Week 5 Abstracts

Speaker: Ruth Gregory

Abstract: Conical deficits in a spacetime with an angular symmetry have long been associated with the presence of a cosmic string in theoretical physics, but to what extent is this association accurate? I will review what a cosmic string is for the mathematical audience, and why the above association is generally accepted as correct. However, I will then show that while the association is broadly correct, there can be sub-leading order deviations. This will be demonstrated by looking at black holes threaded with a cosmic string, for which there are exact conical metrics, as well as solutions with a finite width vortex. Surprisingly, the apparently innocent sub-leading terms in the fall-off to a conical solution do have an impact, as shown by studying geodesics in the pierced black hole background.

Speaker: David Maxwell

Abstract: Fluids have been a standard matter source for gravitation since the beginning days of general relativity. Nevertheless, constructing initial data for this family of matter models is surprisingly nuanced. In this talk we describe a novel approach to building Einstein-fluid initial data based on a recently established phase-space technique for building non-vacuum initial data sets. Compared to prior approaches to working with fluids, the input parameters used here allow for more direct specification of physical quantities, such as the number of particles in any given region. We focus on perfect fluids but also discuss extensions of these ideas to viscous models.

Speaker: Annegret Burtscher

Abstract: The Hopf-Rinow theorem is a fundamental result in Riemannian geometry relating metric completeness, properness, and geodesic completeness. We recall the many reasons why it does not hold in Lorentzian geometry. Then we present our joint work with García-Heveling relating global hyperbolicity and completeness of the null distance and show that and how this result can be viewed as an extension of the metric part of the Hopf-Rinow theorem to Lorentzian geometry. We also discuss our generalization to proper cone structures and end with some thoughts about the semi-Riemannian setting.

Speaker: Aghil Alaee

Abstract: One of the fundamental conjectures in mathematical relativity is the positivity of total mass for complete non-compact Riemannian manifolds assuming appropriate lower bounds on scalar curvature. This conjecture has been proved for AE manifolds using several techniques,

starting with the celebrated results of Schoen-Yau and Witten. There are counterexamples to this conjecture in the AF, ALF, and ALE cases. In this talk, we will refine this conjecture and prove a comparison theorem for the mass of toric 4-manifolds. The proof is robust and can be extended to higher dimensions if additional assumptions are added. This is a joint work with Marcus Khuri and Hari Kunduri.

Speaker: Greg Galloway

Abstract: Constant mean curvature (CMC) spacelike hypersurfaces have played an important role in mathematical relativity, in particular in the study of the Einstein equations, both in terms of solving the initial data constraints, and in evolving CMC initial data. Further interest for us on this topic comes from the Bartnik splitting conjecture. In this talk we review some old, and present some new, existence results for CMC spacelike hypersurfaces in the class of cosmological (spatially closed) spacetimes. Some connections to the causal boundary of spacetime are also discussed. This involves joint work with Eric Ling.

Speaker: Ghazal Geshnizjani

Abstract: Quantum fields can notoriously violate the null energy condition (NEC). In a cosmological context, NEC violation can lead to, e.g., dark energy at late times with an equation-of-state parameter smaller than -1 and nonsingular bounces at early times. However, it is expected that there should still be a limit in semiclasssical gravity to how much "negative energy" can accumulate over time and in space as a result of quantum effects. In the course of formulating quantum-motivated energy conditions, the smeared null energy condition has emerged as a recent proposal. This condition conjectures the existence of a semilocal bound on negative energy along null geodesics, which is expected to hold in semiclassical gravity. In this work, we show how the smeared null energy condition translates into theoretical constraints on NEC-violating cosmologies. Specifically, we derive the implied bounds on dark energy equation-of-state parameters and an inequality between the duration of a bouncing phase and the growth rate of the Hubble parameter at the bounce. In the case of dark energy, we identify the parameter space over which the smeared null energy condition is consistent with the recent constraints from the Dark Energy Spectroscopic Instrument.

Speaker: Melanie Graf

Abstract: The classical singularity theorems of General Relativity show that a Lorentzian manifold with a smooth metric satisfying certain physically reasonable curvature and causality conditions cannot be causal geodesically complete. One approach to proving Hawking's singularity theorem is via first studying the influence of Ricci curvature bounds on the volume of certain spacetime regions and spacelike hypersurfaces. In this talk I will present improved geometric estimates of this type based on a segment type inequality (joint work with E.-A. Kontou, A. Ohanyan and B. Schinnerl). As we will see these estimates have been instrumental in the recent proof of Hawking's singularity theorem for spacetime metrics of local Lipschitz regularity assuming a distributional version of the strong energy condition and an almost everywhere mean curvature bound along a local flowout of the initial hypersurface (joint work with M. Calisti, E. Hafemann, M. Kunzinger and R. Steinbauer). This result directly extends known C^1 results and complements versions of Hawking's theorem for timelike non-branching Lorentzian length spaces.

Speaker: Yihan Li

Abstract: In this talk, I will present the application eta invariant and spectral flow on the proof of the odd-dimensional part of Llarull’s Theorem and two of its extensions. Generally speaking, Atiyah-Singer index theory is one of the major tools in the study of Riemannian metrics of positive scalar curvature. In odd dimensions, the spectral flow of a family of twisted Dirac operators on a compact spin manifold can be used to provide a direct proof of Llarull’s rigidity theorem and the so called “spin-area convex extremality theorem”. Furthermore, combining with the deformed Dirac operator introduced by Bismut and Cheeger, this method can be used to prove noncompact extension of Llarull’s theorem, which provides a final answer to a question by Gromov. This talk is based on joint works with Guangxiang Su, Xiangsheng Wang and Weiping Zhang.