

Mathematics Colloquium

Thursday, October 26th, 2023

4:00pm - 5:00pm

Math Tower P-131

Speaker: Sergei Tabachnikov, Penn State University

Title: Billiards in conics revisited

Abstract: Optical properties of conics have been known since the classical antiquity. The reflection in an ideal mirror is also known as the billiard reflection and, in modern terms, the billiard inside in ellipse is completely integrable. The interior of an ellipse is foliated by confocal ellipses that are its caustics: a ray of light tangent to a caustic remains tangent to it after reflection (“caustic” means burning).

I shall explain these classic results and some of their geometric consequences, including the Ivory lemma asserting that the diagonals of a curvilinear quadrilateral made by arcs of confocal ellipses and hyperbolas are equal (this lemma is in the heart of Ivory's calculation of the gravitational potential of a homogeneous ellipsoid). Other applications include the Poncelet Porism, a famous theorem of projective geometry that has celebrated its centennial, and its lesser known ramifications, such as the Poncelet Grid theorem and the related circle patterns and configuration theorems.