SIMONS CENTER
FOR GEOMETRY AND PHYSICS

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spring/summer 2021

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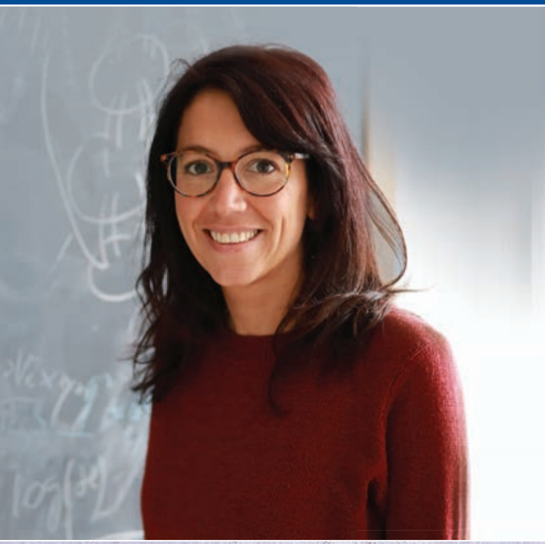

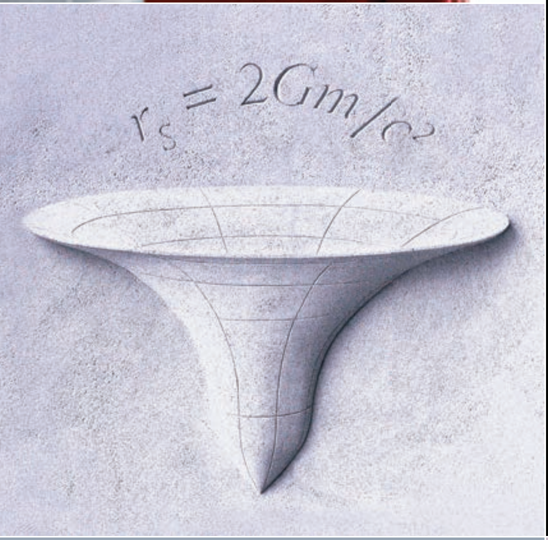

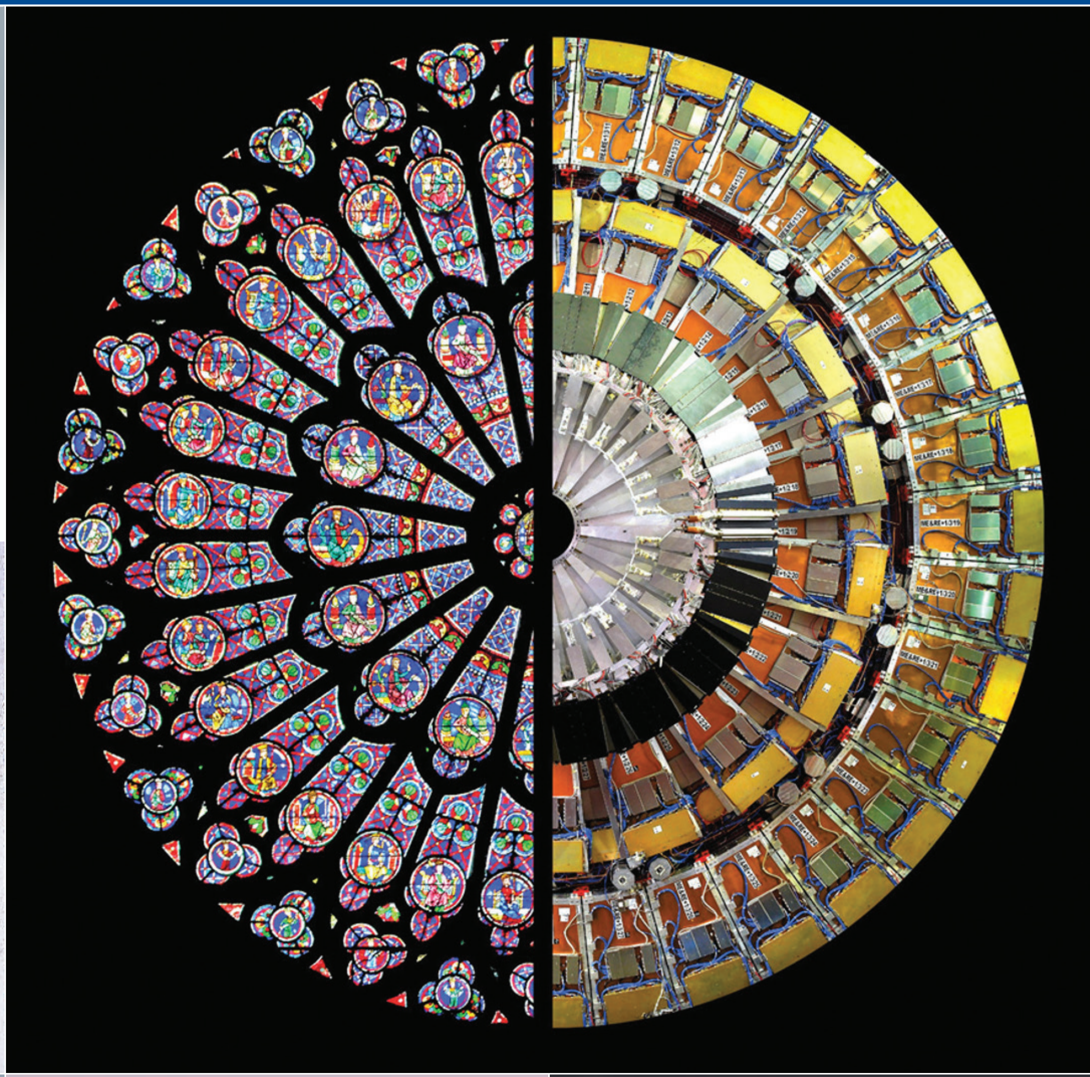

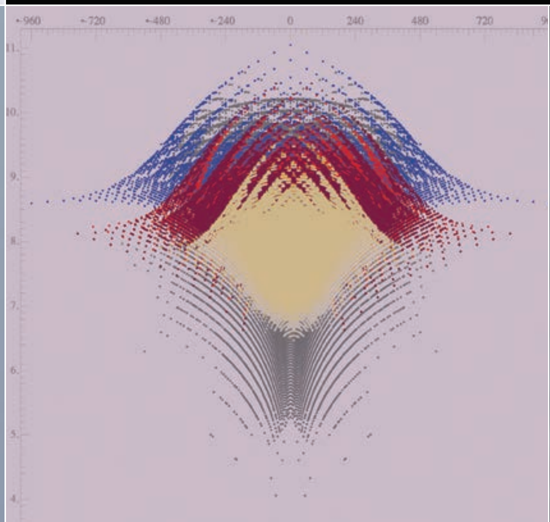

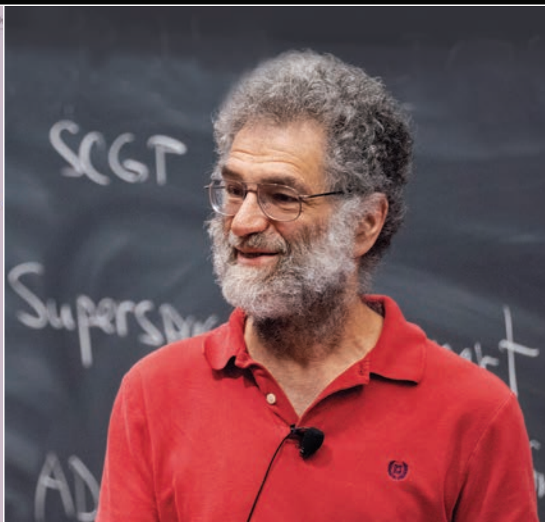

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SIMONS CENTER FOR GEOMETRY AND PHYSICS

Stony Brook University

SCGP NEWS
Vol XVII

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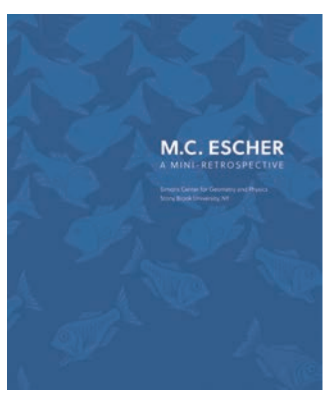
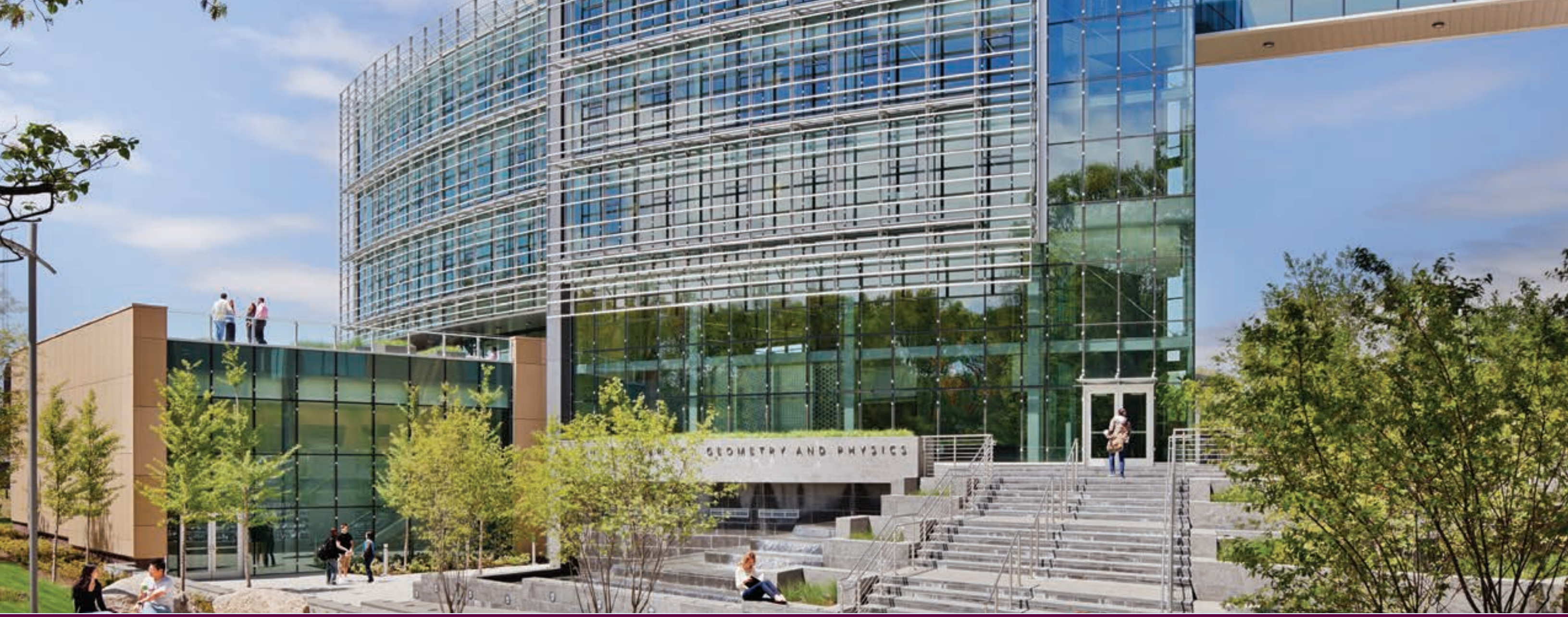


A full review of the Center's fall 2021 exhibition *M.C. Escher®: A Mini-Retrospective* will appear in the next issue of SCGP NEWS.

Image: Exhibit catalogue cover. Design by Ellen Lynch and Lorraine Walsh



Programs & Workshops

Photo: Bob Giglione

CONTINUED: RENORMALIZATION AND UNIVERSALITY IN CONFORMAL GEOMETRY, DYNAMICS, RANDOM PROCESSES, AND FIELD THEORY

February 22 - March 19, 2021
Workshop: March 4-12, 2021

Organized by Dzmitry Dudko, Mikhail Lyubich, Kostya Khanin, David Campbell, and Dennis Sullivan

The program resumed online in the fall of 2020 with a mini-course by Yusheng Luo, Sabya Mukherjee, and Dimitrios Ntalampekos on the fascinating interplay between the dynamics of (anti-) rational maps, Kleinian groups, and Schwarz reflections. The latter theme is a fresh direction of research (related to the geometry of quadrature domains, Coulomb gas ensembles and the Hele-Shaw flow) that sheds new light on the classical Fatou-Sullivan Dictionary between rational maps and Kleinian groups.

