My Career in Physics

A Biographical Research Summary

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This brief history of my career in physics was written in response to an invitation from the SCGP. I would like to thank the SCGP for this invitation. The research results are a selection, and I apologize in advance to coauthors on our papers that are not mentioned here for lack of space.

I was born in 1950 in Boston and grew up in Lexington, MA. My father was a professor of geology at MIT. When I was young, he would often take me in to MIT on weekends and show me labs in the geology and meteorology departments, ranging from paleontology to geophysics, geochemistry, and weather radar. He was also an all-round naturalist and taught me much about the natural sciences. He imparted in me a strong sense of the attraction of research and discoveries in science. I still remember his excitement when compelling evidence was obtained in the mid 1960s for plate tectonics and, separately, for microfossils dated to times long before the Cambrian era.

I graduated from Lexington High School in 1967 and then attended Harvard College, earning an A.B. *summa cum laude* in 1971, majoring in physics. I recall many interesting courses and professors in various subjects from those years. Lab projects included one in oceanography to model wind-driven ocean currents and work on microwaves and a Lamb shift experiment. Awards from this time included Detur and Herschel Prizes, Phi Beta Kappa, Woodrow Wilson Fellowship, National Merit Scholarship, and a NSF Predoctoral Fellowship.

I attended Princeton University for graduate school, earning a Ph.D. in physics in 1975. My thesis work was in theoretical particle physics. An outstanding goal for many years had been the unification of electromagnetic and weak interactions. These both involved perturbative interactions and currents, but differed in several ways, including the fact that electromagnetic interactions involved a massless photon and conserved parity, while weak interactions violated parity and were hypothesized to involve exchange of a massive vector boson. Furthermore, quantum electrodynamics had been shown to be renormalizable, but the existing weak interaction theory with a Fermi current-current effective Lagrangian was not renormalizable, so one could not calculate higher-order weak processes. These processes included the weak contribution to the anomalous magnetic moment of a charged lepton or neutrino; decays that occur at loop level, such as $\mu \to e\gamma$ and $\mu \to ee\bar{e}$, and the $K^0 - \bar{K}^0$ transition. An important advance occurred in 1971, when G. 't Hooft, building on previous work with M. Veltman, showed in [1] that a renormalizable unified electroweak theory can be constructed by starting with a theory that is invariant under a gauge symmetry, using the Higgs mechanism [2], in which a scalar field transforming as a nonsinglet under the gauge symmetry has a potential that is minimized at a nonzero Higgs vacuum expectation value, thereby spontaneously breaking the gauge symmetry. This can be arranged so as to yield a massless photon, while the $W^{\pm}$ and Z vector bosons pick up masses. This advance enabled one to calculate finite results for various processes that occur at the loop level. It gave rise to a period of intensive research on unified electroweak gauge theories.

Having been fascinated by quantum electrodynamics during my undergraduate years, I was thus naturally drawn to this area for my Ph.D. thesis research. While this advance suggested the use of a gauge group for the unification of electromagnetic and weak interactions, it did not determine which gauge group or which representations for fermions or Higgs would be a correct description of nature. Hence, a major effort during the early and mid 1970s was to calculate predictions of various electroweak gauge models and compare these with experiments. My Ph.D. thesis was devoted to this endeavor and included a one-loop calculation of the radiative electroweak decay of a neutral heavy lepton [3] and papers on calculations of cross sections for various neutrino reactions.

After completing my Ph.D. at Princeton, I was a postdoctoral research associate at the Fermi National Accelerator Laboratory, Fermilab. From my office window, I could look out on the ring of the 400 GeV proton accelerator, and I often bicycled around the site, past various experiments. It was inspiring to be part of an active high-energy physics laboratory and hear of new results obtained by the various experiments. My first paper as a postdoc involved a clarification of a subtle point in a one-loop calculation of the amplitude for $K_L^0 \to \mu^+\mu^-$ [4]. In [5], written with C. Albright, C. Quigg, and J. Smith, we tested the predictions of various models for unified electroweak interactions with inclusive and elastic data on neutral-current neutrino reactions. I coauthored papers [6, 7], calculating rates for a number of processes that occur as a consequence of nonzero neutrino masses and lepton mixing, including $\mu \to e\gamma$ $\mu \to ee\bar{e}$, $K_L^0 \to \mu^{\pm}e^{\mp}$, $K^+ \to \pi^+\mu^{\pm}e^{\mp}$, and $(\mu\bar{e}) \leftrightarrow (e\bar{\mu})$. In [7] with B. W. Lee, we determined the conditions for the natural suppression of these processes and showed that this suppression applies in what is now called the Standard Model of particle physics extended to include neutrino masses. Another work, with W. Bardeen and B. W. Lee, during this period was an exact nonperturbative solution of a certain quantum field theory, namely the O(N) nonlinear sigma model in $d = 2 + \epsilon$ dimensions in the large-N limit [8]. This involved a summation of an infinite number of Feynman diagrams that are dominant in this limit.

