Title: Exponentially small phenomena in analytic convex billiards

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Abstract:
Many periodic and heteroclinic billiard trajectories occur in groups, so it is natural to compare their lengths. We shall describe two scenarios
where such length differences are exponentially small in some parameter.
Analiticity is a key hypothesis for such phenomena.

First, we consider heteroclinic billiard trajectories inside some symmetric
analytic perturbations of ellipses with small eccentricity and we check
numerically that length differences are exponentially small in the eccentricity
provided the perturbation is sufficiently small in the eccentricity.

Second, we consider (1,q)-periodic billiard trajectories inside analytic convex
curves and we prove analytically that the length differences are exponentially
small in the period q. We also check numerically that they satisfy relatively
simple asymptotic formulas in some model symmetric perturbations of ellipses
and circles. Perturbed circles are harder to deal with than perturbed ellipses.

Numerical computations require a multiple-precision arithmetic and have been
programmed in PARI/GP.

The classical Melnikov method provides asymptotic exponentially small formulas
for these length differences in perturbative settings, but we will present
examples where the prediction of the Melnikov function is wrong. The exponent
in the true asymptotic formulas, if they exist, should be related to the
imaginary part of the closest complex singularity to the real line of some real
analytic function that has to be correctly identified, which is not an easy
task. This is a work in progress in collaboration with Tere M. Seara.