It followed with a mini-course by Pierre Berger and Dmitry Turaev *Zoology in the Hénon Family: Renormalization of homoclinic tangencies.* Bifurcations near homoclinic tangencies is a classical topic going back to the 1970s, with a discovery of the Newhouse phenomenon on persistent co-existence of infinitely many attracting cycles (which was the final blow to Smale's program on generic hyperbolicity in dimension two and higher). It was followed up in the 1980-90s by a construction and exploration of Benedicks-Carleson stochastic attractors, supporting a physical SRB measure that governs behavior of almost all orbits of the system. In their mini-course, Berger and Turaev explained that any conceivable behavior can be born in the homoclinic bifurcation, including abundant supply of conservative maps with positive characteristic exponent (confirming Herman's Conjecture) and construction of Hénon maps, real and complex, that have wandering domains (settling a long-standing problem).

In the next mini-course, *Renormalization of maps of the circle and related topics*, Michael Yampolsky elaborated on the results announced in his earlier Colloquium talk *(construction of a Renormalization Horseshoe of circle diffeomorphisms and its applications)* and also discussed a conjectural interplay between two circle renormalizations of diffeomorphisms and of critical circle maps (with the latter appearing on the boundary of the former).

The last mini-course, *Renormalization of quasi-periodic dynamics* was delivered by Raphael Krikorian right before Thanksgiving. It gave a thorough conceptual introduction to the Herman-Yoccoz Rigidity Theory of circle and annulus diffeomorphisms.

After Thanksgiving, Dzmitry Dudko gave a mini-course on *A priori bounds for neutral quadratic polynomials*, where he gave a detailed outline of recent results by Dudko and Lyubich that give a chance of full classification of neutral dynamics for arbitrary rotation numbers.

After the New Year's break, the program resumed in January 2021, still in the online regime, with a mini-course by Pavel Bleher and Roland Roeder on Renormalization in the classical hierarchical models, the Dyson model, and the Migdal-Kadanoff model. The Dyson model was the first occasion when the stat-physical renormalization theory was developed, by Bleher and Sinai, in a coherent mathematical way. This theory was explained in the first part of the mini-course. The second part was devoted to the Migdal-Kadanoff model where renormalization appears as a rational map of the projective space. It allowed Bleher, Lyubich, and Roeder to describe completely the asymptotic distribution of the Lee-Yang zeros that govern phase transitions in the model.

The second mini-course, by Theodore Drivas, was devoted to the theory of turbulence. Classical Kolmogorov phenomenological theory predicted critical exponents for the power laws characterizing scaling behavior in the turbulent regime of vanishing viscosity. It follows that the velocity field in the turbulence regime has Hölder exponent 1/3. L. Onsager has shown that the same Hölder exponent can be also viewed as the point of transition from the regime of conservation of energy to anomalous dissipation. In his mini-course, T. Drivas described recent results on construction of weak solutions to the Euler equation with the Hölder exponent arbitrarily close to the exponent 1/3. He also discussed problems related to the phenomenon of intermittency, which can be viewed as multifractal behavior of the velocity field. In his final lecture, he introduced the concept of intrinsic stochasticity and presented new results in this direction obtained in the setting of shell models.

In early March, the Feigenbaum memorial conference was launched. It was opened with a Colloquium talk by Predrag Cvitanovic with an intriguing title, *Do clouds solve PDE?*. This talk contained a mixture of personal recollections and scientific remarks on a possible approach to turbulence. The following morning, the focus of the conference was dedicated to physics and math physics related to Feigenbaum's legacy. In particular, Jean-Pierre Eckmann told us about Feigenbaum's striking observations from his unpublished book on Optics.

The afternoon session of that day was an informal exchange of memories about Feigenbaum. Many warm stories about his character and his way of thinking illuminated the unique personality of this remarkable man.

The final day of the conference was dedicated to a mathematical retrospective of the Renormalization ideas in the past 45 years, particularly in the *One-Dimensional and Holomorphic Dynamics* talk, given by Kostya Khanin, Misha Lyubich, and Dennis Sullivan.

The Feigenbaum memorial conference followed up immediately with a Workshop, *Many Faces of Renormalization.* It presented a broad panorama of various aspects of the Renormalization phenomenon in mathematics and physics including, but not limited to: one-dimensional dynamics, real and complex, conservative and dissipative two-dimensional dynamics, Teichmüller flows and billiards, Schrödinger cocycles and almost Mathieu equations, turbulent flows, and renormalization aspects of Quantum Field Theory. High points of the conference were Colloquium talks by Sasha Polyakov *On praise of Quantum Fields Theory,* with a broad retrospective of this area since the mid-20th century, and by Curt McMullen "Billiards and arithmetic of non-arithmetic groups," presenting several beautiful examples of the interplay between dynamics, Teichmüller theory, and number theory. The latter talk was used as an opportunity to have a toast in honor of Dennis Sullivan, one of the key players of the remarkable Renormalization story. The program did not culminate with the workshop. It followed up with a talk by Stefano Marmi on the automorphic features of the Bruno and related number-theoretic functions, among several other presentations, and a mini-course by Vadim Kaimanovich on *Self-similarity of groups and random walks* related to the Schur Spectral Renormalization for self-similar groups, one of early themes of the program. A few more talks on a variety of renormalization-related topics followed up in the late spring, concluded with a mini-course by Davoud Cheraghi giving a full topological classification of post-critical sets for neutral fixed points of quadratic polynomials (based upon the Inou-Shishikura Renormalization Theory). ♦

SUMMER SEMINAR SERIES: APPLICATIONS OF GAUGE TOPOLOGY, HOLOGRAPHY AND STRING MODELS TO QCD

June 1 - August 16, 2021

Organized by Massimo D'Elia, Jeff Greensite, Elias Kiritsis, Zohar Komargodski, Edward Shuryak, Jacob Sonnenschein, and Ismail Zahed.

This past summer, the Applications of Gauge Topology, Holography, and String Models to QCD program continued its weekly seminar series from June 1 - August 16, 2021. Following a similar format as last year's series, the lectures were held via zoom twice a week. The list of speakers and their talks are listed below, and all the video recordings are available on the SCGP Video Portal.

The number of participants last summer was between 40 and 60. This summer it was smaller, oscillating around 20, possibly due in part to the number of zoom-based meetings significantly increasing in universities and research centers. The discussions that followed were intense and sometimes took well over the allocated time of the seminars.

The organizers encouraged speakers to give pedagogical reviews rather than standard conference talks on their latest papers, and these reviews comprised a significant fraction of the talks. The two main subjects,gauge topology and holographic models, were equally represented. All the talks were generally attended by people with a wide range of interests on various aspects of QCD-related theory. The series contributed a lot to the field during this difficult time of the quarantine.

The organizers are grateful to the Simons Center for Geometry and Physics for taking this experiment under its wing.

The posting of talks and reminders with zoom coordinates, as well as zoom sessions themselves and the placement of recordings to the video portal, worked flawlessly throughout the summer. We understand that SCGP personnel perhaps were not in their offices all the time due to COVID restrictions, and we thank them for making the whole operation run so smoothly. ♦

June 7, 2021	Dmitri Kharzeev	Chiral magnetic effect: from quarks to quantum computers
June 10, 2021	Romuald Janik	A simple description of holographic domain walls in confining theories-extended hydrodynamics
June 14, 2021	Dallas DeMartini	Confinement in the SU(3) Instanton-dyon Ensemble
June 17, 2021	Kiminad Mamo	Electro-production of heavy vector mesons: probing the spin-2 (gravitational) and resummed spin-j (Pomeron) of the proton
June 21, 2021	Amos Yarom	Stochastic gravity and turbulence
June 24, 2021	Umut Gursoy	Hydrodynamics of spin currents
June 28, 2021	Erich Poppitz	The mixed 0-form/1-form anomaly in Hilbert space: pouring the new wine into old bottles
July 1, 2021	Andrea Guerrieri	S-matrix Bootstrap for effective field theories
July 7, 2021	Massimo D'Elia	Thermal monopoles and color confinement in full QCD
July 8, 2021	Matti Jarvinen and Jacob Sonnenschein	Many phases of generalized 3D instanton crystals
July 12, 2021	Mithat Unsal	Graded Hilbert spaces and quantum distillation in QFT
July 19, 2021	Christian Weiss	Instanton vacuum, chiral symmetry breaking, and the gluonic structure of hadrons
July 22, 2021	David Mateos	Real-Time Dynamics of Plasma Balls (and Cosmological Bubbles) from Holography
July 26, 2021	Lorenz von Smekal	Quark free energies and electric fluxes
July 29, 2021	Miguel Costa	Regge theory in holographic QCD
August 2, 2021	Micheal Teper	SU(N) gauge theories in 3+1 dimensions: glueball spectra, running couplings, k-strings, topology.
August 5, 2021	Francesco Nitti	Holographic theories at finite theta-angle, glueball spectra and instanton condensation
August 9, 2021	Jeff Greensite	Stable field excitations around static fermions in gauge Higgs theories
August 12, 2021	Yizhuang Liu	"Holographic pentaquarks"
August 16, 2021	Edward Shuryak	Instanton effects in hadronic structure

VIRTUAL WORKSHOP: QUANTUM HALL EFFECT: STATUS REPORT

May 3-7, 2021

Organized by Andrey Gromov, Gabor Csáthy, F. Duncan M. Haldane, Steven Simon, and Dam Thanh Son.

This workshop took place over Zoom. The main goal of the workshop was to bring together theorists and experimentalists working on fractional quantum Hall effect. We particularly wanted to emphasize experimental talks. Almost every session consisted of a mix of an experimental talk and a theoretical talk. Put together, this produced a very lively atmosphere and stimulated plenty of discussions.

The major objective was to communicate to theorists the major achievements of the experimentalists over the last few years. While it is too early to say, we hope that this workshop will lead to more theoretical work grounded in experiment as well as new experimental work inspired by questions that were asked by theorists.

The notable experimentalists included: Moty Heiblum, Gabor Csathy, Mansour Shayegan, Mitali Banerjee, Loren Pfeiffer. Notable theorists were: Bert Halperin, Eduardo Fradkin, Jainendra Jain, F. Duncan, M. Haldane, Dam Thanh Son, Michael Zaletel, Kun Yang, Paul. Wiegmann. ♦

VIRTUAL WORKSHOP: C*-ALGEBRAS, K-THEORIES AND NONCOMMUTATIVE GEOMETRIES OF CORRELATED CONDENSED MATTER SYSTEMS

May 17-21, 2021

The overall objective of the workshop was to explore the extent to which operator algebra theory and noncommutative geometry might supply a mathematical framework for the study of phases of gapped many-body systems, especially in the context of interactions, where one faces many difficulties. Needless to say, this complex issue was not resolved within the course of a one-week meeting, but experts from both physics and mathematics were introduced to one another and made aware of the techniques available and the problems that need to be addressed. We judge the meeting to have been a success.

The workshop was held over Zoom, and it was given a special format in order to combat online fatigue. The central event of each day was a two-hour discussion session, while two or three overview talks were scheduled beforehand to serve as introductions to topics and problems. Each session was chaired by a specialist, who gave a brief synopsis, listed several open questions, and moderated the discussion. Participation and engagement were high, the discussion was lively, and the sessions continued beyond their allotted times each day. The speakers, moderators, and discussion topics were as follows:

Day 1	
Nigel Higson	Talk: What can NCG do?
Pieter Naaijkens	Talk: Split property and long-range entanglement
Johannes Kellendonk	Moderated discussion: Bulk-defect correspondence principle in the KK-theoretic framework
Day 2	
Chris Bourne	Talk: KK-theory and free-fermionic topological phases from the viewpoint of SPT phases
Lukasz Fidkowski	Talk: Classification of correlated phases of matter
Anton Kapustin	Moderated discussion: Towards classification of interacting short-range entangled phases of matter
Day 3	
Hermann Schulz-Baldes	Talk: Index theory in disordered topological insulators and semimetals
Maissam Barkeshli	Talk: (2+1)D topological phases of matter and G-crossed braided tensor categories
Emil Prodan	Moderated discussion: Roe/groupoid-algebras and disordered topological phases
Day 4	
Yoshiko Ogata	Talk: Classification of symmetry protected topological phases of one-dimensional fermion systems
Alexander Engel	Talk: Propagation of operators and (uniform) Roe algebras
Amanda Young	Talk: Gap stability of topologically ordered ground states in the infinite volume setting
Bruno Nachtergaele	Moderated discussion: Gapped phases of correlated matter in the framework of operator algebras
Day 5	
Sven Bachmann	Talk: Topological indices in the presence of interactions
Bram Mesland	Talk: Groupoid algebras and topological phases of matter
Francesca Arici	Moderated discussion: Beyond K-Theory/KK-theory/NCG

VIRTUAL WORKSHOP: NEW DIRECTIONS IN TOPOLOGICAL PHASES: FROM FRACTONS TO SPATIAL SYMMETRIES

May 24-28, 2021

Organized by Jennifer Cano, Dominic Else, Andrey Gromov, Siddharth Parameswaran, and Yizhi You

Our workshop took place over Zoom. The main goal of the workshop was to bring together experts in fractons, topological crystalline materials, topological phases, soft condensed matter physics and quantum field theory broadly defined. The talks have covered an extraordinary variety of topics and allowed participants to learn about applications of fractons far outside of their immediate domain of expertise.