I next took on a faculty position at Princeton. By this time, there was compelling evidence from data in favor of SU$(2) \otimes$ U$(1)_Y$ as the correct electroweak gauge group, [9], comprising part of the Standard Model, together with color SU(3)$_c$. The discoveries of the τ lepton in 1976 and the b quark in 1977 suggested that there are three generations of quarks and charged leptons. Kobayashi and Maskawa had discussed quark mixing for three generations [10], generalizing the previous Cabibbo theory [11]. There was thus motivation for using data to constrain the new mixing angles that parametrize the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and I contributed to this in [12, 13]. (At this time, the possibility of more than three generations of Standard-Model fermions was still open; later, the measurement of the invisible width of the Z at the CERN LEP collider showed that there were three such generations.) I also proposed a method to search for detection of a long-lived or effectively stable neutral lepton [14] that was later performed by a Fermilab experiment [15].

I started on the faculty at Stony Brook University (then called SUNY Stony Brook) in 1979. During fall, 1979, I taught a graduate course on theoretical particle physics and recall several excellent students, including Luis Alvarez-Gaumé, the current SCGP Director. Regarding research, I continued my work on effects of neutrino masses and mixing. Up until that time, upper limits on neutrino masses had been quoted, e.g., by the Particle Data Group (PDG), as applying to "$m(\nu_e)$", "$m(\nu_\mu)$", and "$m(\nu_\tau)$". In fall, 1979, I submitted a note [16] to the PDG in which I pointed out that these limits were not consistent as stated, since ν_e, ν_μ, and ν_τ are interaction eigenstates and not mass eigenstates. Specifically, $|\nu_e\rangle = \sum_{i=1}^{3+n_s} U_{ei}|\nu_i\rangle$ and $|\nu_\mu\rangle = \sum_{i=1}^{3+n_s} U_{\mu i}|\nu_i\rangle$, where $|\nu_i\rangle$ are neutrino mass eigenstates, U_{ei} and $U_{\mu i}$ are lepton mixing matrix coefficients, and n_s denote possible additional mass eigenstates that are mainly contained in electroweak-singlet ("sterile") neutrinos. In this note, which was included in the 1980 PDG Review of Particle Properties [16], I gave a precise definition of the neutrino mass limits from various experiments. I served for a number of years afterwards on the PDG, responsible for the section on neutrinos. In 1980, with K. Fujikawa, I calculated the magnetic moment of a massive (Dirac) neutrino [17], which arises at the one-loop level, obtaining $\mu_\nu = 3eG_F m_\nu/(8\pi^2\sqrt{2})$. I later generalized this with calculations of transition magnetic and electric neutrino dipole moments for both Dirac and Majorana neutrinos [18].

Related to [16], I realized that one could test in a new way for neutrino mass effects. In 1980 I suggested a new class of tests for neutrino masses and mixing, namely to search for massive neutrino emission, via lepton mixing, in two-body leptonic decays of charged pseudoscalar mesons, including $\pi^+ \to \mu^+\nu_\mu$, $\pi^+ \to e^+\nu_e$, $K^+ \to \mu^+\nu_\mu$, $K^+ \to e^+\nu_e$, and corresponding heavy-quark meson decays, as well as in nuclear beta decays [19]. I applied these retroactively to existing data to derive the first upper bounds on such emission of heavy neutrinos via lepton mixing [19, 20, 21]. These tests with pseudoscalar meson decays are especially powerful, because the signal is monochromatic, and in the case of decays of the form $M^+ \to e^+\nu_e$, where $M^+ = \pi^+, K^+, D^+$, D_s^+, and B^+, a decay mode to a neutrino with substantial mass avoids the helicity suppression of the decay modes to neutrinos of negligibly small mass. The tests with meson decays that I suggested were carried out in a series of experiments at a number of labs and have yielded extremely stringent upper limits on $|U_{ei}|^2$ and $|U_{\mu i}|^2$ involving a possible heavy neutrino, ν_i. Some of the early searches included [22, 23] at SIN and KEK; others included [24] at TRIUMF, [25] at BNL, and recently, results from the PIENU experiment at TRIUMF [26], and the NA62 experiment at CERN [27]. I also calculated how the ratio of branching ratios $BR(\pi^+ \to e^+\nu_e)/BR(\pi^+ \to \mu^+\nu_\mu)$ would change in the presence of heavy neutrinos [20]. I have continued to work on the physics of neutrino masses and mixing, including constraints on total lepton number violation. Some of these studies were fruitful collaborations with experimentalists, such as [28]-[32]. For example, in [28] with D. Ayres et al., we performed an early study showing the sensitivity of a deep underground experiment searching for atmospheric neutrino oscillations [28]. This type of experiment was later used in the SuperKamiokande experiment [33] that obtained decisive evidence for neutrino oscillations and hence neutrino masses and lepton mixing. With a Ph.D. thesis student, I. Mocioiu (now a professor at Penn State University), we calculated matter effects in long-baseline neutrino oscillation experiments [34]. These are relevant for the Deep Underground Neutrino Experiment, DUNE. Over the years, I have contributed to a number of multi-author high-energy physics planning documents and white papers, such as [35]-[39]; references to these can be found on the arXiv. I organized a conference on neutrino physics at Stony Brook in Oct., 2002, where one of the pioneers of the subject, Ray Davis, was present. By coincidence, the announcement of the Nobel Prize to Davis and Koshiba occurred at the time of this conference. Recently, in [31] with D. Bryman, spokesperson of the TRIUMF PIENU experiment, we derived new constraints on possible electroweak-singlet (sterile) neutrinos. At present, I am a theorist member of the PIONEER collaboration planning a future experiment called PIONEER at the Paul Scherrer Institute that is designed to measure $BR(\pi^+ \to e^+\nu_e)/BR(\pi^+ \to \mu^+\nu_\mu)$ with a factor of 10 improvement in precision relative to the best present measurement and hence can probe for possible heavy neutrino effects with commensurately greater sensitivity [32].

