Topics in Homotopy Theory and Geometry

September 14, 2018

1   Overview and Prerequisites

I am plan to give a series of lectures on different topics in homotopy theory and differential topology aimed at graduate students. I will assume basic background which is typically taught in first year graduate courses in algebraic topology and differential topology.

Background Material
In algebraic topology these include:

- Simplicial complexes, CW complexes, path spaces and Serre fibrations,

- Definition of singular homology and cohomology of a topological space and simplicial homology and cohomology of simplicial complexes, Cech homology,

- Notions of homotopy and homotopy equivalence, homotopy groups

- Hurewicz Theorem, Whitehead Theorem, and Universal Coefficient Theorem,

and in differential topology:

- definition of topological manifolds, smooth manifolds, orientablilty,

- classification of compact surfaces

- covering spaces,

- transversality of submanifolds, Sard's theorem,

- Poincré duality (I will cover this briefly when it becomes necessary),

- deRham's theorem

There are many good references that cover this background material — "Algebraic Topology" by Hacther, and "Differential Forms in Algebraic Topology" by Bott and Tu and good ones.

2 Prelimiary List of Topics (subject to change as the semester progresses)

- Lecture 1: Obstruction Theory, Eilenberg-MacLane spaces, and Postnikov towers.

- Lecture 2: Simplicial sets and simplicial homotopy theory

- Lecture 3: Spectral Sequences, double complexes, Cech to singular SS, Atiyah-Hirzebruch SS, Serre SS, Hodge-to-deRham SS.

- Lecture 4: Topics in cohomology — Cup products, higher order products, Milnor construction and cohomology of groups, equivariant cohomology.

- Lecture 5: Steenrod squares, chain approximations to diagonal, higher order step functions, Adem relations and Cartan formulae, cohomology of K(Z/2,n).

- Lecture 6: Manifold topology: Poincaré duality, intersection theory, transversality, Sard's theorem

- Lecture 7: Quillen's Rational Homotopy Theory

- Lecture 8: Differential forms and deRham's theorem

- Lecture 9: Differential forms and rational homotopy theory

- Lecture 10: Kähler metrics on smooth complex projective varieties

- Lecture 11: Rational Homotopy Theory of smooth complex projective varieties.