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Speaker: Sam Punshon-Smith

Title: Chaotic mixing of scalars and Batchelor spectrum in stochastically forced fluid mechanics

Abstract: The Lagrangian trajectories associated with general time dependent fluid motions are often chaotic. When the velocity field is not too rough, the associated stretching and folding of trajectories is the main mechanism responsible for the exponential mixing of quantities that are passively advected by the flow (i.e temperature or some chemical concentration). Yet despite this relatively intuitive picture, it is notoriously difficult to prove that even relatively explicit flows are chaotic or mixing. In this talk I will discuss how one can use tools from ergodic theory of random dynamical systems, particularly a theorem due to H. Furstenberg, to show that Lagrangian chaos and mixing is, in some sense, a 'generic' property of time dependent fluid motions. Specifically, I will discuss how solutions to the 2d Navier-Stokes equation (among other Markovian fluid models) subject to a non-degenerate, white-in-time stochastic forcing have a chaotic Lagrangian flow, almost surely, and mix scalars exponentially fast, uniformly in the diffusivity. As a consequence, we are able to rigorously prove a well known prediction, due to George Batchelor, on the power spectrum of passive scalars. This is based on joint work with Jacob Bedrossian and Alex Blumenthal.