Notable speakers included Maissam Barkeshli, Michael Hermele, Jeongwan Haah, Nathan Seiberg, Masaki Oshikawa, Roderich Moessner, T. Hughes, Leo Radzihovsky, Fiona Burnell, and X.G. Wen.

We chose the 15-minute per talk format to allow as many junior researchers as possible to present their work as well. The talks were separated by discussion breaks most of which were utilized to the full extent.

Lastly, the workshop played a crucial role in keeping the community together and talking to each other during the pandemic. ♦

Simons Summer Workshop annual banquet at Avalon Nature Preserve, Stony Brook.
Photo: Martin Rocek

Simons Summer Workshop

By Martin Rocek
Edited by Lorraine Walsh

Summer workshop participants heading to the beach.
Photo: Martin Rocek

The theme of the 2021 Simons Summer Workshop that took place from July 12th to August 6th was *Strings and Geometry.* After a COVID induced hiatus, the annual event convened with an enthusiastic group of researchers enjoying lectures, discussions, Café lunches, the weekly evening parties, Tuesday concerts—and of course the beach.

It was evident how much we missed personal interactions—Zoom is not enough! And while there were fewer visitors from abroad due to COVID, local participation made the workshop lively and productive.

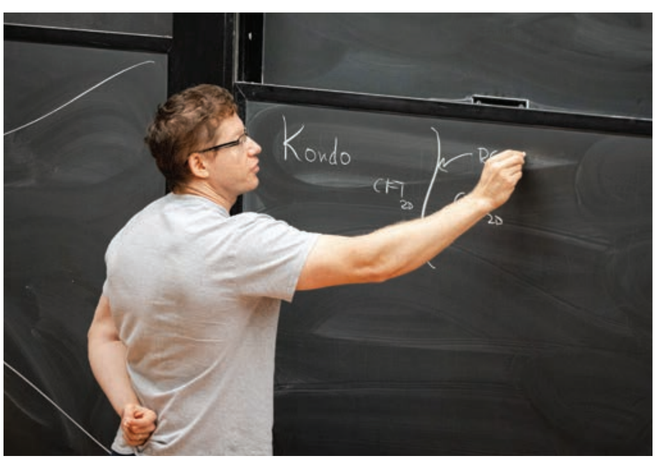
The first week included physical talks: Simeon Hellerman's lecture on amazing results for CFT's at large charge, and more formal approaches such as Amihay Hanany's contribution on magnetic quivers. Lectures that explicitly related string theory and geometry were Cumrun Vafa's beach talk on the mathematical aspects of the Swampland, Du Pei's new invariants of manifolds from string theory, and mathematical perspectives on physics by Ron Donagi.

The second week began with two talks on black holes: Juan Maldacena's comments on string scale black holes and Mark Mezei's discussion of the interior volume inside a black hole. Sergei Gukov provided the beach talk and spoke about theories that are not standard CFT's, nevertheless interesting, with many fascinating connections between VOA's, geometry of 3-manifolds, and TQFT's. The last two talks of the week focused on aspects of dualities.

The third week commenced with a discussion of R-matrices and gauge theories on spaces with interfaces. In the afternoon, we enjoyed a wonderful presentation on mosque architecture by Dr. Valérie Gonzalez, complete with fantastic pictures of geometric examples (please see page 24 for more information). Zohar Komargodski discussed line defects in CFT's, and thereafter at the beach,

Top row: Cinzia Da Vià and Luis Álvarez-Gaumé; Nathan Haouzi and a summer workshop feast; Cumrun Vafa, Afarin Sadr, and son. Photos: Martin Rocek. Bottom row: Nikita Nekrasov; Cumrun Vafa; Martin Rocek. Photos: Joshua Klein

Geoff Pennington talked about the information problem in a tractable toy model (JT gravity). The final lecture of the week by Irene Valenzuela returned to the Swampland to discuss geometrical aspects of the distance conjecture.



Zohar Komargodski. Photo: Joshua Klein

The final week featured mostly local speakers, including Mike Douglas on numerical methods for finding metrics on higher dimensional geometries, Mario Martone's lecture on the classification of SCFT's, Nikita Nekrasov's beach talk on nonperturbative methods, and Sahand Seifnashri's beautiful talk on non-invertible symmetries.

At the beach. Photo: Martin Rocek

The Formal Side of String Theory: From Mirror Symmetry to Black Holes

By Alba Grassi,
Professor of Mathematical Physics
University of Geneva and CERN

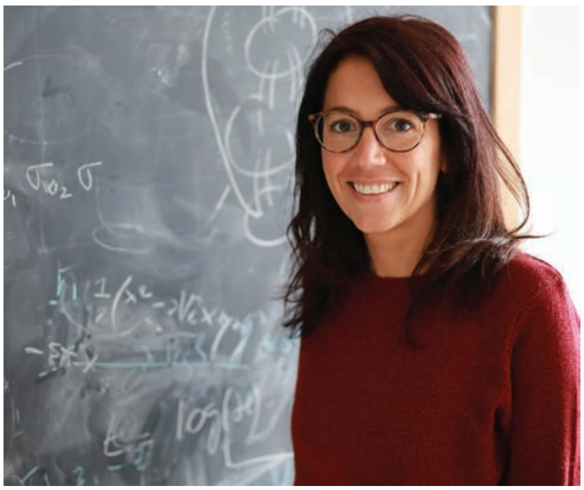


Photo courtesy Alba Grassi

A crucial question in theoretical physics is to understand how Einstein's theory of gravity and quantum physics are related. In fact, we know that general relativity is an accurate description of gravity at distances much larger than the Planck length ($\ell_{\text{Planck}} \approx 10^{-35}$ m). However at small scales quantum effects become important and the smoothness of general relativity begins to conflict with the randomness of quantum mechanics. Therefore, in this regime we expect general relativity to make way for a new quantum theory of gravity, and thus a corresponding quantum version of geometry.

The challenge of understanding gravity and the geometry of spacetime at quantum scales has led theoretical physicists to the study of string theory. Even though we still do not have a complete answer to this question, string theorists have developed great ideas and powerful tools leading to many interesting applications. This includes applications in nuclear physics, condensed matter and mathematical physics. Here we will focus on the latter. Let me stress that the impact of string theory in mathematical physics is actually quite vast, and in this article we only discuss a tiny subset of recent developments.

Mathematics and physics have a long history of cross fertilisation and one of the novelty that string theory brought to this interaction is the phenomenon of duality. String dualities are relations between objects and theories that appear very different, but when we think about them from the perspective of string theory, they surprisingly show some hidden connections. A powerful example of string duality is mirror symmetry [17] which introduces a new connection between pairs of Calabi-Yau (CY) manifolds. These spaces look quite different at first glance, for example they can have different topologies. Yet, string theory on each of these geometries behaves similarly. This observation has increasingly attracted the attention of mathematical physicists leading to a precise correspondence between these manifolds. Beyond its formal beauty, mirror symmetry is also very useful to solve concrete problems since it allows us to map very challenging computations on one space to much simpler problems on its mirror partner. This type of situations, where we can map hard problems to simpler ones, arises frequently in the string duality context and it is one of the reasons why these dualities are so powerful.

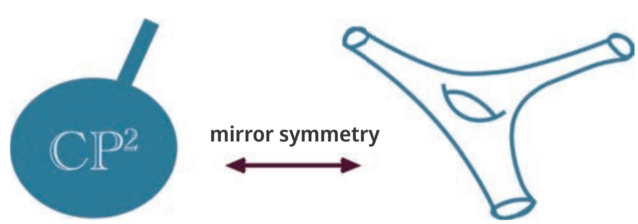


Figure1. Mirror symmetry relates pairs of geometries.

In formal applications of string theory, we often use a simplified model called topological string theory. As any other string theory, topological string can be viewed as a two dimensional conformal field theory coupled to gravity. What makes it simpler is the property of the underlying conformal field theory is topological; specifically here we consider the topologically twisted supersymmetric sigma models [11]. There are two twists we can consider that are related by mirror symmetry: the A twist and the B twist. Let us consider the A twist. In this model the topological string free energy F_X encodes the counting of holomorphic maps from Riemann surfaces into the target geometry X, Fig. 2. From the point of view of string theory, Riemann surfaces emerge when we consider periodic trajectories of a closed string and represent the so-called string worldsheet, Fig. 3. The counting of holomorphic maps in this context can be made mathematically precise and it is encoded in the enumerative geometry of the underlying target space X; more precisely, in the Gromov-Witten (GW) invariants of X. Schematically we write the perturbative free energy as

$$F_{\rm X} = \sum_{g\geq 0} g_s^{2g-2} F_{\rm X}^g(t, \ell_s), \tag{1}$$

with

$$F_{\rm X}^g(t, \ell_s) = \sum_{d\geq 1} N_g^d \mathrm{e}^{-dt/\ell_s^2} \tag{2}$$

where N_g^d are the genus g, degree d GW invariants of X; t is the area of the degree one map, ℓ_s is the string length and g_s is the string coupling constant. The computation of these invariants has been a very challenging problem in mathematics, but today, thanks to the contribution of string theorists, we can compute them systematically for a relatively large class of geometries. See for instance: [19, 1, 3, 18, 6, 8].

Recently it was also noticed that, when X is a toric CY manifold, we can extract the corresponding GW invariants from the spectrum of certain functional-difference operators called quantum mirror curves. These are quantum mechanical operators of the form

$$\mathcal{O}_{\rm X}(\hat{x}, \hat{p}, \hbar) = \sum_{i=1}^{n} c_i^{a_i\hat{x}+b_i\hat{p}} \qquad [\hat{x}, \hat{p}] = i\hbar, \quad a_i, b_i, c_i \in C\ . \tag{3}$$

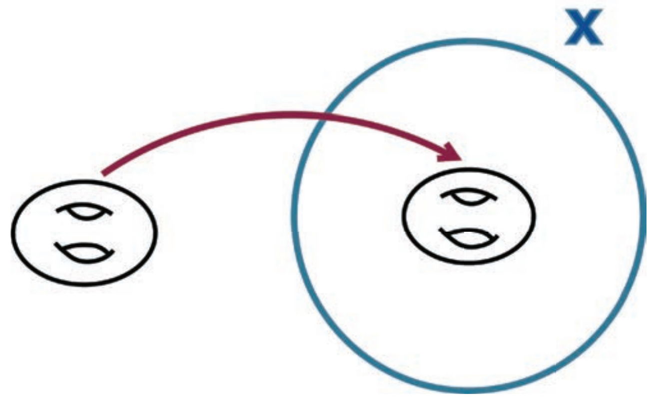


Figure 2. Topological string theory is the physical counterpart of (modern) enumerative geometry. The fundamental objects in this model are embedding of Riemann surfaces in a target space geometry X.

To extract the GW invariants we need to study (3) in a highly quantum regime where $\hbar \to \infty$ [14, 9, 21]. In turn, this means that enumerative geometry is in fact emerging from the spectrum of such quantum operators, which therefore provide a dual description of topological string. In several string dualities, from Witten's work on intersection theory [26] to the AdS/CFT correspondence [20], geometry and gravity can be viewed as an emergent phenomena. This perspective is interesting for several reasons. For example, it provides us with a powerful guide toward a non-perturbative definition of string theory. Indeed, if we consider for instance the free energy (1), one can show that (1) is in fact a divergent series [25, 15]. This means we are missing some key non-perturbative effects in our string description. The problem is that it is usually hard to study them directly within string theory. At the same time, when we relate (3) to topological string we have $g_s = \hbar^{-1}$. This means that non-perturbative effects in topological string (g_s large) are captured by the WKB expansion ($\hbar$ small) on the operator side, and therefore are in good control.