I have also been interested in a number of other subjects in particle physics. For example, in 1982, with M. Suzuki, we pointed out the possibility of invisible decays of the Higgs boson and calculated rates for these decays [40]. The ATLAS and CMS experiments at the Large Hadron Collider have searched for this type of Higgs decay [41, 42, 43]. With a Ph.D. thesis student, A. Kusenko (now a professor at UCLA), we studied quark mixing in supersymmetric grand unified theories [44], and performed a related study with a YITP postdoc, V. Jain in [45]. I coauthored papers from the early 1980s to the recent years on possible neutron-antineutron (n-$\bar{n}$) transitions, e.g., [46, 47, 48]. (S. Rao and S. Girmohanta were Ph.D. thesis students, while S. Nussinov is a professor at Tel Aviv University with whom I have also written a number of other works, e.g., [49].) A new experiment searching for n-$\bar{n}$ transitions is planned at the European Spallation Source

[50]. This search is complementary to searches for proton decay, since if n-$\bar{n}$ oscillations were to occur, they would violate baryon number, B by two units, in contrast to proton decay, which would violate B by one unit. An interesting question concerns the properties that a hypothetical model with the gauge and fermion content of the Standard Model would have if it did not contain a Higgs boson. With C. Quigg we discussed these in [51].

I met my wife, I-Hsiu Lee, when she was a graduate student in physics at Berkeley and I was a visiting professor there. I-Hsiu's Ph.D. thesis was in supersymmetry, involving papers on $K^+ \to \pi^+\tilde{\gamma}\tilde{\gamma}$ [52] and $\mu \to e\gamma$ [53], but she later switched to research in lattice field theory. At this time I was also doing some work in lattice gauge theory, e.g., [54, 55], so it was natural to collaborate with her, and we wrote a number of papers together on lattice field theory in the 1980s and 1990s, e.g., [56]. I-Hsiu and I also wrote papers with other coauthors, including [57] with S. Aoki, who was a postdoc at BNL and YITP, and is now a professor at Kyoto University, and [58, 59] with J. Shigemitsu of Ohio State University. Ref. [59] showed symmetric fermion mass generation, the subject of a SCGP workshop in May, 2024. Lattice gauge theory has played a crucial role in understanding the nonperturbative properties of quantum chromodynamics (QCD), and experiments with the Relativistic Heavy Ion Collider, RHIC, have elucidated the properties of QCD at finite temperature and density. In [60] with J. Verbaarschot in the Stony Brook nuclear group, a YITP postdoc, M. Stephanov (now a professor at the University of Illinois at Chicago), and other coauthors, we studied the QCD phase diagram as a function of these two parameters. Later, after having a faculty position at Rockefeller University, I-Hsiu switched to computational biology and is now working in this area.

