Inaugural Distinguished Stony Brook Lectures in Algebraic Geometry

Speaker: Professor Jacob Tsimerman (University of Toronto), "Transcendence in Algebraic Geometry"

Tuesday, November 7: Colloquium

Title: Transcendence of period integrals over function fields

Abstract: Periods are integrals of differential forms, and their study spans many branches of mathematics, including diophantine geometry, differential algebra, and algebraic geometry. If one restricts their attention to periods arising over Q, then the Grothendieck Period Conjecture is a precise way of saying that these are as transcendental as is allowed by the underlying geometry. While this is a remarkably general statement (and very open), it does not include another major (also open!) conjecture in transcendence theory - the Schanuel conjecture. In particular, e is not a period, even though it can described through periods via the relation that the integral from 1 to e of dx/x is 1. We shall present a generalization due to André which unifies the two conjectures in a satisfactory manner.

In the (complex) function field case, a lot more is known. The Grothendieck Period Conjecture has been formulated and proved by Ayoub and Nori. We shall explain the geometric analogue of the André - Grothendieck Period Conjecture and present its proof. It turns out that this conjecture is (almost) equivalent to a functional-transcendence statement of extreme generality known as the Ax-Schanuel conjecture, which has been the subject of a lot of study over the past decade in connection with unlikely intersection problems. The version relevant to us is a comparison between the algebraic and flat coordinates of geometric local systems. We will explain the ideas behind the proofs of this Ax-Schanuel conjecture and explain how it implies the relevant period conjecture.

Wednesday, November 8

Title: Definable o-minimal structures and applications to Periods

Abstract: O-minimal geometry is a theory of `tame functions' which has had tremendous applications in number theory over the past few decades. We explain how it is also naturally a home for Hodge theory. Specifically, the `period domains' which form the moduli spaces of hodge structures are not algebraic, but they do have definable o-minimal structures. This often allows one to mimic algebraicity arguments in this context. We explain this fact and explain some consequences for hodge theory, including a proof of Griffiths algebraicity conjecture, and a statement relating periods in families to derivatives of their hodge co-ordinates.

Thursday, November 9

Title: Unlikely Intersection problems and Functional Transcendence

Abstract: We explain the role transcendence plays in so-called unlikely intersection problems in number theory, focusing on more geometric examples. We discuss in particular the following problem: Given a very generic abelian variety A of dimension g, what is the smallest genus of a curve that A contains? We explain what is known over C, and how functional transcendence techniques are required to upgrade the problem to Qbar.