)$ magnetic fields F_a, F_b in a 2-plane. For simplicity, we consider the dynamics only in the 2-plane, whose coordinates we denote X^4, X^5. Sketchily, in the conformal gauge we have

$$S = \frac{1}{4\pi\alpha'} d^2\xi \int_\Sigma \partial_a X^m \partial_a X^m + \frac{1}{2\pi\alpha'} \left[\int dt\, (A_a)_m \partial_t X^m \,|_{\sigma=0} - \int dt\, (A_b)_m \partial_t X^m \,|_{\sigma=\ell} \right] \quad (42)$$

For constant magnetic fields, we may use $A_m = \frac{1}{2} F_{mn} X^n$ In order to find the appropriate boundary conditions, we require that upon variation of this action, the boundary terms drop. The variation, keeping carefully the boundary terms from integration by parts, is given by

$$\begin{aligned}
\delta S &= \frac{1}{2\pi\alpha'} \int_\Sigma d^2\xi\, \partial_a X^m \partial_a \delta X^m + \\
&+ \frac{1}{2\pi\alpha'} \left[\int dt\, (F_a)_{mn} \delta X^n \partial_t X^m \,|_{\sigma=0} - \int dt\, (F_b)_{mn} \delta X^n \partial_t X^m \,|_{\sigma=\ell} \right] = \\
&= \frac{1}{2\pi\alpha'} \int_\Sigma d^2\xi\, \delta X^m \partial_a \partial_a \delta X^m - \frac{1}{2\pi\alpha'} \left[\int dt\, \left(\partial_\sigma X^m \delta X^m + (F_a)_{mn} \delta X^m \partial_t X^n \right) |_{\sigma=0} + \right. \\
&\left. - \int dt\, \left(\partial_\sigma X^m \delta X^m + (F_b)_{mn} \delta X^m \partial_t X^n \right) |_{\sigma=\ell} \right] \quad (43)
\end{aligned}$$

Since the variations δX^m are arbitrary, the boundary conditions are

$$\partial_\sigma X^m + F_{mn}\, \partial_t X^n = 0 \quad \text{at}\, \sigma = 0, \ell \quad (44)$$

For simplicity let us center in the particular case of $F_a = 0$, and denote $(F_b)_{45} = \tan\theta$, we have

$$\begin{aligned}
\sigma = 0 \qquad & \partial_\sigma X^4 = 0 \quad , \quad \partial_\sigma X^5 = 0 \\
\sigma = \ell \qquad & \cos\theta \partial_\sigma X^4 + \sin\theta \partial_t X^5 = 0 \\
& - \sin\theta \partial_t X^4 + \cos\theta \partial_\sigma X^5 = 0 \quad (45)
\end{aligned}$$

Now recall that T-duality interchanges Neumann and Dirichlet boundary conditions (and hence $\partial_\sigma X$ and $\partial_t X$). Using T-duality along X^5, these boundary conditions are related to the boundary conditions for open strings stretching between two D-branes, labelled a, b, at angles 0 and θ with respect to the X^4 axis. In general, any D-brane with magnetic field F in a 2-plane is related to a D-brane at angle $\theta = \tan^{-1} F$ in the T-dual.

This mapping facilitates the computation of the open string spectrum, by relating it to a known answer (we leave the direct computation using the boundary conditions (44) as an exercise). An open string stretched between two D-branes with magnetic fields F_a, F_b leads to the same spectrum as an open string stretching between two D-branes with relative angle $\theta = \tan^{-1} F_b - \tan^{-1} F_a$. The generalization of these ideas to several factorized 2-planes is straightforward.

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