Returning to my research, in the area of quantum field theory, an interesting subject to consider is an asymptotically free vectorial non-Abelian gauge theory with a given gauge group G and a set of N_f massless fermions transforming according to a representation R of G. The asymptotic freedom condition imposes an upper bound on N_f, denoted N_u, depending on G and R. Let us denote the value of the gauge coupling measured at a reference momentum μ as $g = g(\mu)$, and also denote $\alpha = g^2/(4\pi)$. For a range of $N_f < N_u$, this theory evolves from weak coupling in the ultraviolet to an infrared (IR) fixed point of the renormalization group (IRFP) [61]. At this IRFP, the beta function $\beta = d\alpha/d\ln\mu$ vanishes, so the theory is scale-invariant and is inferred to be conformally invariant, hence the term "conformal window" (CW) for this regime. The properties of the theory at this IRFP are of fundamental interest. In a series of papers, many with a YITP postdoc, T. Ryttov (now a professor at the Southern Denmark University), we have calculated anomalous dimensions of various operators at this IRFP for various G, R, and N_f, e.g., [62, 63, 64]. As N_f decreases toward the lower end of the conformal window, the value of the coupling at the IRFP increases. Hence, to maintain accuracy, one needs to calculate to higher-loop order, and we have done this, using inputs up to five-loop order. We have compared our perturbative calculations with measurements from lattice gauge simulations, e.g., [65], finding good agreement. One output of these results is an estimate of the lower end of the conformal window, at N_{cr}. If $N_f < N_{cr}$, then as the theory evolves to the IR, the coupling eventually gets large enough to break the chiral symmetry and hence also the conformal symmetry, generating a dynamical mass for the fermions. A theory with N_f slightly below N_{cr} behaves in a quasi-conformal manner, and the dynamical breaking of the approximate conformal symmetry generates a light pseudo-Nambu-Goldstone boson, the dilaton. Such theories may be relevant to models of dynamical electroweak symmetry breaking and the solution of the Higgs mass hierarchy problem [66]. I explored related models in a series of papers, e.g, [67, 68] with T. Appelquist, [69] with a Ph.D. thesis student, N. Christensen, and [70] with a YITP postdoc, M. Kurachi. With T. Ryttov, I co-organized a SCGP conference on this subject in Jan. 2018 [71]. In a different, but related direction, I studied beta functions of IR-free theories, including 4D U(1) and $\lambda\phi^4$ at higher-loop order to search for possible UV zeros, finding evidence against such zeros (e.g., [72, 73, 74]).

Finally, I have also done research in statistical and mathematical physics. This work includes papers with a YITP colleague, B. McCoy, and two YITP postdocs, J. Perk and G. Müller (now faculty at the University of Oklahoma and the University of Rhode Island, respectively), on quantum spin models (e.g., [75, 76]). Several studies were collaborations with mathematicians. Among these, two were with former colleagues in the Stony Brook math department, C.-H. Sah [77] and R. Roeder [78]. One interesting area that involves a confluence of statistical physics and mathematical graph theory is the study of the Tutte polynomial, which encodes important information about a graph [79], and is equivalent to the partition function of a certain spin model called the Potts model. For a graph G, the Tutte polynomial is defined as $T(G,x,y) = \sum_{G'} (x-1)^{k(G')-k(G)}(y-1)^{c(G')}$, where G' is a spanning subgraph of G, $k(G')$ denotes the number of connected components in G', and $c(G')$ denotes the number of independent circuits in G'. This is related to, and generalizes, the q-state Potts model partition function $Z(G,q,v)$, where v is a temperature-dependent variable: $Z(G,q,v) = (q/v)^{k(G)}v^n T(G,x,y)$, where $x = 1+(q/v)$, $y = v+1$, and n is the number of vertices of G. Special cases of Tutte/Potts polynomials include chromatic polynomials describing graph coloring and reliability polynomials describing connectivity of networks. Since these are polynomials, it is of interest to calculate the zeros in the complex q plane for fixed v and the zeros in the complex v plane for fixed q, and we have done this in a series of papers. The q-state Potts antiferromagnet has the interesting property that for sufficiently large q on a given lattice, it exhibits nonzero ground-state entropy per vertex, $S_0 = k_B \ln W$, where $W = \lim_{n\to\infty}[P(G,q)]^{1/n}$, an exception to the third law of thermodynamics. A sampling of some of our papers includes [80]-[89]. Ref. [84] was in collaboration with a YITP colleague, M. Roček, and much of this work involved collaborations with several Ph.D. thesis students, including V. Matveev, S.-H. Tsai, and S.-C. Chang, all of whom are now faculty at their respective universities. As $n \to \infty$, zeros merge to form curves which can separate regions of these respective complex planes. Limiting functions such as the reduced free energy $f = \lim_{n\to\infty} n^{-1}\ln[Z^{(}G,q,v)]$ have different analytic forms in these different regions. The study of these sets of zeros and their accumulation sets for various families of graphs, including lattice graphs, involves a very interesting confluence of statistical physics, graph theory, complex variables, and algebraic geometry. Indeed, in one of their classic works [91], C. N. Yang and T. D. Lee pioneered the study of statistical mechanical models with physical variables (the magnetic field in their case) generalized from real to complex values. M. E. Fisher discussed the analogous generalization of temperature-dependent variables from physical to complex values [92]. The Potts model also has connections with percolation, and we have worked on this subject, obtaining new exact analytic results for average cluster numbers in bond percolation [93].