This connection between topological string and operator theory is also useful from the spectral theory perspective. Indeed, we currently have many efficient ways to compute GW invariants which do not rely on operator theory[1].

[1] Insights coming from operator theory have been nevertheless useful for the computation of (refined) GV invariants of elliptic (non-toric) CYs [16].

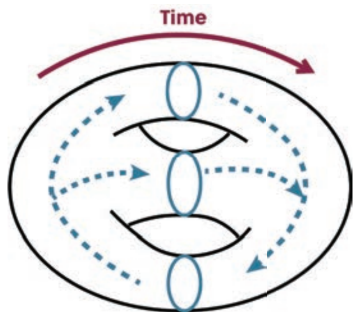


Figure 3. A Riemann surface of genus two. Physically it represents the periodic trajectory of a string (in blue) that, after one splitting and one joining, comes back to itself.

On the contrary, in spectral theory it is typically very hard to go beyond the perturbative WKB analysis. Hence, at the practical level, the above connection can be viewed as a new tool to compute non-perterturbative effects ($\hbar$ large) in the theory of difference equations. These results are part of a bigger program where stringy and gauge theoretic tools are used to study and exactly solve the spectral theory of new families of quantum mechanical operators, see for instance [14, 24, 12]. The interest behind this line of research lies in the fact that beyond a few toy models, like the harmonic oscillator, it is generically difficult to find analytic solutions to spectral problems. Nevertheless, thanks to supersymmetric gauge theory and topological string theory, we can nowadays overcome some of the technical obstacles and provide new families of solvable models. Among the operators that we can study within this framework we have the so-called four dimensional quantum Seiberg-Witten curves. These objects play an important role in several domains of theoretical and mathematical physics, from integrable systems [22, 24] to black hole quasinormal modes [2, 4, 10, 5, 7], Fig. 4.

In such applications to spectral theory, a central role is played by the Nekrasov and the Nekrasov-Shatashvili (NS) functions [23, 24]. These are a new class of special functions which were originally introduced in the context of instanton counting in Seiberg-Witten theory and in close connection with topological string theory. However, they have now proven to be useful tools in various research fields. For example, it turns out that the NS functions are precisely the functions that solve the eigenvalues problem associated to the Regge-Wheeler and Teukolsky equations [2], or that compute the finite frequency greybody factor for Kerr black holes [5]. Beyond spectral theory and black holes, another field where Nekrasov functions are nowadays frequently used is the one of isomonodromic deformations. In the simplest case isomonodromic deformation equations are Painlevé equations. These have been studied and classified at the turn of the of twentieth century and over the years several special solutions have been found. Nevertheless the construction of generic solutions in an explicit form is a very non-trivial problem which has been addressed only recently. More precisely, it was found in a remarkable work by the Kiev school [13] that Nekrasov functions are indeed the main building blocks for constructing such generic solutions.

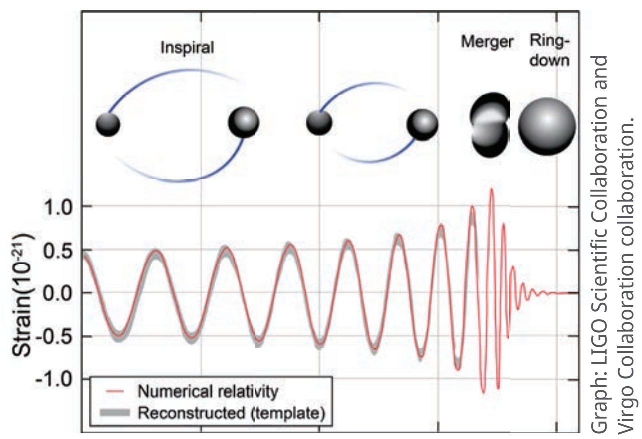


Figure 4. Gravitational wave signal of two merging black holes. Quasinormal modes are responsible for the damped oscillations characterising the ringdown phase of the signal. Image from [26].

Therefore we can fairly say that, although the original goal of unifying Einstein's theory of gravity and quantum physics has not yet been achieved, on the way to the solution we are developing many powerful ideas, such as string dualities, and new tools, like Nekrasov functions, with a wide range of unexpected applications going well beyond the original purpose of string theory.

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A Mirror into the Higher Dimensional World

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Photo: Claudia Landgrover

1 String Theory and Mirror Symmetry

I learn about the new from the classical. My research is in mirror symmetry [HKK+03]. The "new" is symplectic geometry [MS17] and the "classical" is complex geometry [Huy05], while "mirror symmetry" bridges the two. I am more familiar with the mathematics, so I appreciate the wonderful opportunity to work at the Simons Center with experts on the physics origin of the theory. The story of mirror symmetry starts in string theory, as follows.

We seek to make sense of our world by understanding its processes as physical systems, specifically, how small, medium, and large things move in time. Physics has evolved to explain increasingly more complex behaviors. On a day-to-day basis, visible objects in our life move according to classical Newtonian mechanics (think $F = m$a). On large scales, Einstein formulated the theories of special and general relativity to describe how very fast and very large objects move, such as how planets and moons affect gravity and space-time (three space dimensions and one time dimension). The relativity theories generalize Newtonian mechanics in that they agree with the classical theory at day-to-day speeds and medium scales but also work for fast objects or at large scales. For small scales, quantum mechanics was formulated to describe particles, such as photons in light, as wave functions (think Schrödinger's equation).

But what about unifying relativity and quantum mechanics? One theory to do so involves going smaller. Consider the difference in magnitude from the universe to an atom, but instead from an atom to something much smaller. There is a candidate called a "string." String theory is conjectured to unite all these theories, and strings would be the constituents of quarks, which in turn are constituents of atoms.

In order to be consistent with our current physical theories, these small strings must vibrate in 6 extra dimensions! We can't see the dimensions because they are wrapped up very compactly. It's similar to how a loop of string may look like a flat two-dimensional circle far away, but up close the start and finish of the piece of string may be a small distance apart in a third dimension. This compact 6-dimensional space, where we expect the strings to live, is known as a Calabi-Yau manifold. In what follows, we will see how a mirror pair of Calabi-Yau manifolds gives the same physical theory of strings. This mirror symmetry is one of several types of dualities, all of which are conjecturally related through something called "M-theory."

How does one mathematically measure when two systems give the same "physical theory"? Take a weighted sum of all the possible configurations of the system, in the form of an integral, and show the integrals for both are equal. Let's define terms. Another name for what I am calling a "physical system" is a "field theory," and "fields" represent possible configurations (also called states) that the physical system can take. For example, in the case of strings, the type of fields that we take are "sigma models,"

$$\text{Sigma Model} \qquad \sigma : \Sigma \to M,$$

where the name "sigma" comes from the σ notation in this map. Here M is the 6-dimensional Calabi-Yau manifold and Σ is a cylinder $S^1 \times \mathbb{R}$. S^1 is the notation for a circle. The image of the map $\sigma(S^1 \times \{t\})$ indicates where the circular string at time t maps to in M. So each σ contains the information of how a string vibrates over time in the manifold M. It carries a certain amount of energy, and integrating this energy over all possible σ for a fixed Σ and M gives the "partition function" for the field theory. Theoretical physicists found that 6-dimensional Calabi-Yau manifolds come in mirror pairs M and $\check{M}$ which have the same partition function. Mathematicians became interested in this because if one can mirror geometric information about M in $\check{M}$, it is a chance to learn more about geometry. I will describe this now.

2 What is Geometry?

As a geometer, I study different types of manifolds [Lee13], which are geometric spaces. A manifold is a geometric space that is characterized by being smooth and having the same dimension everywhere. Most tangible objects around us are low-dimensional manifolds. In physics, one often talks about coordinate charts, which are coordinates describing a particular frame of reference. For example, if you see someone standing 5 feet to your right and you consider yourself at the point $(x, y) = (0, 0)$, then that person has the coordinates $(5, 0)$. From their coordinate chart, they are at $(0, 0)$ and you are at $(-5, 0)$. The change in coordinates from one chart to the other is a coordinate transformation.

These charts are the building blocks of a manifold. For an n-dimensional manifold, you start with charts in n coordinates and then glue them together on their overlap by "transition functions," the coordinate transformations. So at any point in a manifold, if you were standing there, it would look like $\mathbb{R}^n$ locally around you. For example, on the Earth when you look around, it looks flat like the xy-plane $\mathbb{R}^2$. But we know when we zoom out, the surface of the Earth is a sphere, not $\mathbb{R}^2$. That's the beauty of manifolds; you can build a lot of different types of geometric spaces from these local charts.

For more examples, space-time is a four-dimensional manifold. The two-dimensional manifolds which are compact and don't have boundary consist of spheres, the surface of a bagel, the surface of two bagels attached together, and the surface of g bagels attached together in a line for any natural number g. In fact, a surface is completely classified by its number of holes. That is, any compact 2-dimensional manifold without boundary, if it were made of clay, could be continuously deformed into a surface that looks like g bagels attached to one another in a row. "Continuously deformed" means without breaking or gluing. If $g = 0$, it's topologically a sphere. Topology is a field of math, similar to geometry, where one studies geometric spaces up to continuously deforming them. There is a joke that a topologist can't tell the difference between a coffee mug and a bagel because both have one hole.

This number of holes g is called the "genus" of the surface and is an invariant used to classify surfaces [Hat02, pg 5]. In higher dimensions than two, a lot more can happen geometrically. We need more math and types of invariants to classify higher dimensional manifolds. Two branches of geometry are complex geometry and symplectic geometry. The former is when you replace $\mathbb{R}$ with $\mathbb{C}$ in the definition of a manifold, where $\mathbb{C}$ denotes the complex numbers $(a, b) = a + \sqrt{-1}b$ for $a, b \in \mathbb{R}$. The second branch is the geometry of phase space; the trajectory of a particle moving under a potential, with coordinates describing both its position and momentum, can be understood as a mathematical set-up in symplectic geometry. One may ask, what types of geometries are there besides symplectic and complex? This is a problem in algebra and has also been classified, by studying the structure of the aforementioned coordinate transformations, known as the G-structure. Mathematicians classified these algebraic objects G, called "groups" [Smi15, §2.3.1], [SS65].

3 Mirror symmetry of Geometries

3.1 The example of a bagel

It can be hard to picture six dimensions, the candidate for where strings vibrate, so let's consider a two-dimensional analogue of mirror symmetry. If you understand mirror symmetry for a bagel, you know a lot of what is needed to understand my research. So I will describe the mirror correspondence between complex and symplectic geometries for a bagel.

A bagel is a Calabi-Yau manifold, which means a certain type of curvature of the manifold is zero. In particular, a bagel has the property that it can be constructed from a flat rectangle. See Figure 1.

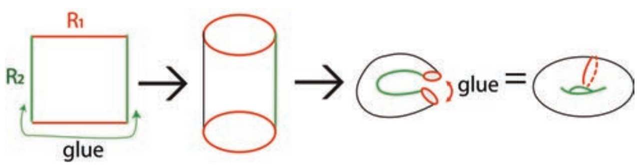

Figure 1

In two dimensions, symplectic geometry and complex geometry can be understood by area and scaling, respectively. Suppose the sides of the bagel's rectangle have lengths R_1 and R_2. Then its symplectic geometry is encoded by the area R_1R_2. Its complex geometry is described by R_2/R_1, the amount that length gets scaled when we rotate the horizontal side of the rectangle with length R_1 by $90°$ (in other words, multiplication by $\sqrt{-1}$) to the vertical side of length R_2. Denote this bagel by M. To indicate the rectangle lengths, we can write

$$M = S^1_{R_1} \times S^1_{R_2}.$$

The manifold mirror to M is also a bagel, but R_1 is replaced by $1/R_1$.

$$\check{M} = S^1_{\frac{1}{R_1}} \times S^1_{R_2}.$$

Why is this? Let's see what the symplectic and complex geometric invariants are on $\check{M}$. The area is R_2/R_1 while the scaling factor is R_1R_2. This is swapped from M! This is a preview of mirror symmetry for a bagel; it matches the symplectic geometry on M to the complex geometry on $\check{M}$ and the complex geometry on M to the symplectic geometry on $\check{M}$. This phenomenon of inverting the length R_1 is "T-duality." For a more technical treatment of T-duality in the torus, see [HKK$^+$03, §11.2.3] and [Qui15]. Three mathematicians formulated T-duality as mirror symmetry in [SYZ96]. Hence a mirror found in this way is an "SYZ mirror."

