Abstract

Multi-matrix integrals in planar, large N limit are genuine functional integrals, in general very difficult to compute. Apart from some known solvable examples (one-matrix model, two-matrix model with specific interaction, etc) one has to rely on perturbation theory or Monte-Carlo at large enough N. Matrix bootstrap (MB), initiated by Anderson and Kruczenski, and Lin, is an interesting alternative to these methods. MB deals with the planar loop equations for loop moments -- the averages of traces of "words" built out of products of matrix variables. The number of unknowns - loop moments - grows quicker then the number of loop equations. The needed extra conditions come from the positivity of correlation matrix of loop moments. This allows to establish the upper and lower limits for particular, lowest loop moments, sometimes with an excellent precision. The main difficulty of the previous works is the non-linearity of loop equations, leading to a non-convex optimization procedure. In our recent paper with Zechuan Zheng, we propose to complete this scheme with the relaxation procedure: the non-linear loop equations are incorporated into the relaxation matrix as linear inequalities. The problem becomes the standard SDP, allowing for longer loops and thus a better precision. We demonstrate the relaxed matrix bootstrap (RMB) on the example of an analytically unsolvable 2-matrix model. The RMB for Z_2 symmetric states gives a very satisfactory precision for generic parameters, up to 6 digits. We also managed to apply RMB to more challenging, Z_2 symmetry breaking solutions, though with less of precision. We also prove analytically, using the Hamburger problem, that MB for the 1-matrix model converges to physical solutions, with eigenvalues distributed only on the real axis.