

Hyperuniformity and its connection to number theory and geometry

The study of hyperuniform states of matter is an emerging multidisciplinary field, influencing and linking developments across the physical sciences, mathematics and biology. A hyperuniform many-particle system in d -dimensional Euclidean space is characterized by an anomalous suppression of large-scale density fluctuations relative to those in typical disordered systems, such as liquids and amorphous solids. As such, the hyperuniformity concept generalizes the traditional notion of long-range order to include not only all perfect crystals and quasicrystals, but also exotic disordered states of matter, thus providing a unified framework to quantitatively categorize such phases of matter. Disordered hyperuniform states have attracted great attention across many fields over the last two decades because they have the character of crystals on large length scales but are isotropic like liquids. I will begin by reviewing the hyperuniformity concept. Then I will give an overview of how hyperuniformity is linked to the Epstein zeta function of number theory, sphere packing problems, eigenvalues of random matrices, nontrivial zeros of the Riemann zeta function, spatial distribution of the prime numbers, free fermions, Laughlin's incompressible quantum states, and photoreceptor mosaics in avian retina.