3.2 Picturing higher dimensions

Mirror symmetry may occur between manifolds of any even dimension. One way to picture higher dimensions is in pieces. Consider the Cartesian xy-plane, also denoted $\mathbb{R}^2$. You can build it by taking a horizontal line $\mathbb{R}$, the base, and at each point on the horizontal line attaching a vertical line $\mathbb{R}$, the fiber. The base and the fiber are each one dimensional, but together their product is two dimensional. The bagel can be seen with a base and fiber too. Take a circle as the base, and at each point attach another S^1, tracing it around the base circle as in Figure 2. A description of a manifold in terms of a base and fibers is called a fibration [Hat02, Fiber Bundles]. Furthermore, a bagel is a torus fibration, because its fibers are one-dimensional tori, namely circles.

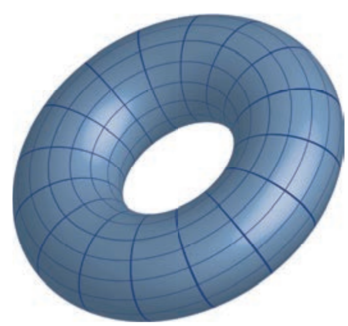

Figure 2

I envision higher dimensions as products of lower-dimensional manifolds. In my research, I think about the symplectic geometry of a mirror to the genus 2 surface [Can20], [ACLL]. Recall this is when two bagels are attached together. The mirror I consider is six dimensional and is obtained by adapting a method called generalized SYZ mirror symmetry [HV00], [AAK16], [AA21]. Earlier, we mentioned mirror symmetry for Calabi-Yau manifolds. The case of the genus 2 surface and its six-dimensional mirror is a natural next step after Calabi-Yau manifolds; the curvature quantity that was zero for a Calabi-Yau is negative for the genus 2 surface. The consequence of this nonzero curvature is that the mirror is unbounded, because its base is $\mathbb{C}$, instead of being compact such as S^1 or $S^1 \times S^1$.

Specifically, its base is the two-dimensional complex plane $\mathbb{C} = \mathbb{R} \times \sqrt{-1}\mathbb{R}$. All fibers except one are four-dimensional tori, which can be pictured by taking a bagel as the base and another as the fiber. (Or start with a circle and pull it around three other circles in different dimensions $S^1 \times S^1 \times S^1 \times S^1$.) One fiber of this six dimensional fibration is "degenerate," meaning some of the circles in the four-torus collapse. This degenerate fiber encodes much of the geometry of the total space.

If one found a mirror to a point in the same way I find a mirror to the genus 2 surface, it would look like the right-most image in Figure 3. The first two images illustrate the recipe of generalized SYZ mirror symmetry that produces the third image. There, the base is $\mathbb{C}$ and a fiber is a two-dimensional torus (genus 1 surface), except over $0 \in \mathbb{C}$ one circle of the torus gets pinched to a point. It seems like a lot is going on to be mirror to just a point. In fact, the key behavior that is mirror to the point is the pinched torus fiber over 0. Specifically, the point in that fiber where it is pinched can be thought of as mirroring the original point. For the mirror of the genus 2 surface, most fibers are four-tori T^4, and instead of a string of spheres over the $\times$ as in the middle picture below, it looks like a configuration of honeycombs!

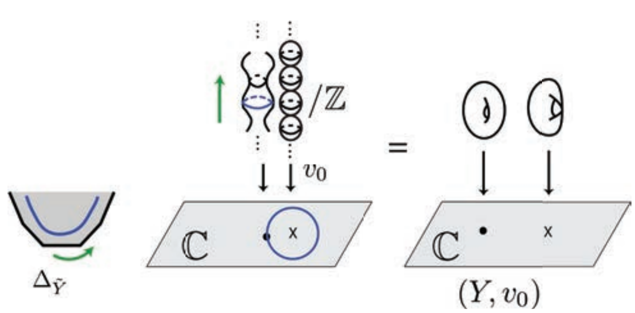

Figure 3

4 Mirror Symmetry of Algebras

We've just seen how geometries can mirror each other on two manifolds. One can further explore, are invariants equal on the manifolds too? This leads to my field of research in homological mirror symmetry (HMS), first conjectured by Kontsevich [Kon95]. Above, we saw one example of an invariant for surfaces, genus. For HMS, the invariants are more complicated and involve the following two categories.

The invariant on the complex manifold M is called the bounded derived category of coherent sheaves, denoted $D^bCoh(M)$ [Huy06]. In agriculture, a sheaf is a bundle of stalks all aligned and bundled together. This illustrates the notion of a sheaf in math too. Often to understand $D^bCoh(M)$ it suffices to consider line bundles on M. These are when you attach a line at every point on the manifold M in a smoothly varying way. It's a fibration with base M and fiber $\mathbb{R}$. Think: straight hairs on a head, if the head had hair everywhere and each hair was actually an infinitely long line. Maybe a less alarming example is the farmer's sheaf bundle of stalks: take the base as a circle lying flat on the ground and each fiber is a vertical line, like a stalk. That gives a cylinder, and is an example of a line bundle on a circle. To describe a line bundle on a torus, it suffices to say how many times the line rotates as it moves around the two circles in a torus (its base circle and fiber circle). In fact, one usually considers complex lines, replacing $\mathbb{R}$ with $\mathbb{C}$, and this winding number is encoded in an invariant called the first Chern class c_1 of the complex line bundle [Huy05].

The invariant on the symplectic manifold $\check{M}$ is called the Fukaya category $Fuk(\check{M})$ [FOOO09], [Aur14]. For a bagel, while $D^bCoh(M)$ consisted of line bundles that wind k times, $Fuk(\check{M})$ consists of submanifolds called Lagrangians, which in this case includes straight lines of slope k passing through the center of the rectangle in Figure 1. This exhibits homological mirror symmetry of objects on the two-torus:

$$D^bCoh(M) = Fuk(\check{M}).$$

Line bundles of winding number k on M are mirror to lines of slope k on the mirror torus $\check{M}$. In a category, though, there are both objects and maps between objects. Recall that algebra came up in our earlier discussion of manifolds, in maps between charts.

Similarly, homological algebra describes maps between objects of these categories. In the bagel example, HMS is an equivalence of maps between line bundles on M with maps between integer-slope lines on $\check{M}$, defined by their points of intersection. This was proven in [PZ98]. It was proven for all tori in [Fuk02] and described for the four-dimensional torus in our work [Can20], [ACLL], because we view the genus 2 surface as a submanifold of the four-torus.

There are many more examples of homological mirror symmetry and symplectic manifolds to understand through their Fukaya categories. I look forward to advancing this active and interesting area of research. I have more information on my webpage, https://sites.google.com/view/ccannizzo.

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Symmetry: A Deep Affinity Between Art and Science

By Philip F. Palmedo

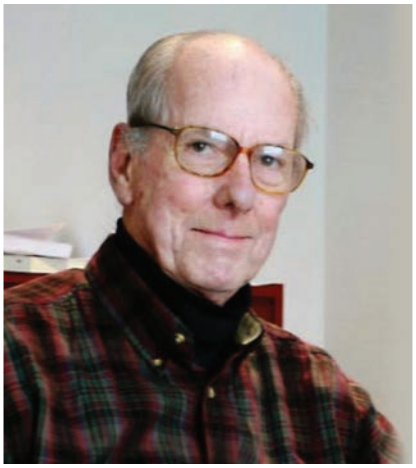


Photo: Howard Romero

From its earliest days I have admired the vision inherent in the Simons Center, including Jim Simons's idea of incorporating art in the Center's program. In that spirit, I discuss here one of many deep affinities between art and science.

Early humans, our hunter-gatherer ancestors, were faced with a baffling, chaotic natural world. Their survival depended on figuring out nature, and those who had a greater curiosity and sought regularities and patterns in nature had a higher probability of survival. That early seeking of order in nature was at the origin of both art and science.

Finding the order in nature was necessarily accompanied by its complement: representation. Early evidence of representation appears on the walls of caves painted during the Upper Paleolithic era, some 40,000 to 15,000 years ago. Although the painters of those walls are often referred to as the first artists, we must go further back to discover the true first artists and the origin of an essential relationship between art and science.

Hand axes evolved with humans over something like a million years, but around 700,000 BC, for some individuals in some places, the form itself took on more importance than the function.

The clean shapes and perfect symmetry of these hand axes were achieved at a huge cost of time and effort with no practical benefit. They were the first aesthetic objects, the first sculptures. Symmetry, as eloquently represented in this early sculpture, would be a fundamental affinity between art and science.

The appealing symmetries found in nature, from flowers to beetles to snowflakes, derive from the symmetrical laws of physics and biophysics that control the processes of growth. The laws of physics determined aesthetic preferences.

Symmetry is a recurrent theme in the art of diverse cultures throughout time and space. There are few classical architectural buildings, from the

Fig. 1. Hand ax from Kathu Pan, South Africa, ironstone, 9¼ × 4½ in. c. 600,000 BC. McGregor. Museum, Kimberley, South Africa.

Fig. 2. The Taj Mahal

Parthenon to the Taj Mahal, that don't display some degree of symmetry.One remarkable example of complex symmetry was produced some 1000 years ago in Peru by a culture without a written language. This is the front and back of a double-woven band about ¾ in. wide. It was included in a show at the Simons Center featuring Professor Anthony Phillips's collection where he demonstrated seven types of symmetry used by Peruvian weavers.

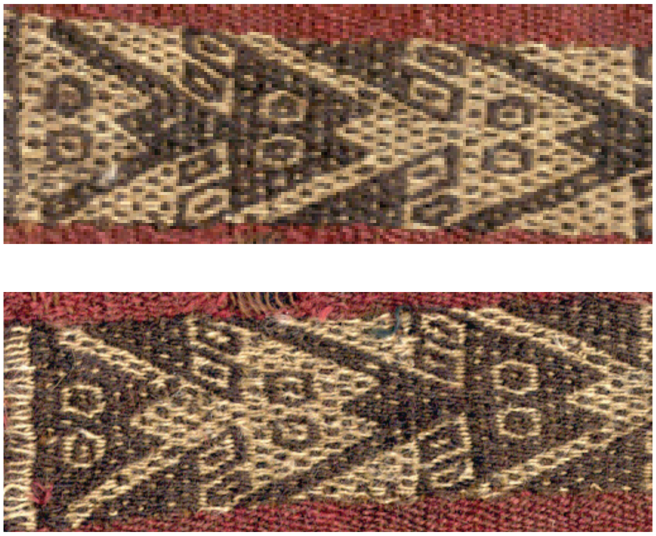


Fig. 3. Textile with double-woven bands, front and back, Peru, 1000–1400 AD . Image courtesy Anthony Philips

Ever since the days of Newton, the concept of symmetry has played an important role in science and is a fundamental tenet of modern physics. It was the basis of Paul Dirac's prediction in 1931 of the existence of a positively charged particle to complement the electron. The prediction was based on a belief that since the equation describing the electron had that solution, that particle had to exist. The positron was discovered the next year. The hypothetical model that would reconcile the two descriptions of the world–of the very small and the very large–is defined as Supersymmetry.

Humanity's first conceptual models of the universe were based on the symbols of nature's perfection: the circle and the sphere. By the sixteenth century we have artistic representations of those models, with the earth at the center. These celestial maps are some of the first examples of the use of artistic means to present scientific concepts.

Johannes Kepler struggled for years to explain why Tycho Brahe's observations deviated from Copernicus's circular planetary orbits. God, in his perfection, would use only the perfectly symmetrical shape when he designed his universe. But Kepler's belief in mathematics, and the simpler explanation of the observations, finally convinced him. The ellipse is symmetric in one of its dimensions.

The life of the circle in art was accentuated with the birth of modernism. In 1913, at the origins of pure abstraction, Kazimir Malevich radically presented a painting of a black circle as a work of art. Even today there are few modern museums or gallery spaces where a circle doesn't appear. In March of 2021, New York's Metropolitan Museum of Art installed on its façade sculptures by Carol Bove featuring circles.

Not all art is symmetric, of course. But other attributes of art that are satisfying, like balance and compositional integrity, are closely related. When Archaic Greek sculpture developed into Classical works, it was the asymmetries that brought vitality to the works. But the implied reference point was still symmetry.