In my time at Stony Brook, I have been quite fortunate to collaborate with many very smart and hard-working thesis students and postdocs, as well as professors. I owe a great deal to the collaborations with these physicists. It has been an enjoyable and productive time. During this time, theoretical physics has undergone many advances, and it has been inspiring to be a part of this scientific endeavor.

References

[1] G. 't Hooft, "Renormalizable Lagrangians for Massive Yang-Mills fields", Nucl. Phys. B **33**, 173-199 (1971).

[2] P. W. Higgs, "Broken Symmetries, Massless Particles and Gauge Fields", Phys. Lett. **12**, 132-133 (1964).

[3] R. Shrock, "Decay $L^0 \to \nu_\ell \gamma$ in Gauge Theories of Weak and Electromagnetic Interactions", Phys. Rev.D **9**, 743-748 (1974).

[4] M. K. Gaillard, B. W. Lee, and R. E. Shrock, "Comment on Calculations of the $K_L \to \mu\bar{\mu}$ Rate in Gauge Theories", Phys. Rev. **D13**, 2674-2679 (1976).

[5] C. H. Albright, C. Quigg, R. E. Shrock, and J. Smith "Neutrino-Proton Elastic Scattering: Implications for Weak Interaction Models" Phys. Rev. **D14**, 1780-1798 (1976).

[6] B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, "Muon- and Electron-Number Nonconservation in a $V-A$ Gauge Model" Phys. Rev. Lett. **38**, 937-939 (1977).

[7] B. W. Lee and R. E. Shrock, "Natural Suppression of Symmetry Violation in Gauge Theories: Muon- and Electron-Number Nonconservation", Phys. Rev. **D16**, 1444-1473 (1977).

[8] W. A. Bardeen, B. W. Lee, and R. E. Shrock, "Phase Transition in the Nonlinear σ Model in a $2+\epsilon$ Dimensional Continuum", Phys. Rev. **D14**, 985-1005 (1976).

[9] S. L. Glashow, "Partial Symmetries of Weak Interactions", Nucl. Phys. B **22**, 579-588 (1961); S. W. Weinberg, "A Model of Leptons", Phys. Rev. Lett. **19**, 1264-1266 (1967); A. Salam, "Weak and Electromagnetic Interactions", in *Elementary Particle Theory*, ed. N. Svartholm (Almquist and Wiksell, Stockholm, 1969).

[10] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interactions", Prog. Theor. Phys. **49**, 652-657 (1973).

[11] N. Cabibbo, "Unitary Symmetry and Leptonic Decays", Phys. Rev. Lett. **10**, 531-532 (1963).

[12] R. E. Shrock and L. L. Wang, "New, Generalized Cabibbo Fit and Applications to Quark Mixing Angles in the Sequential Weinberg-Salam Model", Phys. Rev. Lett. **41**, 1692-1695 (1978)

[13] R. E. Shrock, S. B. Treiman, and L. L. Wang, “Bounds on Quark Mixing Angles in the Standard Six-Quark Model” Phys. Rev. Lett. **42**, 1589-1592 (1979).

[14] R. E. Shrock, “Test for the Existence of Effectively Stable Neutral Heavy Leptons”, Phys. Rev. Lett. **40**, 1688-1691 (1978).

[15] E. Gallas et al., “Search for neutral weakly interacting massive particles in the Fermilab Tevatron wideband neutrino beam”, Phys. Rev. D **52**, 6-14 (1995).

[16] R. E. Shrock, “Note on Neutrino Mass Limits”, in Particle Data Group, Review of Particle Properties, 1980 Edition, Rev. Mod. Phys. **52**, pp. S63-S64 (1980).

[17] K. Fujikawa and R. E. Shrock, “Magnetic Moment of a Massive Neutrino and Neutrino Spin Rotation” Phys. Rev. Lett. **45**, 963-966 (1980).

[18] R. E. Shrock, “Electromagnetic Properties and Decays of Dirac and Majorana Neutrinos in a General Class of Gauge Theories”, Nucl. Phys. B **206**, 359-379 (1982).

[19] R. E. Shrock, “New Tests for and Bounds on Neutrino Masses and Lepton Mixing”, Phys. Lett. **96B**, 159-164 (1980).

[20] R. E. Shrock, “General Theory of Weak Processes Involving Neutrinos. I. Leptonic Pseudoscalar Meson Decays, with Associated Tests for and Bounds on Neutrino Masses and Lepton Mixing”, Phys. Rev. **D24**, 1232-1274 (1981).

[21] R. E. Shrock, “General Theory of Weak Processes Involving Neutrinos. II. Pure Leptonic Decays”, Phys. Rev. **D24**, 1275-1309 (1981).

[22] R. Abela et al., “Search for an Admixture of Heavy Neutrino States in Pion Decay”, Phys. Lett. **105B**, 263-266 (1981).

