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Airy ideals and topological recursion: An investigative tool for enumerative geometry, VOAs, and gauge theories

Abstract: The Chekhov-Eynard-Orantin topological recursion is an abstract framework that appears in many contexts, from enumerative geometry to mathematical physics. In these lectures I will introduce the concept of Airy ideals in the Rees Weyl algebra, which provides a clean reformulation of the topological recursion in the language of D-modules. After introducing the foundations of the theory, including the existence and uniqueness theorem originally proved by Kontsevich and Soibelman, I will focus on applications in enumerative geometry, VOAs and gauge theories. Such applications include the construction of Whittaker vectors for various  $W$ -algebras and Gaiotto vectors for supersymmetric gauge theories, ELSV-type formulae for Hurwitz numbers,  $W$ -constraints for various enumerative invariants, etc. The lectures are meant to be introductory; my hope is to convey why I believe that the formalism of topological recursion and Airy ideals should be in the toolbox of all geometers and mathematical physicists!