For physics, any seeming violation of the grand symmetry of mathematically defined nature is deeply troubling. How to explain the dominance of matter over antimatter in the universe (which incidentally accounts for the fact that we exist)? Explaining this seeming asymmetry, or "CP violation," has stimulated a great deal of creative

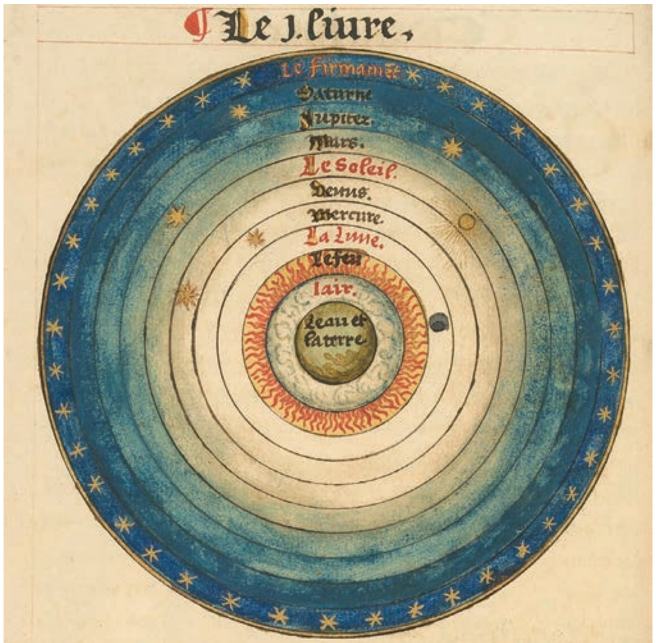


Fig. 4. Oronce Fine, "Le Sphère du Monde," 1549. Houghton Library, Harvard University

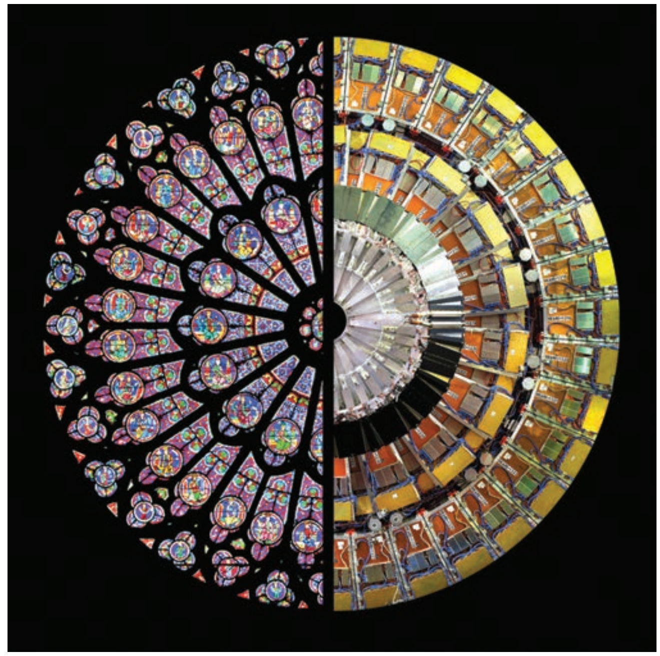


Fig. 5. Circularity: Window at Notre Dame de Paris and the Compact Muon Solenoid at the Large Hadron Collider at CERN
Image courtesy Philip F. Palmedo

research. But the assumption that has driven discovery, remains dominant: that the universe is defined by mathematics and is fundamentally symmetric.

Many other characteristics connect art and science. At their highest levels, creativity in both areas involves understanding the current state of the field and taking a leap beyond its boundary. Although artists now avoid the word beauty, scientists often cite the search for beauty as their motivation. Einstein often evoked beauty as the ultimate criterion for the validity of a scientific explanation.

When success arrives, when scientists achieve the "eureka" moment, they often describe their reaction in aesthetic terms. In 1957, Richard Feynman and Murray Gell-Mann developed a theory of the interactions between fundamental particles. The theory did not agree with some recent experiments but had aesthetic qualities that convinced its authors. Feynman described the moment of discovery: "There was a moment when I knew how nature worked. It had elegance and beauty. The goddamn thing was gleaming."[1]

Just like the relationship between art and science.

Philip F. Palmedo studied art history and physics as an undergraduate at Williams College, and received his PhD in nuclear engineering from MIT. Retired from a distinguished career as a research scientist and entrepreneur, Palmedo is the author of several books on art including *The Experience of Modern Sculpture: A Guide to Enjoying Works of the Past 100 Years.* His most recent book *Deep Affinities: Art and Science* was published by Abbeville Press in 2020. Here, Palmedo expands upon affinities in art and science that are rooted in certain common human instincts, which we might call aesthetic: an appreciation of symmetry, balance, and rhythm; the drive to simplify and abstract natural forms, and to represent them symbolically.

1 Lawrence M. Krauss, Quantum Man: Richard Feynman's Life in Science. New York: W. W. Norton, 2011, p. 216.

The Simons Center Welcomes New Deputy Director

The SCGP is pleased to announce the appointment of Samuel Grushevsky, Professor of Mathematics at Stony Brook University, as the new Deputy Director Fall, 2021. He will work alongside the current Deputy Director, Alexander (Sasha) Abanov, for the 2021-2022 academic year. Grushevsky's research interests include algebraic and complex geometry, relations with number theory, integrable systems and mathematical physics.

More information about Samuel Grushevsky's research and recent selection as a Fellow of the American Mathematical Society, class of 2022, will be published in the next issue of SCGP News.


Photo: Joshua Klein

New Research Assistant Professors at The Simons Center

Mathematics


Photo courtesy Chris Gerig

Chris Gerig

Chris Gerig grew up very close to Stony Brook University before attending college. Greg received his BS in applied & engineering physics at Cornell University in 2011. He began a PhD in experimental physics at UC Berkeley and then changed path and received his PhD in pure math at UC Berkeley in 2018. Chris was an NSF postdoctoral fellow and lecturer at Harvard University for 3 years. He works under the umbrellas of Differential Geometry and Geometric Analysis, at the crossroads of symplectic geometry and gauge theory. Outside of math Chris likes to play sports, swim, sing, and spend time with friends and family.

Mathematics

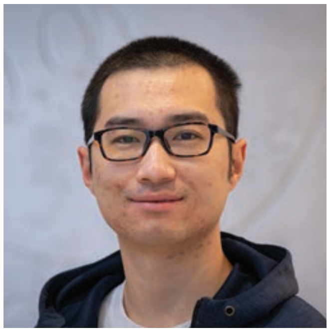


Photo: Claudia Landgrover

Donghao Wang

Donghao Wang received his PhD in mathematics from Massachusetts Institute of Technology in 2021, under the supervision of Tomasz Mrowka. His research interests lie in gauge theory and low dimensional topology. In his thesis, he extended the construction of the monopole Floer homology from closed oriented 3-manifolds to certain 3-manifolds with toroidal boundary. This work relies on an unexpected relation between the Seiberg-Witten equations and Landau-Ginzburg models, which allows us to transport modern techniques from symplectic topology to further advance the study of the Seiberg-Witten theory and pursue applications toward 3-manifold topology. At SCGP, Donghao will continue to explore the deep connection between gauge theory and symplectic topology. In particular, he plans to construct the Fukaya-Seidel category in an infinite dimensional setting and develop a partial bordered monopole Floer theory based on this relation.

Physics

Photo: Claudia Landgrover

Petr Kravchuk

Petr Kravchuk received his PhD from the California Institute of Technology in 2018. He was a member of the Institute for Advanced Study before joining SCGP in 2021. Petr is primarily working on theoretical aspects of conformal field theory, using both numerical and analytical methods. As of recently, he is particularly interested in the properties of the light-ray operators and their connection to infrared divergences of scattering amplitudes, as well as in applications of numerical bootstrap to 3-dimensional systems with fermions. At SCGP, he is planning to continue working in this direction and to collaborate with other SCGP researchers on various aspects of conformal field theory.

Photo: Claudia Landgrover

Gabi Zafrir

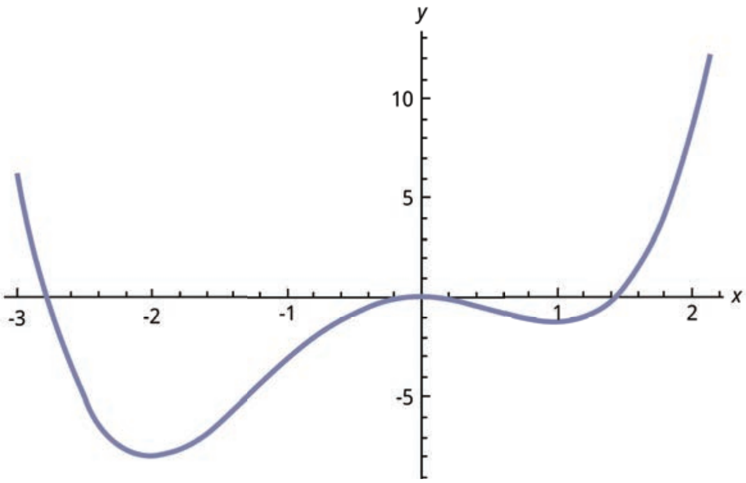
Gabi obtained his PhD in Physics in 2016 from the Technion in Israel, under the supervision of Oren Bergman. From 2016 to 2019, he worked as a postdoctoral researcher at the Kavli Institute for the Physics and Mathematics of the Universe situated in Kashiwa, Japan, following which he worked as a postdoctoral researcher in the University of Milano-Bicocca. He is mainly interested in quantum field theory, string theory, and the intricate relation that exists between these fields. One of his focuses in recent years has been the relationship between quantum field theories in different dimensions and what it can teach us about the dynamics of such theories.

Puzzle Time

Quartic Oscillators is contributed by **Boris Khesin,** Professor of Mathematics, University of Toronto, Canada.

The solution will be published in SCGP News, Volume XVIII.

Consider a particle of unit mass moving on the line according to Newton's equation $\ddot{x} = -\partial U/\partial x$ in a potential, which is a 4th degree polynomial $U(x)$ with two wells, a deep and a shallow one. Find explicitly the period of oscillations of the particle in the deeper well at the energy level corresponding to the bottom of the shallow one.



For instance, for Newton's equation $\ddot{x} = -3x(x+2)(x-1)$, corresponding to the quartic potential $U(x) = 3x^4/4 + x^3 - 3x^2$, with minima at $x = -2$ and $x = 1$, find this period of oscillations in the left well around the point $x = -2$ at the energy level $E = U(1) = -5/4$, see Figure 1.
(Hint: Compare the periods of oscillations in two wells at the same energy levels.)

In Volume XVI of SCGP News the puzzle titled **Quantum Work** was contributed by **Alexander Abanov,** Deputy Director, Simons Center for Geometry and Physics, Stony Brook University.

Reminder

The problem was to compute the maximal work extracted from the quantum system consisting of spin-1/2 atoms separated by an impenetrable partition. There are N atoms on each side of the partition. The atoms are identical and spin-polarized so that the polarization axis of atoms on the left side of the partition forms an angle α with the one on the right-hand side.

Quantum Work: *The Solution*

Let us start with the classical gas problem. Imagine that the impenetrable partition separating the volume consists of two semi-transparent partitions. The one on the left is transparent to left particles but is impenetrable for the right ones. Similarly, the right partition is transparent to the right particles but is impenetrable for the left ones. One can imagine the following two-stage reversible process. First, we move the left partition slowly to the left wall of the container. The partial pressure of the right gas on the partition is $P = Nk_BT/V$, and the work performed by the gas during the expansion is $W_R = \int_V^{2V} P\,dV = Nk_BT\int_V^{2V} dV/V = Nk_BT\ln 2$. Then, repeating the same process with the left gas (right partition), we obtain $W = W_R + W_L = 2W_R = 2Nk_BT\ln 2$ - the result quoted in the statement of the problem and is known from the Gibbs paradox. We remark here that the possibility to perform the processes described is based on the ability of an observer to distinguish between particles of the left and right gases (the ability to construct "semi-transparent partitions"). The latter remark provides a resolution of the Gibbs paradox. Even if right and left gases are not quite the same, but the observer cannot distinguish them, no work can be extracted from the mixing.

In the case of quantum polarized gases, semi-transparent partitions cannot be constructed as a matter of principle if $\alpha \neq 180^\circ$. Indeed, there is a non-zero overlap of spin states for left and right particles, and any partition transparent to the left gas will transmit the part of the right gas corresponding to the projection of the right spin states to the left ones.

Let us denote the polarization state of the left gas as $|L\rangle = |\uparrow\rangle$. Then the state of the spins of the right gas can be represented as $|R\rangle = \cos\frac{\alpha}{2}|\uparrow\rangle + \sin\frac{\alpha}{2}|\downarrow\rangle$. As we are not allowed to manipulate spins, we can think of the problem as of the classical one with N particles of U-type (i.e., up-spins) on the left and the mixture of $N\cos^2\frac{\alpha}{2}$ of U-particles and $N\sin^2\frac{\alpha}{2}$ of D-particles on the right.

