

Speaker: Greg Parker, MIT

Title: Gluing \mathbb{Z}_2 -Harmonic Spinors

Abstract: \mathbb{Z}_2 -harmonic spinors are singular generalizations of classical harmonic spinors and appear in two contexts in gauge-theory. First, they arise as limits of sequences of solutions to equations of Seiberg-Witten type on low-dimensional manifolds; second, they are the simplest type of (singular) Fueter section-- objects which arise naturally on calibrated submanifolds in the study of gauge theory on manifolds with special holonomy. These two pictures are related by proposals for defining invariants of manifolds with special holonomy.

In this talk, after giving an introduction to these ideas, I will discuss a gluing result that shows a generic \mathbb{Z}_2 -harmonic spinor on a 3-manifold necessarily arises as the limit of a family of two-spinor Seiberg-Witten monopoles. Due to the singularities of the \mathbb{Z}_2 -harmonic spinor, the relevant operators in the gluing problem are only semi-Fredholm and possess an infinite-dimensional cokernel. To deal with this, the proof requires the analysis of families of elliptic operators degenerating to a singular limit, and the study of deformations of the singularities which are used to cancel the infinite-dimensional cokernel. At the end, I will discuss related problems in gauge theory and calibrated geometry.