

Title: Hodge theory of Lagrangian fibrations I,II,III

Abstract: For a family of smooth projective varieties, we can obtain local systems on the base that calculate the cohomology of the fibers. These local systems underly variation of Hodge structures (VHS), which “linearize” the smooth family we start with. In these lectures, we will first discuss interesting symmetries carried by these local systems/VHS when the family is formed by a Lagrangian fibration associated with a symplectic variety. Then we will discuss the (much more interesting) case when the Lagrangian fibration admits singular fibers. The theory of perverse t-structure by Beilinson-Bernstein-Deligne-Gabber allows us to extend the local systems described above to perverse sheaves; then Saito’s theory of Hodge modules further extends the VHS of the smooth fibers to the total base. We will present results and conjectures concerning structures of these perverse sheaves and Hodge modules. Our theory applies to both Lagrangian fibrations associated with compact hyper-Kähler manifolds, and non-compact Lagrangian fibration (e.g. Hitchin’s integrable systems). Part of these lectures is based on joint work in progress with Davesh Maulik and Qizheng Yin.