Random Tilings
Workshop Schedule

Monday, February 18

10:15am **Fillipo Colomo**
A third-order phase transition in random tilings

11:30am **Bernard Nienhuis**
Solvable incommensurable random tilings

12:30pm **Lunch**

2:30pm **Greta Panova**
Asymptotics of symmetric functions with applications in statistical mechanics

3:30pm **Tea Time**

4:00pm **Dan Betea**
Boxed plane partitions and Aztec diamond via Schur and Macdonald processes

Tuesday, February 19

10:15am **Dan Romik**
Connectivity patterns in loop percolation and constant term identities

11:30am **Luigi Cantini**
Two boundary refinement of the Razumov-Stroganov correspondence

12:30pm **Lunch**

3:00pm **Anita Ponsaing**
The 2-boundary Brauer mode

3:30pm **Tea Time**

4:00pm **Paul Zinn-Justin**
TBA

5:30pm **Wine and Cheese Reception**
Complimentary to participants

6:00pm **Materializing Information Gallery Closing**

Wednesday, February 20

9:15am **David Wilson**
Local statistics of the abelian sandpile model

10:15am **Nicholas Witte**
Discrete Orthogonal Polynomial Ensembles arising in Random Tiling Problems and the discrete Painleve equations

11:30am **Marta Mazzocco**
Quantum Painleve Cubics

12:30pm **Lunch**

2:00pm **Zhongyang Li**
Critical Temperature of Periodic Ising Models

2:30pm **Sevak Mkrtchyan**
Skew plane partitions with non-homogeneous weights

3:30pm **Tea Time**

5:30pm **Wine and Cheese Reception**
Complimentary to participants

6:30pm **Banquet Dinner for Workshop Participants**

Thursday, February 21

10:15am **Andrea Sportiello**
Classification of patterns in the deterministic Abelian Sandpile Model

11:30am **Christoph Richard**
Tilings, Point sets, and Dynamical Systems

12:30pm **Lunch**

2:00pm **Nicolai Reshetikhin**
Bethe vectors and solutions to the reflections q-KZ equation

3:00pm **Vladimir Korepin**
Six vertex model: A counterexample in statistical mechanics

3:30pm **Tea Time**

4:00pm **Hjalmar Rosengren**
Special polynomials related to elliptic lattice models and Painlevé VI

5:30pm **Wine and Cheese Reception**
Complimentary to participants

6:00pm **Nowhere Differentiable Gallery Closing Receptio**

Friday, February 22

9:30am **Alexander Glazman**
Connective constant for a weighted self-avoiding walk on a rhombi tiling

10:15am **Vadim Gorin**
From tilings to general-beta matrix ensembles: appearance of the Gaussian Free Field

11:30am **Leonid Petrov**
Random tilings of polygons by rhombi, and their Gaussian Free Field fluctuations

12:00pm **Fredrik Viklund**
The Virasoro algebra and discrete Gaussian free field

12:30pm **Lunch**

3:30pm **Tea Time**

Abstracts:

Dan Betea:

We will be talking about the combinatorics of tilings of a hexagon by rhombi (boxed plane partitions) and of the Aztec diamond by dominoes in the context of the Schur and Macdonald processes. In particular, various natural measures on such objects arise in this formalism (as first observed by Okounkov and Reshetikhin in the unboxed plane partition case) and for such measures it is very easy to compute the partition functions. The Schur formalism also leads to fast exact sampling algorithms for random boxed plane partitions and Aztec diamonds similar to the shuffling algorithms already known. If time permits, we will describe elliptic generalizations of such measures which lead to the so called elliptic Macmahon formulae. In our context, this is roughly based on the fact that multivariate elliptic functions due to Rains generalize Macdonald polynomials.

Luigi Cantini:

We refine the Razumov Stroganov correspondence between enumerations of Fully Packed Loop configuration on a square and components of the ground state of the dense O(1) loop model on asemi-infinite cylinder, by introducing boundary weights on the top and bottom row. Work done in collaboration with A. Sportiello.