[23] Y. Asano et al., “Search for a Heavy Neutrino Emitted in $K^+ \to \mu^+ \nu$ Decay”, Phys. Lett. **104B**, 84-88 (1981).

[24] D. I. Britton et al., “Improved Search for Massive Neutrinos in $\pi^+ \to e^+ \nu$ Decay”, Phys. Rev. D **46**, R885-R887 (1992).

[25] A. V. Artamonov et al., (BNL E949 Collab.), “Search for Heavy Neutrinos in $K^+ \to \mu^+ \nu_H$’ Decays”, Phys. Rev. D **91**, 052001 (2015).

[26] A. Aguilar-Arevalo et al., “Improved Search for Heavy Neutrinos in the Decay $\pi \to e\nu$, Phys. Rev. D **97**, 072012 (2018).

[27] E. Cortina Gill et al. (NA62 Collab.), “Search for Heavy Neutral Lepton Production in K^+ Decays to Positrons”, Phys. Lett. B **807**, 135599 (2020).

[28] D. Ayres, B. Cortez, T. Gaisser, A. K. Mann, R. E. Shrock, and L. R. Sulak, “Neutrino Oscillation Search with Cosmic Ray Neutrinos”, Phys. Rev. **D29**, 902-910 (1984).

[29] L. S. Littenberg and R. E. Shrock, “Upper Bounds on Lepton Number Violating Meson Decays”, Phys. Rev. Lett. **68** 443-446 (1992).

[30] L. S. Littenberg and R. Shrock, “Implications of Improved Upper Bounds on $|\Delta L| = 2$ Processes”, Phys. Lett. **B491**, 285-290 (2000) [hep-ph/0005285].

[31] D. A. Bryman and R. Shrock, “Constraints on Sterile Neutrinos in the MeV to GeV Range”, Phys. Rev. D **100**, 073011 (2019) [arXiv:1909.11198].

[32] PIONEER Collab., “PIONEER: Studies of Rare Pion Decays”, arXiv:2203.01981.

[33] Y. Fukuda et al., “Evidence for Oscillation of Atmospheric Neutrinos”, Phys. Rev. Lett. **81**, 1562-1567 (1998).

[34] I. Mocioiu and R. Shrock, “Matter Effects on Neutrino Oscillations in Long Baseline Experiments” Phys. Rev. **D62**, 053017 (2000).

[35] C. Albright et al., “Physics at a Neutrino Factory”, hep-ex/0008064.

[36] K. N. Abazajian et al., “Light Sterile Neutrinos: a White Paper”, arXiv:1204.5379.

[37] R. Mohapatra et al., “Theory of Neutrinos: a White Paper” Rept. Prog. Phys. **70**, 1757-1867 (2007).

[38] S. Alekhin et al., “A Facility to Search for Hidden Particles at the CERN SPS: the SHiP Physics Case”, Rept. Prog. Phys. **79**, 124201 (2016).

[39] B. Abi et al. (DUNE Collab.), “DUNE Far Detector Technical Design Report, Volume II: DUNE Physics” arXiv:2002.03005.

[40] R. E. Shrock and M. Suzuki, “Invisible Decays of Higgs Bosons”, Phys. Lett. **110B**, 250-254 (1982).

[41] G. Aad et al. (ATLAS Collab.), “Search for invisible Higgs-boson decays in events with vector-boson fusion signatures using 139 fb^{-1} of proton-proton data recorded by the ATLAS experiment”, JHEP 08 (2022) 104 [arXiv:2202.07953].

[42] G. Aad et al. (ATLAS Collab.) “Combination of searches for invisible decays of the Higgs boson using 139 fb^{-1} of proton-proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment”, Phys. Lett. B **842**, 137963 (2023). [arXiv:2301.10731].

[43] A. Tumasyan et al. (CMS Collab.), “Search for invisible decays of the Higgs boson produced via vector boson fusion in proton-proton collisions at $\sqrt{s} = 13$ TeV”, Phys. Rev. D **105**, 092007 (2022)

[44] A. Kusenko and R. Shrock, “A New Model for Fermion Masses in Supersymmetric Grand Unified Theories ”, Phys. Rev. **D49**, 4962-4965 (1994).

[45] V. Jain and R. Shrock, “Models of Fermion Mass Matrices Based on a Flavor- and Generation-Dependent U(1) Gauge Symmetry”, Phys. Lett. **B352**, 83-91 (1995).

[46] S. Rao and R. E. Shrock, “$n-\bar{n}$ Transition Operators and their Matrix Elements in the MIT Bag Model”, Phys. Lett. **116B**, 238-242 (1982).

[47] S. Nussinov and R. Shrock, “$n-\bar{n}$ Oscillations in Models with Large Extra Dimensions”, Phys. Rev. Lett. **88**, 171601 (2002).

