

## Abstract

We introduce a family of geometrical lattice models generalising the well-known loop model on the hexagonal lattice. These models have a  $U_q(\mathfrak{sl}_n)$  quantum group symmetry, the loop model being the  $n=2$  case. The general models give rise to branching webs and describe, at a special point, the interfaces in  $Z_n$  symmetric spin models. We mainly discuss the  $n=3$  case of bipartite cubic webs, which is based on the Kuperberg  $A_2$  spider. We exhibit a local vertex-model reformulation, analogous to the well-known correspondence between the loop model and the nineteen-vertex model. The local formulation allows us in particular to study the model by means of transfer matrices and conformal field theory. We find that it has a rich phase diagram, including a dense and a dilute phase that generalise those known for the loop model.