

Speaker: Chris Brav

Title: From Calabi-Yau categories to the symplectic geometry of moduli spaces

Abstract: First, we survey some ideas from the non-commutative geometry of differential graded categories, in particular so-called Calabi-Yau structures on categories, and how they can be used to construct symplectic structures on various moduli spaces appearing in algebraic geometry, representation theory, and symplectic topology. Second, we use deformation theory of Calabi-Yau structures to show how cyclic homology classes on 2d Calabi-Yau categories give rise to Hamiltonians on such moduli spaces and a calculation of Poisson brackets between them. Examples include Goldman's Hamiltonians on character varieties, Hitchin's Hamiltonians on the moduli space of Higgs bundles, and Hamiltonians on Nakajima quiver stacks generalising the Calogero-Moser Hamiltonians from integrable systems. These results give chain-level refinements of more classical results on the Goldman Lie bracket of loops and the necklace Lie algebra of a quiver, put into a general framework and admitting generalizations to higher dimensions, where one obtains shifted symplectic structures and chain-level string brackets generalising those of Chas-Sullivan. This first part is joint work with Tobias Dyckerhoff and the second part with Nick Rozenblyum.