Fillipo Colomo:

We consider the six-vertex model with (suitably modified) domain wall boundary conditions, with a rectangular region erased away from a corner (`L-shaped' six-vertex model). The partition function of the model is computed, and the free-energy is evaluated exactly in the scaling limit, in the so-called free-fermion case (corresponding to the uniform measure on domino tilings).

A third-order phase transition is observed, the control parameter being the size of the erased corner. The transition occurs when the erased corner gets large enough to interfere with the Arctic curve, with the usual Arctic Circle turning suddenly into a concave curve with cusps.

Alexander Glazman:

We consider a self-avoiding walk on a rhombi tiling, all rhombi are cogruent, each has angle~θ between~$\frac{\pi}{3}$ and~$\frac{2\pi}{3}$. The weight of a rhombus depends on the way the self-avoiding walk passes it and the weight of the whole self-avoiding walk is calculated as a product of these weights. If~$\theta=\frac{\pi}{3}$, this can be mapped to a self-avoiding walk on a honeycomb lattice. The connective constant in this case was rigorously proved to be equal to~$\sqrt{2+\sqrt{2}}$ by H.~Duminil-Copin and S.~Smirnov recently. We generalize their proof.

Vadim Gorin:

It is now known that the asymptotic fluctuations of the height function of uniformly random lozenge tilings of planar domains (equivalently, stepped surfaces in 3d space) are governed by the Gaussian Free Field (GFF), which is a 2d analogue of the Brownian motion. On the other hand, in a certain limit regimes such tilings converge to various random matrix ensembles corresponding to beta=2. This makes one wonder whether GFF should also somehow arise in general-beta random matrix ensembles. I will explain that this is indeed true and the asymptotics of fluctuations of classical general-beta random matrix ensembles is governed by GFF. This is joint work with A.Borodin.

Vladimir Korepin:

A common belief in statistical mechanics is that the bulk free energy does not depend on boundary conditions in thermodynamic limit. The six vertex model, also known as ice model, provides a counterexample. In this model the bulk free energy depends on boundary conditions in thermodynamic limit.

Zhongyang Li:

A periodic Ising model has interactions which are invariant under translations of a full-rank sublattice L of Z2. We prove an exact, quantitative characterization of the critical temperature, dened as the supremum of temperatures for which the spontaneous magnetization is strictly positive. For the ferromagnetic model, the critical temperature is the solution of a certain algebraic equation, resulting from the condition that the spectral curve of the corresponding dimer model on the Fisher graph has a real zero on the unit torus.

Marta Mazzocco:

In this talk we will give a short introduction to the Painleve' equations and their monodromy manifolds. We will show the relation between the sixth Painleve' monodromy manifold and the Teichmuller space of a 4-holed Riemann sphere. This will lead to a system of flat coordinates for the PVI monodromy manifold that allows us to quantise. We will then use the confluence procedure to study all other Painleve' equations. We will also investigate the relation with the Cherednik algebra of type \check C_1 C_1 and obtain confluences of it.

Sevak Mkrtchyan:

We study skew plane partitions with non-homogeneous weights. We consider (ordinary) plane partitions with almost periodic weights, and observe a non-differentiable frozen boundary. We also consider two-periodic weights with various boundary conditions.

Unlike the homogeneous case, even when the back wall has non-lattice slopes the plane partitions have finite height in the scaling limit. In addition, under certain boundary conditions we observe two turning points on the outside walls. As a result the local statistics near the turning points is fundamentally different from the homogeneous case. We also compute the correlation kernel in the bulk and observe that it is only $\mathbb{Z}\times2\mathbb{Z}$ translation invariant.

Bernard Nienhuis:

Random tilings of the plane by means of dominos or lozenges have received an increasing amount of attention over the last decades. They stand out in the field of random tilings in the fact that they are solvable, meaning that many expectation values can be found exactly. They can be turned into models of non-interacting fermions.

If one tiles the plane with different tiles with an irrational ratio of their area, the tiling will not be commensurable with a periodic lattice, unless the tiling itself is periodic. This is in contrast to the observation that the vertices of a lozenge tiling are the same as those of the triangular lattice, and that the vertices of the domino tiling are a subset of those of the square lattice.