We replace the partition in the middle by two semi-transparent "up" and "down"partitions (the partitions can be realized, by having barriers made out of strong magnetic field). Then we slowly move the partition transparent for up particles to the left and extract the work $W_1 = Nk_B\sin^2\frac{\alpha}{2}T\ln 2$ from the expansion of the gas of down particles from the left half to the whole container. We are left with N-up particles on the left and $N\cos^2\frac{\alpha}{2}$ particles on the right. We move the remaining partition slowly to its new equilibrium position defined by the equality of partial pressures $\frac{Nk_BT}{V'} = \frac{N\cos^2(\alpha/2)k_BT}{2V-V'}$ or $V' = \frac{2V}{1+\cos^2(\alpha/2)}$. We compute the work of gases on both sides of the partition as $\frac{W_2}{Nk_BT} = \int_V^{V'}\frac{dv}{v} + \int_V^{2V-V'}\frac{\cos^2(\alpha/2)dv}{2V-v} = \sin^2\frac{\alpha}{2}\ln\frac{V'}{V} = \sin^2\frac{\alpha}{2}\ln\frac{2}{1+\cos^2(\alpha/2)}$. Finally, for the total work extracted from the system we obtain $W = W_1 + W_2 = Nk_BT\sin^2\frac{\alpha}{2}\ln\frac{4}{1+\cos^2(\alpha/2)}$.

For opposite initial polarization $\alpha = \pi$, the gases can be treated as different, and the answer reproduces the classical result for different gases. For $\alpha = 0$, the states of the spins are identical on both sides of the partition, and no work can be extracted from the mixing.

The Shah Mosque, Isfahan, Iran. Photo courtesy Valérie Gonzalez

The Simons Center Art and Outreach Program

By Lorraine Walsh
Art Director and Curator for SCGP and Visiting Associate Professor of Art

ART & SCIENCE TALKS

Philip F. Palmedo, a former physicist and member of the SCGP community, provided an illustrated lecture titled *Deep Affinities: Art and Science* in the early spring of 2021. His enlightening talk explored how two defining enterprises of humankind—art and science—are rooted in certain common instincts that are fundamentally aesthetic. It was the Center's first outreach lecture as a hybrid event combining both an in-person and Zoom attendance—and a most welcome one at that.

Palmedo has written extensively on art and science. His research considers how these two disciplines originated in early humans' need to understand a complex natural world on which they were dependent. They shared basic concerns and found efficiencies and solutions in symmetries. The creative process, the criteria of success, and the creators' reactions on succeeding are remarkably similar in both fields.

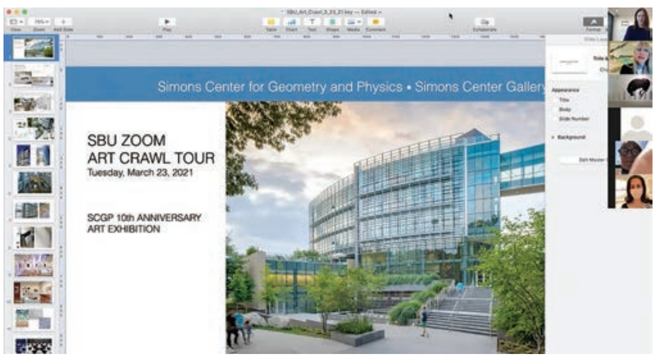
In this issue, we feature Palmedo's illuminating article titled *Symmetry: A Deep Affinity between Art and Science* (page 17.) It is drawn from his most recent book *Deep Affinities: Art and Science*, published by Abbeville Press in 2020.

Another lecture that spoke of symmetry was provided by Valérie Gonzalez during her visit to the Simons Summer Workshop. Her talk, titled *Mosque Architecture and the Experience of the Islamic Sacred*, was a welcoming respite for an in-person talk both aesthetically and mathematically inspiring. Gonzalez talked about form, space, and ornamental design of mosque architecture found in diverse areas of the Muslim world, from the ancient world to the contemporary era.

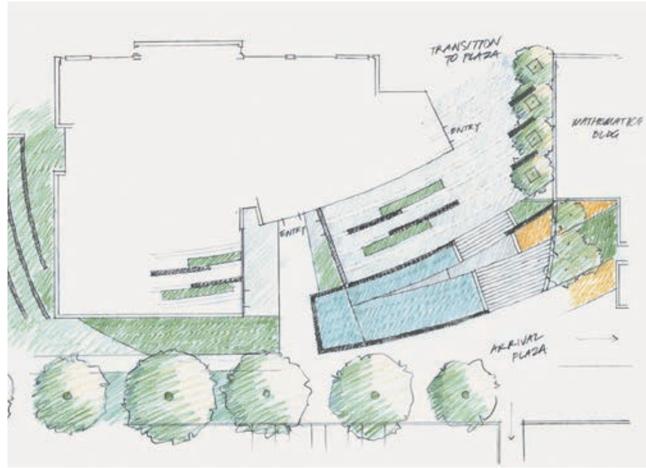
Gonzalez, a specialist of Islamic art history, aesthetics and visual culture, is a Research Associate at SOAS, University of London. She obtained a PhD in Islamic Studies from University of Provence Aix-Marseille, and a Master of Fine Arts at the School of Fine Arts, Marseille-Luminy.

UNIVERSITY GALLERY TOURS

Most of the Center's Outreach activities were curtailed due to Covid last spring and summer. Nevertheless, the University-wide gallery tour went forward for the Stony Brook community. In March, The Art Crawl, as it is known, took place in a virtual environment for the first time. Participating galleries included the Charles B. Wang Center, Paul W. Zuccaire Gallery, Frank Melville Jr. Memorial Library, Alloway Gallery, the North Reading Room, and the Simons Center Gallery. It was moderated by Kristen Nyitray, Director of Special Collections and University Archives, who skillfully guided guests to several Zoom rooms with over eighty visitors in "attendance."



Zoom screenshot



Drawing by Dirtworks Landscape Architecture, PC

EXHIBITIONS

The exhibition *Building the Building*, curated by Sam Rashba and Lorraine Walsh, was extended into the spring and summer of 2021 due to Covid. The exhibit commemorated the ten-year anniversary of the Simons Center's building, which was accompanied by an in-depth publication celebrating the people, history, and accomplishments of the Center. For more information, please see SCGP News XVI.

MUSIC

After a quiet 2020, the Center was grateful for the return of music. The Summer Music Series is an annual tradition held during the Simons Summer Workshop. And in the summer of 2021, four excellent concerts were performed in the Della Pietra Family Auditorium.

On July 13th, Quartet Salonnières performed a program titled *From Vienna to Paris: Beethoven in Context* with violinists Natalie Kress and Aniela Eddy, Majka Demcak on viola, and Cullen O'Neil on cello. Keenan Zach and his band played on July 20th. Zach, a bassist, plays both jazz and classical music. He is currently pursuing his doctorate at Stony Brook University. Iva Casian-Lakos, on cello and voice, received a warm welcome back on July 27th. Casian-Lakos was accompanied by Annie Nikunen on flute for the second half of the performance. The series concluded with a wonderful concert by Peter Watrous and his jazz band on August 3rd.

Top row: Peter Watrous jazz band members on cello and saxophone, Iva Casian-Lakos. Bottom row: Majka Demca, Cullen O'Neil, Natalie Kress, and Aniela Eddy. Photos: Joshua Klein

Photo: Elena Yakubovskaya

SIGMACAMP

By Victoria Bershadsky and Elena Yakubovskaya

SigmaCamp is honored to partner with the Simons Center of Geometry and Physics to bring puzzles, games, and engineering challenges to the bright and curious young members of the Stony Brook community.

In May, SigmaCamp held the Puzzles and Games event. During this event, children from ages two to eighteen were shown the fun of math and science through hands-on challenges. It was a wonderful respite with a brain teaser twist! All community members who attended were most grateful that this event sparked new ideas the families could explore together.

This Halloween, the Center was transformed into a spooky space where SigmaCampers and their friends participated in engineering challenges including making scotch tape buckets, pencil bridges, index card towers, and Keva cars. Teams competed against each other with vigor and excitement. Some even managed to hold gallons of water in a barrel made out of clear postage tape, as well as create bridges that held ten cans of condensed milk. Their amazing achievements were acknowledged with SigmaCamp style awards: games and books that encourage further research and exploration. To challenge the younger members of our community and the families of Friends of Sigma, the Center's space was explored as a maze of various interactive and thought-provoking exhibits. This included activities with everything from building a huge Sierpinski pyramid with Zometools to nano bug races. The Quidditch game was the highlight of the evening.

These events, sponsored by the Simons Center Outreach program, spread the beauty of scientific exploration in the name of the Sigma Spirit.

Photo: Elena Yakubovskaya

Archival digital print series with photography, drawings, and hybrid images. 2021-2022. Images courtesy Lorraine Walsh

MISSOURI BOTANICAL GARDEN EXHIBITION

Lorraine Walsh, SCGP Art Director, exhibited new work at the Stephen and Peter Sachs Museum, Missouri Botanical Garden. Walsh and her collaborator Lei Han created artwork representing environmental shifts as seen through the seemingly disparate practices of ancient horticultural grafting techniques and contemporaneous machine learning (a subset of artificial intelligence). They created a sculpture with laser engravings, drawings, digital images, and three short films for the exhibition. For the exhibit, their art focuses on the native grapevine species Missouri Vitis aestivalis (also known as the Norton grape) and explored varied approaches in order to bring a fruitful awareness of the significant effect climatic change has on life. Grafting the Grape exhibition, curated by Nezka Pfeiffer, the Sachs Museum, May 2021 to March 2022.

A NEW PIANO

The SCGP now has a Steinway grand piano placed in the Della Pietra Family auditorium. We look forward to many stellar concerts in the future—for the Center, Stony Brook University, and the community at large. Stay tuned!

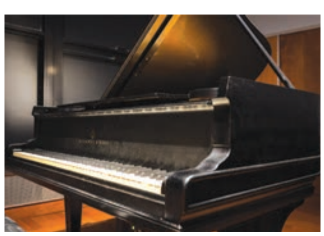
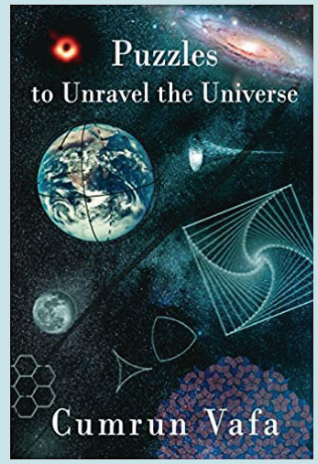


Photo: Joshua Klein

UPCOMING DELLA PIETRA LECTURES 2022

MAY 2-7, 2022

Cumrun Vafa, Hollis Professor of Mathematics and Natural Philosophy at Harvard University, *Puzzles to Unravel the Universe*



TO BE SCHEDULED:

Ian Stewart, Emeritus Professor of Mathematics, Warwick University, UK

Alvaro de Rujula, Emeritus Professor, CERN
Fumiharo Kato, Professor of Mathematics, Tokyo University

UPCOMING EXHIBITIONS

Photo: Manu Palomeque

SPRING 2022

Ander Mikalson and Katie Paterson: *Artworks to Sonify and Colorize the Universe*

2022-2023 Upcoming Programs & Workshops

PROGRAMS

Confronting Large N, holography, integrability and stringy models with the real world: ***February 14, 2022 - April 22, 2022***. Organized by Sergei Dubovsky, Zohar Komargodski, Erich Poppitz, Jacob Sonnenshein, and Mithat Unsal.

Geometrical aspects of topological phases of matter: spatial symmetries, fractons and beyond: ***April 4 - May 27, 2022***. Organized by Jennifer Cano, Dominic Else, Andrey Gromov, Siddharth Parameswaran, and Yizhi You.

The Stokes Phenomenon and its applications in Mathematics and Physics: ***April 25 - June 24, 2022***. Organized by Anton Alekseev, Marco Gualtieri, and Xiaomeng Xu.

Singularity and Prediction in Fluids: ***May 31 - July 1, 2022.*** Organized by Theodore D. Drivas and Dennis Sullivan.

Integrability, Enumerative Geometry and Quantization: ***August 22 - September 23, 2022.*** Organized by Gaëtan Borot, Alexandr Buryak, Chiu-Chu Melissa Liu, Nikita Nekrasov, Paul Norbury, and Paolo Rossi.