[48] S. Girmohanta and R. Shrock, “Baryon-Number-Violating Nucleon and Dinucleon Decays in a Model with Large Extra Dimensions”, Phys. Rev. D **101**, 015017 (2020).

[49] S. Nussinov and R. Shrock, “Some Remarks on Theories with Large Compact Dimensions and TeV Scale Quantum Gravity”, Phys. Rev. D **59**, 105002 (1999).

[50] A. Addazi et al., “New High-Sensitivity Searches for Neutrons Converting into Antineutrons and/or Sterile Neutrons at the European Spallation Source”, J. Phys. G **48**, 070501 (2021).

[51] C. Quigg and R. Shrock, “Gedanken Worlds without Higgs: QCD-Induced Electroweak Symmetry Breaking” Phys. Rev. D **79**, 096002 (2009) [arXiv:0901.3958].

[52] M. K. Gaillard, Y.-C. Kao, I-H. Lee, and M. Suzuki, “$K^+ \to \pi^+ + \tilde{\gamma}\tilde{\gamma}$ in Spontaneously Broken Supersymmetric Models”, Phys. Lett. B **123**, 241-245 (1983).

[53] I-H. Lee, “Lepton Number Violation in Softly Broken Supersymmetry,” Phys. Lett. B **138**, 121-127 (1984).

[54] J. S. Barber, R. E. Shrock, and R. Schrader, “A Study of $d = 4$ U(1) Lattice Gauge Theory with Monopoles Removed” Phys. Lett. **152B**, 221-225 (1985).

[55] R. E. Shrock, “The Phase Structure of SU(2)× U(1) Lattice Gauge Theory”, Phys. Lett. **162B**, 165-170 (1985).

[56] I-H. Lee and R. E. Shrock, "Chiral Symmetry Breaking Phase Transition in Lattice Gauge-Higgs Theories with Fermions", Phys. Rev. Lett. **59**, 14-17 (1987).

[57] S. Aoki, I-H. Lee, and R. E. Shrock, "Study of the Chiral Transition in SU(2) Lattice Gauge-Higgs Theory with Dynamical Fermions", Phys. Lett. B **207**, 471-477 (1988).

[58] I-H. Lee, J. Shigemitsu, and R. E. Shrock, "Lattice Study of a Yukawa Theory with a Real Scalar Field", Nucl. Phys. B **330**, 225-260 (1990).

[59] I-H. Lee, J. Shigemitsu, and R. E. Shrock, "Study of Different Lattice Formulations of a Yukawa Model with a Real Scalar Field", Nucl. Phys. B **334**, 265-278 (1990).

[60] M. A. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov, and J. J. M. Verbaarschot, "Phase Diagram of QCD", Physical Review D **58**, 096007 (1998).

[61] T. Banks and A. Zaks, "On the Phase Structure of Vector-Like Gauge Theories with Massless Fermions", Nucl. Phys. B **196**, 189-204 (1982).

[62] T. A. Ryttov and R. Shrock, "Higher-Loop Corrections to the Infrared Evolution of a Gauge Theory with Fermions", Phys. Rev. D **83**, 056011 (2011) [arXiv:1011.4542].

[63] T. A. Ryttov and R. Shrock, "Infrared Zero of β and Value of γ_m for an SU(3) Gauge Theory at the Five-94Loop Level", Phys. Rev. D **94**, 105015 (2016) [arXiv:1607.06866].

[64] T. A. Ryttov and R. Shrock, "Higher-Order Scheme-Independent Series Expansions of $\gamma_{\bar{\psi}\psi,IR}$ and β'_{IR} in Conformal Field Theories", Phys. Rev. D **95**, 105004 (2017) [arXiv:1703.08558].

[65] A. Hasenfratz and D. Schaich, "Nonperturbative β Function of Twelve-Flavor SU(3) Gauge Theory", JHEP 02 (2018) 132 [arXiv:1610.10004].

[66] T. Appelquist, J. Ingoldby, and M. Piai, "Dilaton EFT Framework For Lattice Data", JHEP 07 (2017) 035 [arXiv:1702.04410].

[67] T. Appelquist and R. Shrock, "Neutrino Masses in Theories with Dynamical Electroweak Symmetry Breaking", Phys. Lett. B **548**, 204-214 (2002).

[68] T. Appelquist and R. Shrock, "Dynamical Symmetry Breaking of Extended Gauge Symmetries" Phys. Rev. Lett. **90**, 201801 (2003).

[69] N. D. Christensen and R. Shrock, "Implications of Dynamical Generation of Standard-Model Fermion Masses Phys. Rev. Lett. **94**, 241801 (2005).

[70] M. Kurachi and R. Shrock, "Study of the Change from Walking to Non-Walking Behavior in a Vectorial Gauge Theory as a Function of N_f", JHEP **12**, 034 (2006).