For example, the tiling of the plane with squares and triangles has vertices that do not match those of any periodic lattice, unless the tiling itself is periodic. At best the configuration can be quasi-periodic. Nevertheless also this tiling is solvable.

It has interesting properties akin to those found in quasicrystals in physics. This tiling model is a member of a family of three: tilings with isosceles triangles and rectangles, all of them solvable. If the ratio of numbers of triangles and rectangles is chosen appropriately the probability distribution of tilings is quasi-periodic and has a respectively twelve- , ten- and eight-fold rotational symmetry.

This talk will have the character of a review of the current knowledge of these tilings, the basis of this knowledge and open questions.

Greta Panova:

We show a new method for studying the asymptotics of certain normalized symmetric functions arising from representation theory (the Schur functions will serve as our main example) as the number of variables tends to infinity. We then apply these results in the setting of several models from statistical mechanics to derive their behavior in the limit. In particular, we study the limit behavior of random lozenge tilings of polygonal (and other) domains and derive the GUE-eignevalue distribution of the positions of horizontal lozenges near the boundary. We also study similar behavior in the 6-vertex model (equivalently, Alternating Sign Matrices). We also derive the asymptotic mean value current in the $O(1)$ dense loop model. The presented results are joint work with Vadim Gorin.

Leonid Petrov:

I will discuss the model of uniformly random tilings of polygons drawn on the triangular lattice by lozenges (= rhombi) of three types (equivalent formulations: dimer models on the honeycomb lattice, or random 3-dimensional stepped surfaces). This model can be formulated as a certain determinantal point process. Asymptotic questions about random tilings (when the polygon is fixed and the mesh of the lattice goes to zero) have received a significant attention over the past years.

Using a new formula for the correlation kernel in the model, I will explain the proof of Kenyon's conjecture (2004) that the large-scale asymptotics of random tilings are asymptotically governed by the Gaussian Free Field. Similar ideas also lead to bulk local asymptotics (governed by the universal "discrete sine" processes) and edge asymptotics (there one sees the Airy(2) process).

Anita Ponsaing:

The Brauer loop model is an integrable lattice model similar to the fully packed O(n) loop model but with crossing loops allowed. In 2005, de Gier and Nienhuis noticed a connection between the ground state of the periodic Brauer loop model and the degrees of some algebraic varieties as calculated by Knutson in 2003. This connection was explored further by Di Francesco and Zinn-Justin in 2006, and proved shortly after by Knutson and Zinn-Justin. In these works a special role was played by the ground state components in the `permutation sector', that is, the components that can be viewed as permutations between points $1,..,n$ and points $n+1,...,2n$.

Around the same time Di Francesco calculated the ground state of the Brauer model with reflecting boundaries, including closed-form expressions for the sum of all components and the sum of all components in the permutation sector. In this talk, we present a work-in-progress in collaboration with Paul Zinn-Justin on similar calculations for the model with non-reflecting boundaries. As in the earlier works, we use the transfer matrix approach to build a possible solution, which we then attempt to prove using recursions. We will discuss the unique difficulties presented by the existence of two non-reflecting boundaries, and compare the model with its non-crossing counterpart.

Nicolai Reshetikhin:

Reflection qKZ equation describes correlation functions and form-factors in integrable systems with integrable boundary conditions. In particular, it describes correlation functions for the 6-vertex model with reflecting boundary conditions. We solve these equations for quantum affine sl_2 and diagonal reflections matrices using "off-shell" Bethe vectors. The talk is based on a joint work with J. Stokman and B. Vlaar.

Christoph Richard:

We will review concepts to describe non-periodic structures such as quasicrystallographic or random tilings. This leads to a description via dynamical systems associated to the tiling. Within this setting, we give an overview of the well-established case of zero entropy tilings and discuss some results and problems in the case of non-zero entropy.

