Dynamics of Peakons, Jetlets and G-strands

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Abstract

The way we view hydrodynamics changed forever when Arnold made his revolutionary discovery [1] that the Euler equations for an ideal fluid represent geodesic motion on SDiff (volume preserving diffeomorphisms) with respect to the L^2 norm on the tangent space TSDiff$\simeq \mathfrak{X}_{div} \times$ SDiff, where $\mathfrak{X}_{div}$ denotes the divergence-free vector fields. Arnold's famous paper has led to many further developments in continuum dynamics. These developments range, for example, from shallow-water solitons to shape analysis for computational anatomy.

The developments of Arnold's discovery that we will discuss in this talk are based on a dual pair of momentum maps that emerge from the Euler-Poincaré theory of Lagrangian reduction by symmetry when the symmetry is the Lie group of diffeomorphisms acting on a smooth manifold M, or on a space of smooth embeddings in M [2].

The examples we shall discuss as variations on the theme of dual momentum maps are:

1. Shallow-water solitons called peakons
2. Jetlets: a new type of coherent particle-like fluid excitation that carries momentum and angular momentum, while preserving its circulation.
3. G-strands: maps from $\mathbb{R}^2$ (or $\mathbb{C}$) into a Lie group G that are determined from Hamilton's principle for a Lagrangian that is *invariant* under G. For continuum dynamics in a domain M, the group is $G = \mathrm{Diff}(M)$.

If time remains, we will also say a word about stochastic extensions of these examples.

References

[1] Arnold, V. I., "Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamique des fluides parfaits," Annales de l'institut Fourier, **6**, No. 1, 319–361 (1966).

[2] Holm, D. D., Marsden, J. E., "Momentum Maps and Measure-valued Solutions," in: The Breadth of Symplectic and Poisson Geometry, J.E. Marsden and T.S. Ratiu, Editors, Birkhäuser Boston, Boston, MA, 2004, Progr. Math., **232**, pp. 203–235.