This course studies the macroscopic behavior and fluctuations of uniformly random lozenge tilings of polygonal domains. We identify tilings with their height functions and focus on central limit theorem-type results for these functions. Our main results establish Gaussian Free Field fluctuations for the associated two-dimensional fields, as well as discrete Gaussian asymptotics for height differences between boundary components of the tiled domain.

The class of domains we consider consists of gluings of elementary building blocks, called trapezoids, along a single vertical line. Such gluings may have complicated topology and, in some cases, may even be non-orientable.

Exact enumeration results and algebraic properties of lozenge tilings of trapezoids provide a rich set of tools for the analysis. A key ingredient is the identification of the model with a discrete log-gas, which allows for the use of Nekrasov equations (also known as discrete loop or discrete Dyson–Schwinger equations).

The course is based on a research monograph written jointly with G. Borot and A. Guionnet.