[71] Simons Workshop on Continuum and Lattice Approaches to the Infrared Behavior of Conformal and Quasiconformal Gauge Theories, Jan. 8-12, 2018, R. Shrock and T. A. Ryttov, organizers; http://scgp.stonybrook.edu/archives/21358.

[72] R. Shrock, "Study of UV Zero of the Beta Function in Gauge Theories with Many Fermions", Phys. Rev. D **89**, 045019 (2014) [arXiv:1311.5268].

[73] R. Shrock, "Study of the Question of an Ultraviolet Zero in the Six-Loop Beta Function of the O(N) $\lambda|\vec{\phi}|^4$ Theory", Phys. Rev. D **96**, 056010 (2017) [arXiv:1707.06248].

[74] R. Shrock, "Search for an Ultraviolet Zero in the Seven-Loop Beta Function of the $\lambda\phi^4_4$ Theory", Phys. Rev. D **107**, 056018 (2023) [arXiv:2301.01830].

[75] B. M. McCoy, J. H. H. Perk, and R. E. Shrock, "Time-Dependent Correlation Functions of the Transverse Ising Chain at the Critical Magnetic Field", Nucl. Phys. **B220** 35-47 (1983).

[76] G. Müller and R. E. Shrock, "Dynamic Correlation Functions for Quantum Spin Chains", Phys. Rev. Lett. **51**, 219-222 (1983).

[77] E. Ramos, C.-H. Sah, and R. E. Shrock, "Algebra of Diffeomorphisms of the N-Torus", J. Math. Phys. **31**, 1805-1816 (1990).

[78] S.-C. Chang, R. K. W. Roeder, and R. Shrock, "q-Plane Zeros of the Potts Partition Function on Diamond Hierarchical Graphs", J. Math. Phys. **61**, 073301 (2020) [arXiv:1911.04012].

[79] See, e.g., B. Bollobás, *Modern Graph Theory* (Springer, New York, 1998).

[80] V. Matveev and R. Shrock, "Complex-Temperature Singularities of the Susceptibility in the $d = 2$ Ising Model: Square Lattice", J. Phys. A **28**, 1557-1583 (1995).

[81] V. Matveev and R. Shrock, "Complex-Temperature Properties of the 2D Ising Model for Nonzero Magnetic Field", Phys. Rev. **E53**, 254-267 (1996).

[82] R. Shrock and S.-H. Tsai, "Upper and Lower Bounds for Ground State Entropy of Antiferromagnetic Potts Models", Phys. Rev. **E55**, 6791-6794 (1997).

[83] R. Shrock and S.-H. Tsai, "Families of Graphs with $W_r(\{G\}, q)$ Functions That Are Nonanalytic at $1/q = 0$", Phys. Rev. **E56**, 3935-3943 (1997).

[84] M. Rocek, R. Shrock, and S.-H. Tsai "Chromatic Polynomials for Families of Strip Graphs and their Asymptotic Limits", Physica **A252**, 505-546 (1998).

[85] R. Shrock and S.-H. Tsai, "Ground State Degeneracy of Potts Antiferromagnets on 2D Lattices: Approach Using Infinite Cyclic Strip Graphs" Phys. Rev. E **60**, 3512-3515 (1999).

[86] R. Shrock, "Exact Potts Model Partition Functions for Ladder Graphs", Physica A **283**, 388-446 (2000).

[87] S.-C. Chang and R. Shrock, "Exact Potts Model Partition Functions on Wider Arbitrary-Length Strips of the Square Lattice", Physica A **296**, 234-288 (2001).

[88] S.-C. Chang and R. Shrock, "Flow Polynomials and their Asymptotic Limits for Lattice Strip Graphs", J. Stat. Phys., **112**, 815-879 (2003).

[89] R. Shrock and Y. Xu, "Weighted-Set Graph Colorings", J. Stat. Phys. **139**, 27-61 (2010).

[90] R. Shrock, "Exact Potts/Tutte Polynomials for Polygon Chain Graphs", J. Phys. A, **44**, 145002 (2011).

[91] C. N. Yang and T. D. Lee, "Statistical Theory of Equations of State and Phase Transitions. 1. Theory of Condensation", Phys. Rev. **87**, 404-409 (1952); T. D. Lee and C. N. Yang, "Statistical Theory of Equations of State and Phase Transitions. II. Lattice Gas and Ising Model", Phys. Rev. **87**, 410-419 (1952).

[92] M. E. Fisher, "The Nature of Critical Points", in *Lectures in Theoretical Physics*, ed. W. E. Britten (Univ. of Colorado Press, Boulder, CO, 1965), vol. 7C, pp. 1-159.

[93] S.-C. Chang and R. Shrock "Exact Results for Average Cluster Numbers in Bond Percolation on Infinite-Length Lattice Strips", Phys. Rev. E **104**, 044107 (2021).