Dan Romik:

Loop percolation, also known as the dense O(1) loop model, can be thought of as a natural variant of critical bond percolation in Z^2. The model has been studied extensively in a cylindrical geometry, where the resulting connectivity pattern of 2n points arranged around the "lid" of the cylinder is a random noncrossing matching (a.k.a. "link pattern") with fascinating properties that appears naturally in connection with the enumeration of Fully Packed Loops (the Cantini-Sportiello-Razumov-Stroganov theorem) and the ground state of the quantum XXZ spin chain.

I will discuss the cylindrical geometry and also consider the model on a half-plane, which is the limit of the cylindrical models. A remarkable rationality phenomenon has been observed by Zuber and others whereby the probabilities of certain connectivity events are simple rational numbers (in the half-plane case) or simple rational functions of n (in the cylindrical case). For example, the probability for two given adjacent endpoints to be connected is 3/2*(n^2+1)/(4n^2-1), or 3/8 in the limiting case of the half-plane. This result and a few other instances of the rationality phenomenon have been proved by Fonseca and Zinn-Justin. One of the new results I will discuss is a much more general formula expressing the probabilities of arbitrary "submatching events" as constant terms of certain multivariate Laurent polynomials. This reduces the problem of proving the rationality phenomenon in the general case to that of proving a concrete conjecture in algebraic combinatorics about a family of constant term identities.

Hjalmar Rosengren:

Recently, various objects related to elliptic lattice models have been described by polynomials in one or two variables, typically with positive coefficients. In particular, we mention eigenvalues of the Q-operator (Bazhanov and Mangazeev), eigenvalues of the XYZ Hamiltonian (B&M, Fendley and Hagendorf, Razumov and Stroganov, Zinn-Justin) and the domain wall partition function for the three-colour model (Rosengren). We describe how these polynomial systems are related, in that they can all be constructed as special cases of certain symmetric polynomials. These polynomials can be identified with affine Lie algebra characters and satisfy a non-stationary Schrödinger equation with elliptic potential. Further specializations of these polynomials include a four-dimensional lattice of solutions to Painlevé VI, related to one of Picard's elliptic solutions by Bäcklund transformations.

Andrea Sportiello:

The Abelian Sandpile Model (ASM) is a non-equilibrium statistical mechanics systems that, after the work of Dhar et al. in the 90's, in its "stochastic" implementation, is known to be related to the Potts Model (in the sector of `connected subgraphs'), and thus, in D=2, to the c=-2 logarithmic CFT. The investigation of its behaviour under "deterministic" dynamics is similarly old, but precise quantitative results are mostly recent.

I present the results of common work with G. Paoletti and S. Caracciolo, that aim at the classification of periodic patterns emerging in the deterministic sandpile, and the proof of the emergence of remarkable exact fractal Sierpinski-like structures.

Fredrik Viklund:

I will discuss recent joint work with C. Hongler (Columbia) and K. Kytola (Helsinki) concerning the discrete Gaussian free field on a square grid. I will indicate how for this model discrete complex analysis can be used to construct concrete (exact) representations of the Virasoro algebra of central charge 1 directly on the discrete level.

David Wilson:

We show how to compute local statistics of the abelian sandpile model on the square, hexagonal, and triangular lattices. The one-site marginals alone on the square lattice took 20 years to determine. We prove that on the square lattice, all local events are rational polynomials in 1/pi, while on the hexagonal and triangular lattices they are rational polynomials in sqrt{3}/pi. The proofs use the Cori and Le Borgne version of Majumdar and Dhar's burning bijection between sandpiles to spanning trees, and the methods of Kenyon and Wilson for computing grove partition functions.

Nicholas Witte:

Many random tiling problems lead to discrete orthogonal polynomial ensembles with weights drawn from the Askey table of hypergeometric and basic hypergeometric orthogonal polynomials. Examples include the Krawtchouk weight in the case of random domino tiling of the Aztec diamond and the Hahn weight in the case of random rhombus tiling of a hexagon. A theoretical framework will be given whereby such ensembles can be characterised by a classical solution to an appropriate discrete Painleve equation, and we also give details of the correspondences between the Askey table and the Sakai scheme.

Paul Zinn-Justin: