Week 6 Abstracts

Speaker: James Lucietti

Abstract: I will survey the classification of extremal horizons in vacuum spacetimes (including a cosmological constant) and present a recent rigidity theorem which shows that the intrinsic geometry of compact cross-sections of such horizons must admit a Killing vector field. In particular, this implies that the extremal Kerr horizon is the most general such horizon in four-dimensional General Relativity, completing their classification. I will also discuss the application of such horizon rigidity to the corresponding black hole classification, in particular, I will present a recent uniqueness theorem which shows that the extremal Schwarzschild de Sitter spacetime (or its near-horizon geometry) is the only analytic Einstein spacetime with positive cosmological constant that contains a static extremal horizon with a compact cross-section.

Speaker: Lydia Bieri

Abstract: We shall discuss radiative spacetimes of various types in asymptotically-flat as well as cosmological settings. Asymptotically-flat systems in General Relativity (GR) are solutions of the Einstein equations tending to Minkowski spacetime at infinity. They model stars, clusters of stars, galaxies and related situations in physics. By studying the Cauchy problem for the Einstein equations using mathematical tools from the analysis of partial differential equations as well as geometry, we aim at answering deep questions from physics. Crucial insights have been revealed into gravitational waves (observed for the first time by Advanced LIGO in 2015 and many times since then). These waves are produced during the mergers of black holes or neutron stars and in core-collapse supernovae. We shall discuss the Cauchy problem for the Einstein equations in this context, explain results on gravitational radiation and the memory effect of gravitational waves, the latter being a permanent change of the spacetime. Gravitational waves carry information about their sources and surrounding environments. It will be important how fast the geometric quantities (metric, curvature components) tend to Minkowski spacetime towards infinity. We shall investigate gravitational radiation for classes of spacetimes with a broad range of different fall-off behavior. Understanding radiation necessitates a focus on relevant asymptotics, given that insights into radiation can be gained by analyzing the behavior of asymptotically-flat spacetimes at future null infinity. Therefore, we will derive and discuss such null asymptotic structures. The cosmological setting is very different from the previous scenarios: In cosmological spacetimes there is no `null infinity’. Rather we will investigate what happens in the `cosmological zone’. Our universe is expanding and on its largest scale is highly inhomogeneous. Thus, we expect expansion as well as large-scale structures to alter radiation and memory. We shall discuss mathematical results on gravitational radiation in ΛCDM cosmology, and mention what happens for de Sitter and Friedman–Lemaître–Robertson–Walker (FLRW). Students will gain an overview of fundamental questions and the mathematical tools to tackle them.

Speaker: Hari Kunduri

Abstract: I will outline a proof of the spacetime Penrose inequality for asymptotically flat 2(n+1)-dimensional initial data sets for the Einstein equations, which are invariant under a cohomogeneity one action of SU(n+1). Analogous results are obtained for asymptotically hyperbolic initial data that arise as spatial hypersurfaces in asymptotically Anti de-Sitter spacetimes. More precisely, it is shown that with the dominant energy condition, the total energy is bounded below by an explicit function of the outermost apparent horizon area. Furthermore, the inequality is saturated if and only if the initial data isometrically embed into a Schwarzschild(-AdS) spacetime. The proof, based on work with M Khuri, relies on the generalized Jang equation approach originally developed by Bray and Khuri.

Speaker: Karim Mosani

Abstract: Consider the trapped photon region in the domain of outer communication of sub-extremal Kerr spacetime. Cederbaum and Jahns proved that the canonical projection of this trapped photon region in the (co-)tangent bundle is a five-dimensional submanifold of topology $SO(3)\times \mathbb{R}^2$. By adapting the latter's methodology, we generalize this
result to two stationary axisymmetric classes of spacetimes admitting black hole horizon, namely Kerr-Newman spacetime and Kerr-Sen spacetime. The former is a solution of Einstein-Maxwell equations, while the latter is a solution of Einstein-Maxwell-Dilaton-Axion equations. These include both sub-extremal and extremal cases. The result has potential applications in various areas of mathematical relativity, like black hole uniqueness theorems, black hole dynamical stability, gravitational lensing, and black hole shadows. (This is a joint work with Carla Cederbaum).

Speaker: Charalampos Markakis

Abstract: For an equation of state in which pressure is a function only of density, the analysis of Newtonian stellar structure is simple in principle if the system is axisymmetric or consists of a corotating binary. It is then required only to solve two equations: one stating that the "injection energy," k, a potential, is constant throughout the stellar fluid, and the other being the integral over the stellar fluid to give the gravitational potential. An iterative solution of these equations generally diverges if k is held fixed, but converges with other choices. To understand the mathematical reasons for this, we start the iteration from an approximation that is perturbatively different from the actual solution and, for the current study, confine ourselves to spherical symmetry. We extend these methods outlined here: https://pubs.aip.org/aip/jmp/article-abstract/50/7/073505/231898/Iteration-stability-for-simple-Newtonian-stellar to general relativity and include interactive demonstrations.

Speaker: Paula Burkhardt-Guim

Abstract: We show that any L^∞ Riemannian metric g on $\R^n$ that is smooth with nonnegative scalar curvature away from a singular set of finite $(n-\alpha)$-dimensional Minkowski content, for some $\alpha>2$, admits an approximation by smooth Riemannian metrics with nonnegative scalar curvature, provided that g is sufficiently close in L^∞ to the Euclidean metric. The approximation is given by time slices of the Ricci-DeTurck flow, which converge locally in C^∞ to g away from the singular set. We also identify conditions under which a smooth Ricci-DeTurck flow starting from a L^∞ metric that is uniformly bilipschitz to Euclidean space and smooth with nonnegative scalar curvature away from a finite set of points must have nonnegative scalar curvature for positive times.