Geometric and Representation-Theoretic Aspects of Quantum Integrability: ***August 29 - October 21, 2022.*** Organized by Peter Koroteev, Elli Pomoni, Benoit Vicedo, Dmytro Volin, and Anton Zeitlin.

Number Theory and Physics: ***October 24 - November 18, 2022.*** Organized by Brian Conrey, Matilde Lalin, Giuseppe Mussardo, and German Sierra.

Hyperkahler quotients, singularities, and quivers: ***January 2 - February 24, 2023***, organized by Ljudmila Kemnova, Giovanni Mongradi, and Alexei Oblomkov.

Lighting new Lampposts for Dark Matter and Beyond the Standard Model: ***February 27 - March 14, 2023,*** organized by Ranny Budnik, Rouven Essig, and Maxim Pospelov.

SuperGeometry and SuperModuli: ***March 27 - May 19, 2023.*** Organized by Sergio Cacciatori, Samuel Grushevsky, and Alexander Polishchuk.

WORKSHOPS

Flexibility and rigidity in dynamical systems: ***March 7 - 11, 2022***. Organized by Thomas Barthelmé, Aaron Brown, Alena Erchenko, and Ralf Spatzier.

Recent developments in Lagrangian Floer theory: ***March 14 - 18, 2022.*** Organized by Kenji Fukaya, Yanki Lekili, and Chris Woodward.

Flowing into the future. Particle Jets in Quantum Field Theory and Phenomenology: ***March 21 - 25, 2022.*** Organized by Christine Aidala, Yang-Ting Chien, Abhay Deshpande, and George Sterman.

Graduate Workshop: New Trends in Mathematical Fluid Dynamics: ***April 4 - 8, 2022.***. Organized by Theodore D. Drivas, Sam Punshon-Smith, and Francisco Torres de Lizaur.

Jim Fest: ***April 12 - 15, 2022.*** Organized by Ibrahima Bah and Martin Rocek.

Dynamics of SCFTs and Special Functions: ***April 18 - 22, 2022***. Organized by Shlomo Sergei Razamat and Masahito Yamazaki.

Roundtable for Math and Science Summer Programs: ***April 29 - May 1, 2022.*** Organized by Elena Yakubovskaya, Alexander Kirillov, Mira Bernstein, Marisa Debowsky, Daniil Lukin, and Dan Zaharopol.

Geometry, Topology, and Symmetry in Soft and Living Matter: ***May 9 - 13 2022.*** Organized by Oleg Gang, Miranda Holmes-Cerfon, Karen Kasza, and Alexei Tkachenko.

From Representation Theory to Mathematical Physics and Back: ***May 31 - June 4, 2022.*** Organized by Pavel Etingof, Mikhail Khovanov, Alexander Kirillov, Anna Lachowska, Ivan Loseu, Andrew Neitzke, Joshua Sussan, and Anton Zeitlin.

Ergodic Operators and Quantum Graphs: ***June 6 - 10, 2022***. Organized by David Damanik, Jake Fillman, and Selim Sukhtaiev.

Small Scale Dynamics in Fluid Motion: ***June 13 - 24, 2022***. Organized by Theodore D. Drivas, Tarek M. Elgindi, and Dennis Sullivan.

Simons Collaboration on the Many Electron Problem: ***June 17 - 24, 2022.*** Organized by Andrew Millis.

Recent Advances on Scalar Curvature Problems: ***June 27 - July 1, 2022.*** Organized by Alessandro Carlotto, Marcus Khuri, Philippe G. LeFloch, and Rafe Mazzeo.

Forty Years of Ricci Flow: The Geometric-Flow Revolution in Global Differential Geometry: ***July 11 - 15, 2022***. Organized by Eric Bahuaud, Panagiota Daskalopoulos, Christine Guenther, James Isenberg, Dan Knopf, and Claude LeBrun.

2022 Simons Summer Workshop: ***July 25 – August 12, 2022.*** Organized by Cumrun Vafa and Martin Rocek.

Simons Foundation: Higher Dimensional Geometry: ***August 22 - 26, 2022.*** Organized by Paolo Cascini, Ivan Cheltsov, Christopher Hacon, Elham Izadi, János Kollár, Robert Lazarsfeld, James McKernan, Burt Totaro, and Chenyang Xu.

Generalized Global Symmetries, Quantum Field Theory, and Geometry: ***August 29 - September 2, 2022.*** Organized by Michele Del Zotto and Sakura Schafer-Nameki.

Simons Collaboration on Special Holonomy in Geometry, Analysis, and Physics: Progress and Open Problems: Se***ptember 12 - 14, 2022.*** Organized by Mark Haskins.

Geometric Representation Theory, Integrability, and Supersymmetric Gauge Theories: ***September 26 - September 30, 2022.*** Organized by Peter Koroteev, Elli Pomoni, Benoit Vicedo, Dmytro Volin, and Anton Zeitlin.

Geometry of (S)QFT: ***October 10 - October 14, 2022.*** Organized by Ibrahima Bah and Shlomo Razamat.

5d N=1 SCFTs and Gauge Theories on Brane Webs: ***October 17 - October 21, 2022.*** Organized by Amihay Hanany, Marcus Sperling, Antoine Bourget, and Julius Grimminger.

Number Theory and Physics: ***October 24 - October 28, 2022.*** Organized by: Brian Conrey, Matilde Lalin, Giuseppe Mussardo, and German Sierra.

Supersymmetric Black Holes, Holography and Microstate Counting: ***October 31 - November 4, 2022.*** Organized by Cyril Closset, Leopoldo Pando Zayas, Luigi Tizzano, Chiara Toldo, and Alberto Zaffaroni.

Hyperkahler quotients, singularities, and quivers: ***January 30 - February 3, 2023***. Organized by Ljudmila Kamenova, Giovanni Mongardi, and Alexei Oblomkov.

Lighting New Lampposts for Dark Matter and Beyond the Standard Model: ***March 13 - 17, 2023.*** Organized by Ranny Budnik, Rouven Essig, and Maxim Pospelov.

Combinatorics and Geometry of Convex Polyhedra: ***March 20 - 24, 2023.*** Organized by Karim Adiprasito, Alexey Glazyrin, Isabella Novik, and Igor Pak.

SuperGeometry and SuperModuli: ***March 27 - 31, 2023***. Organized by Sergio Cacciatori, Samuel Grushevsky, and Alexander Polishchuk.

Simons Foundation Conference on Higher Dimensional Geometry: ***May 8 - 12, 2023.*** Organized by Paolo Cascini, Ivan Cheltsov, Christopher Hacon, Elham Izadi, János Kollár, Robert Lazarsfeld, James McKernan, Burt Totaro, and Chenyang Xu.

Gauged Linear Sigma Models @30: ***May 22 - 26, 2023***. Organized by Mykola Dedushenko, Heeyeon Kim, Johanna Knapp, Ilarion Melnikov, and Eric Sharpe.

Recent 2020-2021 SCGP Publications

ABSTRACT

The following are the arXiv papers submitted to the arXiv by SCGP members in the academic year 2020-21 (by original submission date). The order of references is roughly inverse chronological.

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[6] Umut Varolgunes, *Kevin Sackel,* Antoine Song, and Jonathan J. Zhu.. *On certain quantifications of Gromov's non-squeezing theorem.* 2021. arXiv: 2105.00586 [math.SG].

[7] Igor Krichever and *Nikita Nekrasov. Novikov-Veselov symmetries of the two dimensional O(N) sigma mode*l. 2021. arXiv: 2106.14201 [math-ph].

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[10] *Michele Del Zotto, Nikita Nekrasov, Nicolló Piazzalunga*, and Maxim Zabzine. *Playing with the index of M-theory.* 2021. arXiv: 2103.10271 [hep-th].

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[12] *Nikita Nekrasov*. *Blowups in BPS/CFT correspondence, and Painlevé* VI. 2021. arXiv: 2007. 03646 [hep-th].

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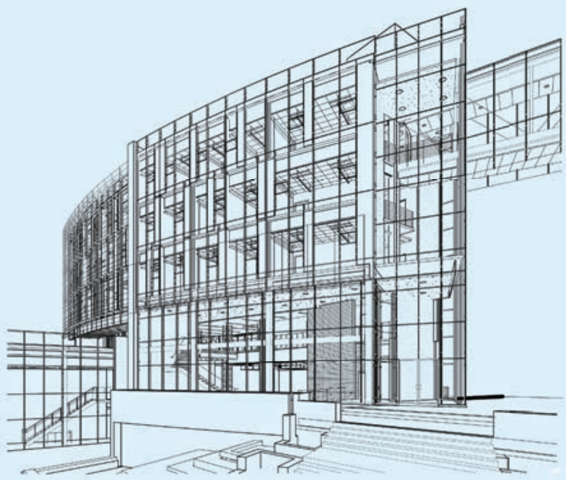

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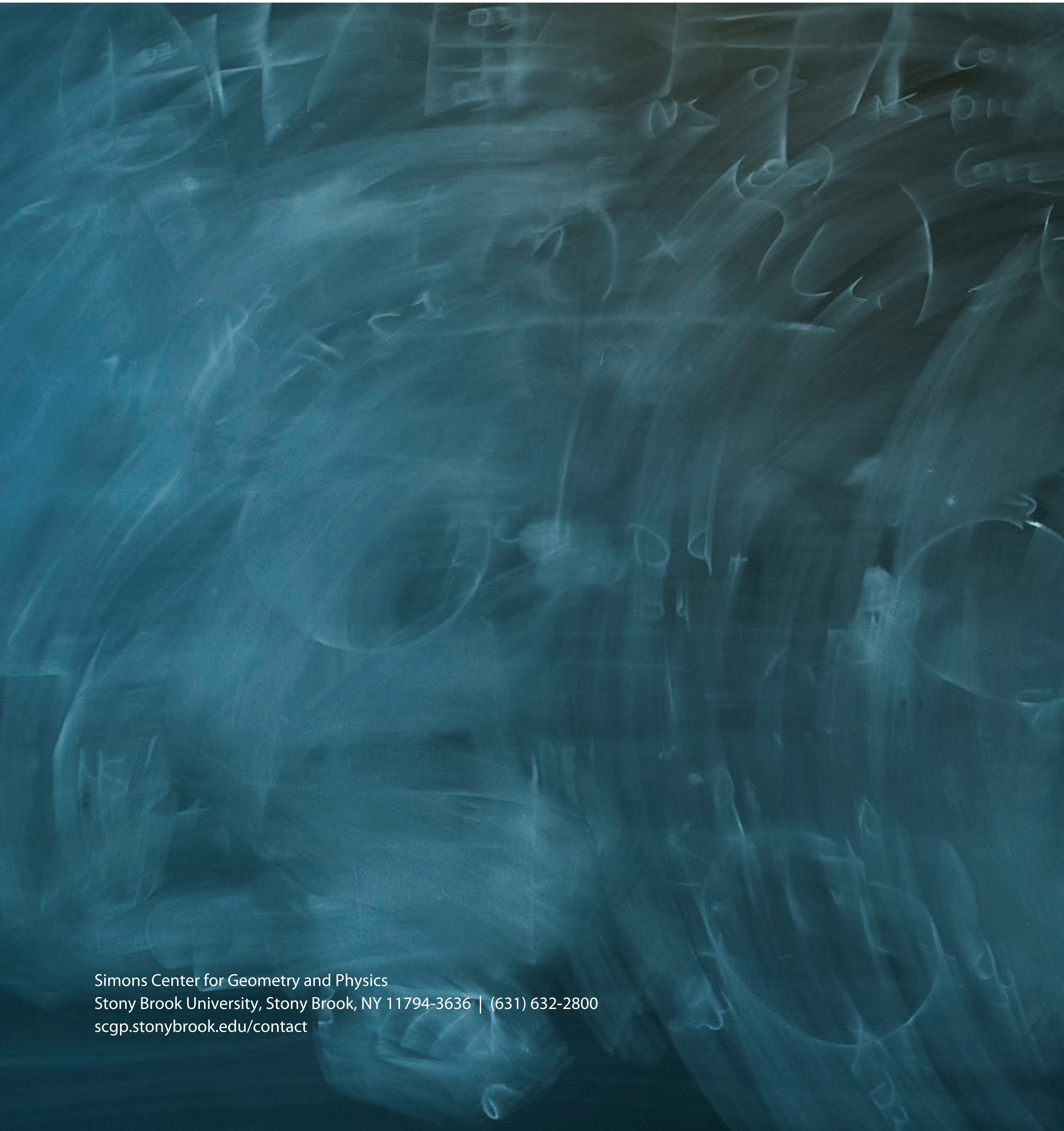

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